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Competition between a public and a data-rich private firm: An application to digital health

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Competition between a public and a data-rich private firm: An application to digital health*

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Abstract

Digital entrants in health care and health insurance often compete against public or mission-oriented organizations rather than only against private rivals. We develop a Hotelling model of mixed competition in which a private data-rich firm chooses the scope of consumer-data collection and then uses the acquired information to personalize offers. The rival supplies a standard service and is either a welfare-maximizing public firm or a profit-maximizing private firm. We characterize equilibrium data collection, prices, consumer surplus, profits, and social welfare. The private digital firm chooses a wider data-harvesting range when its rival is private than when its rival is public, because a public rival uses welfare-oriented pricing to discipline the induced market allocation rather than to maximize its own profit. The welfare ranking is non-monotonic in the value created by personalization. When the benefit from personalization is either small or large, competition against a public rival yields higher welfare; when the benefit is intermediate, competition against a private rival can dominate because it induces a broader rollout of personalized service. These results highlight that the welfare effects of digital entry depend jointly on data-driven personalization and the ownership objective of incumbent health-sector organizations.

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1 Introduction

Digitalization is reshaping sectors in which public or not-for-profit organizations have long played a central role, and health care and health insurance are among the clearest examples. In many countries, data-rich entrants now operate alongside public providers and statutory or not-for-profit insurers. In England, for example, Livi contracts with NHS GP practices and offers app-based GP video appointments to NHS patients.[1] In France, Alan operates as a digital health insurer and care/prevention platform in a system where most residents also hold complementary health insurance and nonprofit mutual insurers remain important providers.[2] In Germany, ottonova entered as a digital private health insurer in a system still dominated by statutory sickness funds.[3] In the United States, digital insurers such as Oscar compete in markets where large not-for-profit plans, such as Kaiser Permanente, remain important benchmark institutions.[4] These markets are neither standard private oligopolies nor traditional public-monopoly environments.

What distinguishes many of these entrants is not only digital delivery per se, but also their ability to collect granular consumer data and use them to tailor offers, service interfaces, and care pathways. Telemedicine platforms can personalize follow-up and navigation, while digital insurers can use app-based interactions and health data to target plan recommendations or concierge services. Alan, for example, states that, with users' explicit consent, it uses information about health care to issue prevention recommendations and survey responses to provide a personalized health and well-being plan.[5] This creates a tension that is central to health economics. Data-driven personalization can improve matching and convenience, but it can also become a strategic tool for demand steering and market stealing. Whether that tool is used aggressively should depend on the objective of the rival organization. A profit-

[1] Care Quality Commission, "Livi UK – Latest inspection summary"; UK Parliament, Health and Social Care Committee, "Written evidence submitted by Livi (FGP0316)."

[2] The Commonwealth Fund, "France" (International Health Systems Profile); Alan, "Your health partner: prevention, insurance and daily support"; Alan, "Press room."

[3] German Federal Ministry of Health, "Statutory health insurance (SHI)"; *BaFin Journal*, September 2017; ottonova homepage and services pages.

[4] Centers for Medicare & Medicaid Services, "Health Insurance Marketplaces"; Oscar Health, 2025 Form 10-K; Kaiser Permanente, "Fast Facts" and "Individual and Family Plans."

[5] World Health Organization, *Global Strategy on Digital Health 2020–2025*; OECD, "Digital health"; Alan, "Privacy Policy," version dated February 16, 2026.

maximizing rival responds differently from a public or mission-oriented rival that internalizes consumer surplus or access considerations.

These examples point to a broader economic question. The debate on competition between public/not-for-profit organizations and private firms (so-called mixed markets) has usually focused on how the objectives of public organizations influence competitive environments (e.g., de Fraja and Delbono, 1989; Matsumura, 1998; Ishida and Matsushima, 2009). It has also examined whether privatization improves efficiency, innovation, and welfare (e.g., Matsumura and Matsushima, 2004). What is new in digital health and insurance is that a digital entrant can collect fine-grained consumer data, tailor service attributes to specific users, and potentially convert that informational advantage into targeted offers. This raises a question that the literature on mixed markets does not address: how does the presence of a public or mission-oriented rival affect a digital firm's incentive to harvest data and personalize its service?

We study competition between a public firm and a private firm in a Hotelling spatial model. The two firms offer a common standard service. In addition, the private digital firm can collect consumer information over an endogenous segment of the market and offer a personalized service that generates an extra benefit for those consumers. The rival offers only the standard service and is either a welfare-maximizing public firm or a profit-maximizing private firm. The public firm's objective matches the standard assumption in the mixed-oligopoly literature (e.g., de Fraja and Delbono, 1989; Matsumura, 1998; Ishida and Matsushima, 2009). After the digital firm chooses the extent of data collection, firms set uniform prices, and the digital firm then tailors its offers within the data-covered segment. This setup captures a key feature of digital health markets: mobile apps, online consultations, remote monitoring, and digital navigation tools can create extra value for some users, but they also allow firms to target those users more precisely. The Hotelling framework is a natural way to analyze this setting and has been widely used in health economics to study competition among heterogeneous providers, including mixed markets with public and private hospitals (Gal-Or, 1997; Ellis, 1998; Biglaiser and Ma, 2003; Sanjo, 2009; Gravelle and Sivey, 2010; Herr, 2011; Bisceglia et al., 2023; Levaggi and Levaggi, 2023).

Our analysis yields three main results. First, the digital firm always chooses a wider data-harvesting range when competing against a private rival than when competing against a public rival. A public rival dampens the incentive to expand data collection because it uses welfare-oriented pricing to discipline the induced market allocation rather than to maximize profit. Second, the welfare ranking between public and private competition is non-monotonic in the value created by personalization. When the extra benefit from personalization is either small or large, the public-rival regime delivers higher welfare; when it is intermediate, the private-rival regime can dominate because it induces a broader rollout of personalized service. Third, consumer surplus, firm incentives, and total welfare do not move together. A market configuration that expands personalized service is not always the one that maximizes welfare, because the gain from better matching must be weighed against the extra mismatch cost generated when more consumers are drawn toward the digital firm.

These findings have two policy implications. First, in sectors such as health care and insurance, the relevant question is not simply whether digital entry should be encouraged or public provision privatized. The more precise question is how public or mission-oriented organizations shape the strategic use of consumer information by digital entrants. Such rivals may restrain excessive data expansion and limit distortions in market allocation, but they may also slow the diffusion of genuinely valuable personalization. The welfare consequences of digital entry therefore depend on the size of the personalization benefit, not only on ownership per se.

Second, low or even zero public-firm profit should not be interpreted as evidence of irrelevance or inefficiency. In our model, low profit can reflect welfare-oriented pricing that disciplines the market by limiting the pricing power of the data-rich entrant and by reducing mismatch costs. Hence privatization should not be treated as an automatic response to low public-sector profitability. When personalization is only moderately valuable, privatization can raise aggregate welfare even though it lowers consumer surplus, creating a genuine policy trade-off; when personalization is very valuable, the case for retaining the public benchmark becomes stronger.

Our paper contributes to three strands of literature. First, it relates to the mixed-oligopoly and privatization literature, beginning with de Fraja and Delbono (1989), de Fraja (1993), and Matsumura (1998), and more specifically to the spatial mixed-market literature, including Cremer et al. (1991) and Matsumura and Matsushima (2003, 2004), which analyze public-private competition under horizontal differentiation.

Second, it contributes to the health-economics literature that uses spatial competition to study hospital and health-service markets under regulation. This literature includes mixed-duopoly analyses of public and private hospitals (Sanjo, 2009; Herr, 2011; Bisceglia et al., 2023; Levaggi and Levaggi, 2023), as well as Hotelling and Salop models of quality, location, waiting times, and mergers in regulated hospital markets.[6] In particular, Brekke et al. (2006) study quality and location choices under price regulation, Brekke et al. (2008) analyze competition and waiting times, Brekke et al. (2011) study hospital competition and quality with regulated prices, and Brekke et al. (2017) analyze hospital mergers with regulated prices. Gravelle and Sivey (2010) also examine hospital competition when patients have imperfect information about quality. However, these papers do not consider data-based price discrimination by a data-rich firm.

A particularly close subset of this health-economics literature studies how ownership form, provider objectives, and financial constraints shape competition when public, private, or not-for-profit organizations coexist. Brekke and Sørgard (2007) analyze public versus private health care in a national health service, Brekke et al. (2012) study quality competition with profit constraints that capture non-profit or regulated providers, and Brekke et al. (2015) examine hospital competition with soft budget constraints. These papers show that differences in objectives and financial discipline can have nuanced effects on quality, access, and welfare. Our paper is close in spirit because it also studies competition against a public or mission-oriented benchmark, but we focus on a data-rich digital entrant that endogenously chooses the scope of consumer data collection and then uses that information to personalize offers.

Third, it connects to the literature on personalized pricing, behavior-based discrimination, and targeted offers in standard oligopoly competition. Foundational contributions include Thisse and Vives (1988) and Shaffer and Zhang (2002). More recent work studies firms' incentives to adopt personalized pricing, the role of cookies and customer recognition, customer-information sharing, and merger-related uses of personalization (Matsumura and Matsushima, 2015; Choe et al., 2018; Chen et al., 2020, 2022; Choe et al., 2022, 2024; Esteves, 2022; Laussel and Resende, 2022; Laussel, 2023). Compared with these studies, we examine a mixed Hotelling market in

[6] Among others, Aiura and Sanjo (2010) explore the competition between urban and rural public hospitals and evaluate the impact of the privatization of rural public hospitals.

which only the digital firm endogenously chooses the scope of data collection and then personalizes service, so the ownership objective of the rival becomes central for equilibrium data collection, pricing, and welfare.

Chen et al. (2022) and Esteves et al. (2025) are closely related papers. Technically, our paper is a direct extension of Chen et al. (2022). Chen et al. (2022, Section 3) also study the effect of the data scope held by a data-rich firm in a Hotelling model. The data-rich firm can offer personalized services under personalized pricing to consumers whose preferences it tracks. Building on their framework, we study a mixed market with a data-rich firm. Esteves et al. (2025) consider quality differentiation in a Hotelling model that captures health services with fixed service prices. They compare uniform quality provision with quality differentiation based on customer types. Our paper instead studies the interaction between a public firm and a data-rich service provider that engages in personalized pricing.

Matsushima and Matsumura (2003, 2006), Heywood and Ye (2009a,b), Heywood et al. (2024), and Heywood and Wang (2024) are also closely related in this respect: they study spatial price discrimination in mixed markets and examine how the presence of a public firm affects firms' location choices when firms can engage in spatial discrimination.[7] In contrast, our paper does not assume that such discrimination is available to all firms. Instead, only the digital firm can endogenously acquire consumer information in part of the market and use it to personalize offers, so the acquisition of customer data and the endogenous scope of personalization are central to our analysis.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 characterizes equilibrium when the rival is public and when it is private. Section 4 compares the two regimes in terms of data harvesting, prices, profits, consumer surplus, and welfare. Section 5 concludes.

2 Model

Consider a product characteristic space $[0, 1]$ with consumers uniformly distributed over the interval. There are two firms in the market, firm 1 and firm 2, and both supply a product at zero marginal cost. Firm 1 is a private firm located at 0. Firm 2 is

[7] Kawasaki (2022, 2023) also consider price discrimination in standard mixed oligopoly models.

located at 1 and is either a public firm or a private firm.

Only firm 1 collects consumer data, that is, information about consumer locations and preferences. This asymmetric capability for data collection reflects the reality that particular data-rich firms can harvest useful data (e.g., Chen et al., 2022). We assume that firm 1 strategically chooses the scope of data collection. The range over which firm 1 collects consumer data is $[0, \delta]$, where δ is determined endogenously at no direct cost. This formulation follows the approach in recent related papers (e.g., Chen et al., 2020, 2022). Firm 1 offers a personalized product to consumers in $[0, \delta]$, , which generates a common extra benefit w, and it sets personalized prices $p(x)$ for those consumers. The parameter w captures the incremental gross service value created by personalization. Consumer data also allow firm 1 to make targeted price offers, so the strategic value of data in the model is not exhausted by w. The extra benefit comes from data analytics that enable personalized services.

If a consumer at x buys the personalized product, her utility is $v + w - tx - p(x)$. Here, v denotes consumers' willingness to pay for the standard product, and $t(> 0)$ measures the degree of mismatch cost. Mismatch cost increases with the distance between the consumer's location and the chosen firm. Firm 1 also offers the standard product to consumers in $[\delta, 1]$ at a uniform price α_1. If a consumer at x buys the standard product from firm 1, her utility is $v - tx - \alpha_1$. Because firm 2 does not collect consumer data, it offers only the standard product to all consumers at a uniform price α_2. Thus, if a consumer at x buys from firm 2, her utility is $v - t(1 - x) - \alpha_2$. Prices are interpreted as net user charges. Negative values can therefore be read as subsidies or reduced copayments; in particular, a negative price by the public firm is interpreted as a subsidy. We abstract from budget constraints and from the shadow cost of public funds. To exclude monopoly outcomes, we assume that $0 \leq w < t$.

Consumers at x buy one unit from the firm that gives the highest net utility, that is, gross utility minus price. To ensure that all consumers buy from one of the firms, we assume that v is sufficiently large. Let $\hat{x}$ be the generalized cutoff on segment $[\delta, 1]$ between buying the standard product of firm 1 and buying that of firm 2. Let $\hat{x}_p$ be the generalized cutoff on segment $[0, \delta]$ between buying the personalized product of firm 1 and buying the standard product of firm 2. Thus, consumers in $[0, \max\{0, \min\{\hat{x}_p, \delta\}\}]$ buy the personalized product from firm 1, whereas consumers in $[\max\{0, \min\{\hat{x}_p, \delta\}\}, \delta]$ buy from firm 2. Likewise, consumers in $[\delta, \min\{1, \max\{\hat{x}, \delta\}\}]$

buy the standard product from firm 1, whereas consumers in $[\min\{1, \max\{\hat{x}, \delta\}\}, 1]$ buy from firm 2. When $0 < \hat{x}_p < \delta$, the marginal consumer at $\hat{x}_p$ is indifferent between firm 1's personalized product and firm 2's standard product. When $\delta < \hat{x} < 1$, the marginal consumer at $\hat{x}$ is indifferent between the two firms' standard products.

We need to consider three possibilities on segment $[0, \delta]$: (i) $\hat{x}_p \geq \delta$, (ii) $0 < \hat{x}_p < \delta$, (iii) $\hat{x}_p < 0$ (note that $\hat{x}_p < 0$ never occurs in equilibrium). Also, we need to consider three possibilities on segment $[\delta, 1]$: (i) $\hat{x} \leq \delta$, (ii) $\delta < \hat{x}$, (iii) $1 \leq \hat{x}$ (note that $\hat{x} \geq 1$ never occurs in equilibrium).

Taking into account the above possibilities, we formulate the profit function of firm 1:

$$\pi_1 = \int_0^{\max\{0, \min\{\hat{x}_p, \delta\}\}} p(x)dx + \alpha_1 \min\{1 - \delta, \max\{\hat{x} - \delta, 0\}\}, \tag{1}$$

Also, we formulate the profit function of firm 2:

$$\pi_2 = \alpha_2(\max\{0, \min\{\delta - \hat{x}_p, \delta\}\} + \max\{0, \min\{1 - \hat{x}, 1 - \delta\}\}). \tag{2}$$

Also, the consumer surplus is

$$\begin{aligned} CS = &\int_0^{\max\{0, \min\{\hat{x}_p, \delta\}\}} (v + w - tx - p(x))dx + \int_{\max\{0, \min\{\hat{x}_p, \delta\}\}}^{\delta} (v - t(1 - x) - \alpha_2)dx \\ &+ \int_{\delta}^{\min\{1, \max\{\hat{x}, \delta\}\}} (v - tx - \alpha_1)dx + \int_{\min\{1, \max\{\hat{x}, \delta\}\}}^{1} (v - t(1 - x) - \alpha_2)dx. \end{aligned} \tag{3}$$

Then, social welfare is defined as the sum of consumer surplus and the firms' profits:

$$SW = CS + \pi_1 + \pi_2. \tag{4}$$

We assume that firm 1 is a profit-maximizing private firm. Also, firm 2 is welfare-maximizing when it is a public firm; it is profit-maximizing when it is a private (privatized) firm. The welfare-maximizing public-firm objective is best understood as a benchmark for a mission-oriented incumbent that places more weight on consumer welfare and allocation distortions than a purely profit-maximizing rival does. The objective of firm 2 follows the manner in the context of mixed oligopoly (e.g., de Fraja and Delbono, 1989; Matsumura, 1998; Ishida and Matsushima, 2009).

The game proceeds as follows. In stage 1, firm 1 chooses the extent of data collection, that is, δ, at no cost. In stage 2, both firms simultaneously and independently set uniform prices α_1 and α_2. In stage 3, after observing the uniform prices, firm 1 chooses the personalized price schedule $p(x)$. The sequential timing of price offers reflects the flexibility of personalized pricing and enables us to solve for the subgame-perfect Nash equilibrium in pure strategies. The assumption is standard in the personalized pricing literature (Thisse and Vives, 1988; Shaffer and Zhang, 2002; Choe et al., 2018).

In the next section, we first analyze the case in which firm 2 is public and then the case in which it is private. We then compare the two scenarios in terms of data harvesting, prices, profits, consumer surplus, and social welfare.

3 Equilibrium

We consider two scenarios: (1) firm 2 is a public firm, and (2) firm 2 is a private firm.

Before turning to these two scenarios, we examine the relationship between $\hat{x}$ and $\hat{x}_p$ in stage 3. Firm 1 can offer $p(x) = 0$ to protect its demand on $[0, \delta]$. Then, given α_1 and α_2, the relationship between $\hat{x}$ and $\hat{x}_p$ at $p(x) = 0$ is:

$$\hat{x} =: \frac{t - \alpha_1 + \alpha_2}{2t} < \frac{t + w + \alpha_2}{2t} = \hat{x}_p\big|_{p(x)=0} =: \hat{x}_0.$$

This inequality implies three possible demand structures. (i) $0 \leq \hat{x}_0 < \delta$: consumers on $[0, \hat{x}_0]$ choose firm 1's personalized product, and those on $[\hat{x}_0, 1]$ choose firm 2's product. (ii) $\hat{x} < \delta \leq \hat{x}_0$: consumers on $[0, \delta]$ choose firm 1's personalized product, and those on $[\delta, 1]$ choose firm 2's product. (iii) $\delta \leq \hat{x} \leq 1$: consumers on $[0, \delta]$ choose firm 1's personalized product, those on $[\delta, \hat{x}]$ choose firm 1's standard product, and those on $[\hat{x}, 1]$ choose firm 2's product.

3.1 Firm 2 is a public firm

In this subsection, firm 2 is public and chooses α_2 to maximize social welfare. This is the benchmark mission-oriented case.

Before the main analysis, consider the two benchmark cases $\delta = 0$ and $\delta = 1$. When $\delta = 0$, the market reduces to the standard Hotelling model, and the socially optimal allocation splits the market equally: consumers on $[0, 1/2]$ (resp. $[1/2, 1]$) buy from firm 1 (resp. firm 2). When $\delta = 1$, the socially optimal cutoff solves

$$\max_x \int_0^x (v + w - ty)dy + \int_x^1 (v - t(1 - y))dy.$$

The optimal cutoff is $x = (t + w)/(2t)$.

We now consider three cases: (i) $\delta \leq 1/2$; (ii) $1/2 < \delta \leq (t + w)/(2t)$; and (iii) $(t + w)/(2t) < \delta$.

In case (i), firm 1 can protect all consumers on $[0, \delta]$, and firm 2 also prefers this outcome. The remaining segment $[\delta, 1]$ is independent of $[0, \delta]$. Firm 1 maximizes $\alpha_1(\hat{x}-\delta)$, and firm 2 chooses α_2 to achieve $\hat{x} = 1/2$. Solving the optimization problems, we obtain the following reaction functions:

$$\alpha_1 = \frac{t(1 - 2\delta) + \alpha_2}{2},\ \alpha_2 = \alpha_1.$$

Solving these reaction functions yields the uniform prices:

$$\alpha^P_{1(i)} = \alpha^P_{2(i)} = t(1 - 2\delta). \tag{5}$$

As δ increases, the segment $[\delta, 1]$ moves farther from firm 1, which leads it to set a lower α_1. Because of strategic complementarity, a lower α_1 also leads to a lower α_2. Given $\alpha^P_{2(i)}$, the equilibrium personalized prices are the maximum $p(x)$ such that $v + w - p(x) - tx \geq v - \alpha^P_{2(i)} - t(1 - x)$:

$$p^P_{(i)}(x) = w + 2t(1 - \delta - x). \tag{6}$$

As a result, firm 1's profit is

$$\pi^P_{1(i)} = w\delta + t\left(\frac{1}{2} - \delta^2\right). \tag{7}$$

An increase in δ involves a trade-off. The benefit is that it expands the segment $[0, \delta]$ over which firm 1 uses personalized pricing. The cost is a decline in the profitability of personalized pricing due to a decrease in α_2 (see (5) and (6)). Therefore, $\pi^P_{1(i)}$ is concave in δ and is maximized at $\delta = w/(2t)(< 1/2)$.

In case (ii), firm 1 can protect all consumers on $[0, \delta]$, and firm 2 also prefers this outcome. The remaining segment $[\delta, 1]$ is independent of $[0, \delta]$. Firm 1 maximizes $\alpha_1(\hat{x} - \delta)$, and firm 2 chooses α_2 to achieve $\hat{x} = \delta$. There are many values of α_2 that achieve $\hat{x} = \delta$. To avoid equilibrium multiplicity, we impose the following assumption:

Assumption 1 *If firm 2 has multiple optimal prices, it sets a uniform price that maximizes consumer surplus.*

Consumer surplus is maximized when the consumer at $x = \delta$ is indifferent between choosing firm 1 at price $p(\delta) = 0$ and choosing firm 2 at price α_2. Because $v + w - 0 - t\delta = v - \alpha_2 - t(1-\delta)$ at $x = \delta$, we have:

$$\alpha^P_{1(ii)} = 0, \;\; \alpha^P_{2(ii)} = t(2\delta - 1) - w. \tag{8}$$

For some off-equilibrium values of δ, $\alpha^P_{2(ii)}$ is negative; under the net-user-charge interpretation in Section 2, this is a subsidy. On the segment $[\delta, 1]$, firm 1 does not supply and firm 2 becomes the monopolist. This monopoly price $\alpha^P_{2(ii)}$ increases with δ because consumers at $x = \delta$ are more likely to accept firm 2's uniform price. Given $\alpha^P_{2(ii)}$, the equilibrium personalized prices are:

$$p^P_{(ii)}(x) = 2t(\delta - x). \tag{9}$$

As a result, firm 1's profit is

$$\pi^P_{1(ii)} = t\delta^2. \tag{10}$$

An increase in δ enlarges the segment $[0, \delta]$ and raises personalized prices, which benefits firm 1.

In case (iii), firm 1 can certainly protect consumers on $[0, (t+w)/(2t)]$, but firm 2 prefers to serve consumers on $[(t+w)/(2t), \delta]$ in order to maximize social welfare. Under the corresponding uniform price of firm 2, consumers on the remaining segment $[\delta, 1]$ choose firm 2 ($\hat{x} < \hat{x}_0$). Firm 1 then earns all of its profit from personalized prices and sets $\alpha^P_{1(iii)} = 0$. Firm 2 chooses α_2 to achieve $\hat{x}_0 = (t+w)/(2t)$, so $\alpha^P_{2(iii)} = 0$.

The equilibrium uniform prices are independent of δ because, when δ is large, the equilibrium market segmentation on $[0, \delta]$ does not depend on δ. The personalized prices of firm 1 are then $p^P_{(iii)}(x) = w + t(1 - 2x)$.

As a result, firm 1's profit is

$$\pi^P_{1(iii)} = \frac{(t+w)^2}{4t}. \tag{11}$$

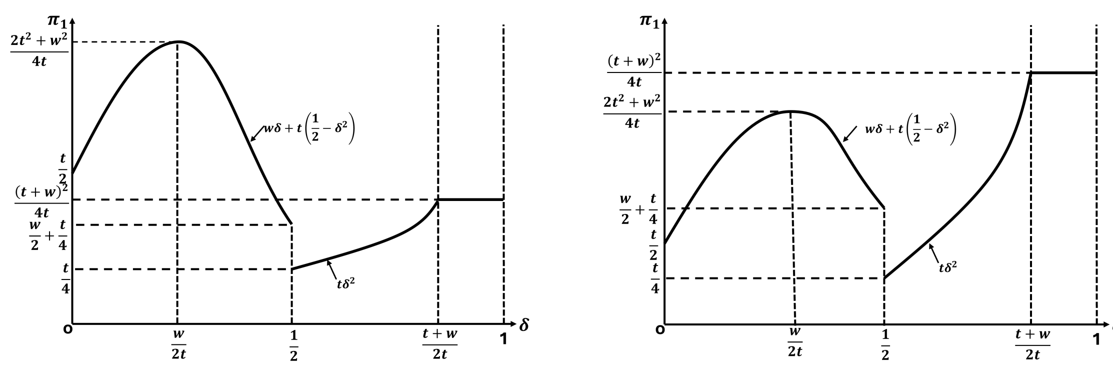
Because firm 2 sets a zero price, firm 1 has no incentive to increase δ when δ is large.

Summarizing the three cases gives Lemma 1:

Lemma 1 *When firm 2 is a public firm, firm 1's profit given δ is*

$$\pi_1^P = \begin{cases} w\delta + t\left(\frac{1}{2} - \delta^2\right) & \textit{if } \delta \le \frac{1}{2}, \\ t\delta^2 & \textit{if } \frac{1}{2} < \delta \le \frac{t+w}{2t}, \\ \frac{(t+w)^2}{4t} & \textit{if } \frac{t+w}{2t} < \delta \le 1. \end{cases} \tag{12}$$

Given this result, we derive the profit-maximizing δ. Figure 1 plots firm 1's profit function in Lemma 1. We impose the following assumption.



(a) For $w \le t/2$

(b) For $w > t/2$

Figure 1: Firm 1's profit when firm 2 is public

Assumption 2 *If firm 1 has multiple optimal δ, it chooses the lowest among those values of δ.*

This tie-breaking rule can be interpreted as the limit of a model with an arbitrarily small linear data-collection cost $c\delta$ as $c \downarrow 0$. In that perturbation, firm 1 would not collect more data than necessary, so among multiple optimal values it would choose the smallest δ. We therefore obtain:

Proposition 1 *(i) Suppose that $w \le t/2$. In equilibrium, firm 1 chooses $\delta_{(i)}^P \equiv w/(2t)$. The equilibrium outcome is case (i) evaluated at $\delta = \delta_{(i)}^P$. Specifically, consumers on $[0, \delta_{(i)}^P]$ purchase personalized products from firm 1, consumers on $[\delta_{(i)}^P, 1/2]$ purchase the standard product from firm 1, and consumers on $[1/2, 1]$ purchase the standard product from firm 2.*

Table 1: Equilibrium outcomes when firm 2 is a public firm

When $w \le t/2$	**When** $w > t/2$
$\delta^P = \frac{w}{2t}, \;\; p^P(x) = 2t(1-x)$	$\delta^P = \frac{t+w}{2t}, \;\; p^P(x) = w + t(1-2x)$
$\alpha_1^P = \alpha_2^P = t - w$	$\alpha_1^P = \alpha_2^P = 0$
$\pi_1^P = \frac{2t^2+w^2}{4t}, \pi_2^P = \frac{t-w}{2}$	$\pi_1^P = \frac{(t+w)^2}{4t}, \pi_2^P = 0$
$CS^P = v - \frac{5t}{4} + \frac{w(2t+w)}{4t}$	$CS^P = v - \frac{t}{2}$
$SW^P = v - \frac{t}{4} + \frac{w^2}{2t}$	$SW^P = v - \frac{t}{4} + \frac{w(2t+w)}{4t}$

(ii) Suppose that $t/2 < w < t$. In equilibrium, firm 1 chooses $\delta^P_{(ii)} \equiv (t+w)/(2t)$. The equilibrium outcome is case (ii) evaluated at $\delta = \delta^P_{(ii)}$. Specifically, consumers on $[0, \delta^P_{(ii)}]$ purchase personalized products from firm 1 and consumers on $[\delta^P_{(ii)}, 1]$ purchase the standard product from firm 2.

There are two local optima of δ: (i) $\delta = \delta^P_{(i)}$ on $[0, 1/2]$ and (ii) $\delta = \delta^P_{(ii)}$ on $[1/2, (t + w)/(2t)]$. The first choice, $\delta = \delta^P_{(i)}$, leads to a higher uniform price for firm 2: $\alpha^P_{2(i)}$ at $\delta = \delta^P_{(i)}$, namely $t - w$, is higher than $\alpha^P_{2(ii)}$ at $\delta = \delta^P_{(ii)}$, namely 0 (see (5) and (8)). The second choice, $\delta = \delta^P_{(ii)}$, gives firm 1 a wider segment over which to use personalized pricing. Because personalized prices are $p(x) = w + \alpha_2 + t(1 - 2x)$, an increase in w reduces the benefit of the first choice but increases the benefit of the second. At the boundary $w = t/2$, the two local optima yield the same profit and Assumption 2 selects the smaller data scope, $\delta^P_{(i)}$; for $w > t/2$, the second effect becomes dominant.

The equilibrium outcomes are listed in Table 1.

3.2 Firm 2 is a private firm

In this subsection, we analyze the case in which firm 2 is a private firm.

First, we derive the condition under which the following outcome arises in equilibrium: consumers on $[0, \delta]$ purchase personalized products from firm 1, consumers on $[\delta, \hat{x}]$ purchase the standard product from firm 1, and consumers on $[\hat{x}, 1]$ purchase the standard product from firm 2. In this outcome, firm 1 fully protects its segment $[0, \delta]$ through personalized prices, and the firms compete in uniform prices over the segment $[\delta, 1]$.

The objectives of firms 1 and 2 are $\alpha_1(\hat{x} - \delta)$ and $\alpha_2(1 - \hat{x})$, respectively. Solving the first-order conditions yields the following reaction functions:

$$\alpha_1(\alpha_2) = \frac{t(1-2\delta) + \alpha_2}{2}, \quad \alpha_2(\alpha_1) = \frac{t + \alpha_1}{2}.$$

The resulting uniform prices and cutoff $\hat{x}$ are

$$\alpha^*_{1(i)} = t - \frac{4t\delta}{3}, \quad \alpha^*_{2(i)} = t - \frac{2t\delta}{3}, \quad \hat{x}^*_{(i)} = \frac{3 + 2\delta}{6}. \tag{13}$$

As in Section 3.1, an increase in δ intensifies competition for the same reason discussed there. Given the equilibrium uniform prices, personalized prices are

$$p^*_{(i)}(x) = w + \alpha^*_{2(i)} + t(1 - 2x) = w - \frac{2t\delta}{3} + 2t(1 - x). \tag{14}$$

As a result, the firms' profits are

$$\pi^*_{1(i)} = \int_0^\delta p^*_{(i)}(x)dx + \alpha^*_{1(i)}(\hat{x}^*_{(i)} - \delta) = \frac{t}{2} + \left(\frac{2t}{3} + w\right)\delta - \frac{7}{9}t\delta^2, \tag{15}$$

$$\pi^*_{2(i)} = \alpha^*_{2(i)}(1 - \hat{x}^*_{(i)}) = \frac{(3 - 2\delta)^2 t}{18}. \tag{16}$$

Note that $\hat{x}^*_{(i)} > \delta$ if and only if $\delta < 3/4$.

As in Section 3.1, an increase in δ involves a trade-off. The benefit is that it expands the segment $[0, \delta]$ over which firm 1 uses personalized pricing. The cost is a decline in the profitability of personalized pricing through a decrease in α_2 (see (13) and (14)). Therefore, $\pi^*_{1(i)}$ is concave in δ. It is maximized at $\delta = 3(2t + 3w)/(14t)(< 3/4)$ if $w < t/2$; otherwise, it is maximized at the upper bound of the relevant range of δ, $3/4$.

Second, we derive the condition under which the following outcome arises in equilibrium: consumers on $[0, \hat{x}_0]$ purchase personalized products from firm 1, consumers on $[\hat{x}_0, \delta]$ purchase the standard product from firm 2, and consumers on $[\delta, 1]$ also purchase the standard product from firm 2. In this outcome, firm 1 cannot fully protect its segment $[0,\delta]$ through personalized prices, and firm 2 captures some consumers on $[0,\delta]$. Over the segment $[\delta,1]$, firm 1 does not supply and sets $\alpha_1 = 0$, while firm 2 serves all consumers there.

We also consider a corner solution of the previous case in which consumers on $[0,\delta]$ purchase personalized products from firm 1 and consumers on $[\delta,1]$ purchase the standard product from firm 2. Over the segment $[\delta,1]$, firm 1 does not supply and sets $\alpha_1 = 0$, while firm 2 serves all consumers in that segment.

We solve firm 2's maximization problem. Anticipating personalized prices and $\alpha_1 = 0$, firm 2 maximizes the following profit function:

$$\begin{cases} \alpha_2(1-\hat{x}_0) & \text{if } \hat{x}_0 \le \delta, \\ \alpha_2(1-\delta) & \text{if } \hat{x}|_{\alpha_1=0} \le \delta < \hat{x}_0, \\ \alpha_2(1-\hat{x}|_{\alpha_1=0}) & \text{if } \delta < \hat{x}|_{\alpha_1=0}. \end{cases}$$

Note that the third case cannot be sustained in equilibrium, so we exclude it below. Solving the maximization problem, we obtain

$$\alpha_2 = \begin{cases} \dfrac{t-w}{2} & \text{if } \dfrac{3}{4} + \dfrac{\sqrt{(2t-w)w}}{4t} < \delta \le 1, \\ t(2\delta-1) & \text{if } \dfrac{3}{4} \le \delta \le \dfrac{3}{4} + \dfrac{\sqrt{(2t-w)w}}{4t}, \\ \dfrac{t}{2} & \text{if } \delta < \dfrac{3}{4}. \end{cases}$$

We then check whether the derived price in each case is sustainable. For the first value of α_2, $\hat{x}_0 < \delta$, so $\alpha_1 = 0$ is firm 1's best response. For the second value of α_2, $\hat{x}|_{\alpha_1=0} = \delta$, so $\alpha_1 = 0$ is again firm 1's best response. For the third value of α_2, however, $\hat{x}|_{\alpha_1=0} > \delta$, so $\alpha_1 = 0$ is not firm 1's best response. Hence only the first two values of α_2 can be sustained in equilibrium. Therefore, when $\delta \ge 3/4$, the equilibrium prices are:

$$\alpha^*_{1(ii)} = 0, \;\; \alpha^*_{2(ii)} = \begin{cases} \dfrac{t-w}{2} & \text{if } \dfrac{3}{4} + \dfrac{\sqrt{(2t-w)w}}{4t} < \delta \le 1, \\ t(2\delta-1) & \text{if } \dfrac{3}{4} \le \delta \le \dfrac{3}{4} + \dfrac{\sqrt{(2t-w)w}}{4t}. \end{cases} \tag{17}$$

$$p^*_{(ii)}(x) = \begin{cases} \dfrac{w+t}{2} + t(1-2x) & \text{if } \dfrac{3}{4} + \dfrac{\sqrt{(2t-w)w}}{4t} < \delta \le 1, \\ w + t(2\delta-1) + t(1-2x) & \text{if } \dfrac{3}{4} \le \delta \le \dfrac{3}{4} + \dfrac{\sqrt{(2t-w)w}}{4t}. \end{cases} \tag{18}$$

In the first case of (17), the equilibrium uniform prices are independent of δ because the equilibrium market segmentation on $[0, \delta]$ does not depend on δ. In the second case of (17), firm 2's price $\alpha^*_{2(ii)}$ allows it to serve the whole range $[\delta, 1]$. As δ increases, firm 2 can more easily attract all consumers on $[\delta, 1]$, so $\alpha^*_{2(ii)}$ rises.

Note that $p^*_{(ii)}(x)$ changes discontinuously at $\delta = 3/4 + \sqrt{(2t-w)w}/(4t)$ because $\alpha^*_{2(ii)}$ changes discontinuously there. The same discontinuity appears in profits.

The firms' profits are

$$\pi^*_{1(ii)} = \begin{cases} \int_0^{\hat{x}_0|_{\alpha_2=\alpha^*_{2(ii)}}} p^*_{(ii)}(x)dx = \dfrac{(3t+w)^2}{16t} & \text{if } \dfrac{3}{4} + \dfrac{\sqrt{(2t-w)w}}{4t} < \delta \leq 1, \\ \int_0^{\delta} p^*_{(ii)}(x)dx = \delta(w+t\delta) & \text{if } \dfrac{3}{4} \leq \delta \leq \dfrac{3}{4} + \dfrac{\sqrt{(2t-w)w}}{4t}. \end{cases} \tag{19}$$

$$\pi^*_{2(ii)} = \begin{cases} \alpha^*_{2(ii)}(1 - \hat{x}_0|_{\alpha_2=\alpha^*_{2(ii)}}) = \dfrac{(t-w)^2}{8t} & \text{if } \dfrac{3}{4} + \dfrac{\sqrt{(2t-w)w}}{4t} < \delta \leq 1, \\ \alpha^*_{2(ii)}(1-\delta) = t(2\delta-1)(1-\delta) & \text{if } \dfrac{3}{4} \leq \delta \leq \dfrac{3}{4} + \dfrac{\sqrt{(2t-w)w}}{4t}. \end{cases} \tag{20}$$

In the second case of (19), $\pi^*_{1(ii)}$ increases monotonically with δ because $\alpha^*_{2(ii)}$ increases.

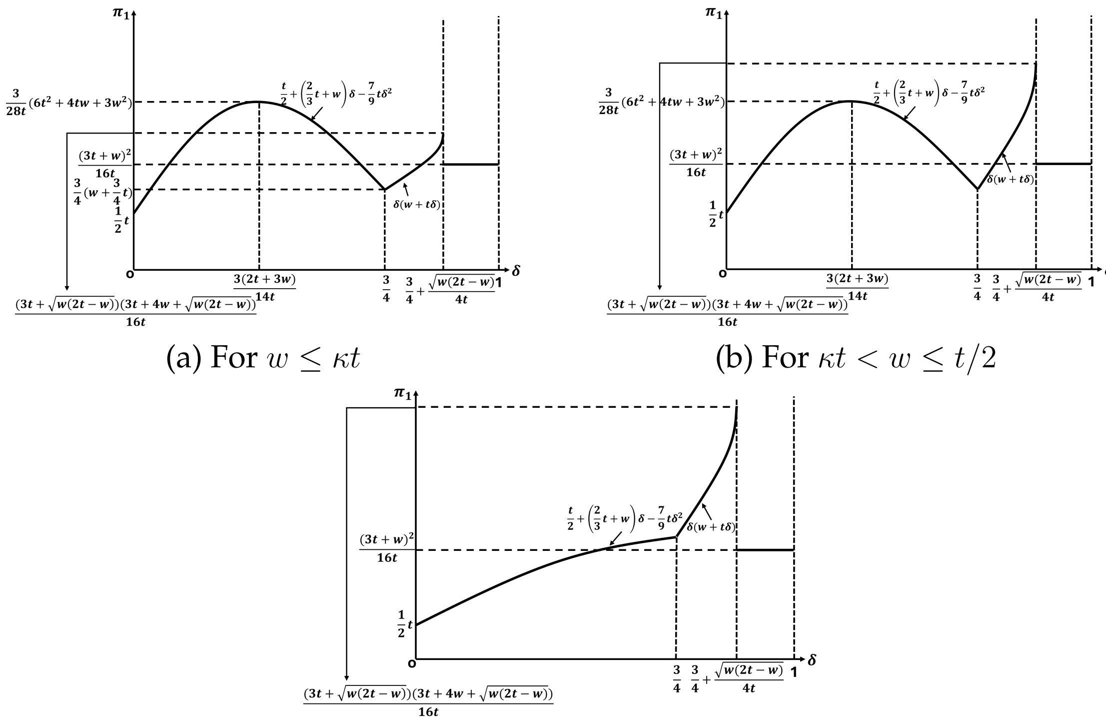
Summarizing the results above, we obtain the following lemma:

Lemma 2 *When firm 2 is a private firm, firm 1's profit is*

$$\pi^*_1 = \begin{cases} \dfrac{t}{2} + \left(\dfrac{2}{3}t + w\right)\delta - \dfrac{7}{9}t\delta^2 & \text{if } \delta < \dfrac{3}{4}, \\ \delta(w+t\delta) & \text{if } \dfrac{3}{4} \leq \delta \leq \dfrac{3}{4} + \dfrac{\sqrt{w(2t-w)}}{4t}, \\ \dfrac{(3t+w)^2}{16t} & \text{if } \dfrac{3}{4} + \dfrac{\sqrt{w(2t-w)}}{4t} < \delta \leq 1. \end{cases} \tag{21}$$

Given this result, we derive the profit-maximizing δ. For convenience, let $\kappa \approx 0.0183132$ denote the threshold characterized formally in Appendix A. Figure 2 plots firm 1's profit function in Lemma 2. Consequently, we obtain the following proposition:

Proposition 2 *(i) Suppose $w \leq \kappa t$. In equilibrium, firm 1 chooses $\delta^*_{(i)} = 3(2t+3w)/(14t)$. The equilibrium outcome is case (i) evaluated at $\delta = \delta^*_{(i)}$. Specifically, consumers on $[0, \delta^*_{(i)}]$ purchase personalized products from firm 1, consumers on $[\delta^*_{(i)}, \hat{x}^*_{(i)}]$ purchase the standard product from firm 1, and consumers on $[\hat{x}^*_{(i)}, 1]$ purchase the standard product from firm 2.*



(a) For $w \leq \kappa t$

(b) For $\kappa t < w \leq t/2$

(c) For $w > t/2$

Figure 2: Profit function of firm 1 when firm 2 is private

(ii) Suppose $w > \kappa t$. In equilibrium, firm 1 chooses $\delta^*_{(ii)} = 3/4 + \sqrt{w(2t-w)}/(4t)$. The equilibrium outcome is case (ii) evaluated at $\delta = \delta^*_{(ii)}$. Specifically, consumers on $[0, \delta^*_{(ii)}]$ purchase personalized products from firm 1 and consumers on $[\delta^*_{(ii)}, 1]$ purchase the standard product from firm 2.

As in Section 3.1, there are two local optima of δ: (i) $\delta = \delta^*_{(i)}$ on $[0, 3/4]$ and (ii) $\delta = \delta^*_{(ii)}$ on $[3/4, 3/4 + \sqrt{(2t-w)w}/(4t)]$. Unlike Section 3.1, firm 2 is private here, so its uniform prices under cases (i) and (ii) are always positive and differ only modestly. As in Section 3.1, however, the second choice $\delta = \delta^*_{(ii)}$ gives firm 1 a wider segment over which to use personalized pricing. As a result, $\delta^*_{(ii)}$ is optimal for firm 1 for $w > \kappa t$, while $\delta^*_{(i)}$ is optimal when $w \leq \kappa t$.

The equilibrium outcomes are listed in Table 2.

Table 2: Equilibrium outcomes when firm 2 is a private firm

When $w \leq \kappa t$	When $w > \kappa t$
$$\begin{aligned} \delta^* &= \frac{3(2+3z)}{14}, \\ p^*(x) &= t\left(\frac{12+4z}{7} - 2x\right), \\ \alpha_1^* &= \frac{3t(1-2z)}{7}, \\ \alpha_2^* &= \frac{t(5-3z)}{7}, \\ \pi_1^* &= \frac{3t(6+4z+3z^2)}{28}, \\ \pi_2^* &= \frac{t(5-3z)^2}{98}, \\ CS^* &= v - \frac{5t}{4} + \frac{t(2+3z)^2}{49}, \\ SW^* &= v - \frac{t}{4} - \frac{t(2+3z)(2-39z)}{196}. \end{aligned}$$	$$\begin{aligned} \delta^* &= \frac{3+s(z)}{4}, \\ p^*(x) &= t\left(z + \frac{3+s(z)}{2} - 2x\right), \\ \alpha_1^* &= 0, \\ \alpha_2^* &= \frac{t(1+s(z))}{2}, \\ \pi_1^* &= \frac{t(3+s(z))(3+s(z)+4z)}{16}, \\ \pi_2^* &= \frac{t(1-z)^2}{8}, \\ CS^* &= v - t\left(1 + \frac{s(z)}{2}\right), \\ SW^* &= v - \frac{t}{4} + \frac{tz(3+s(z))}{4} - \frac{t(1+s(z))^2}{16}. \end{aligned}$$

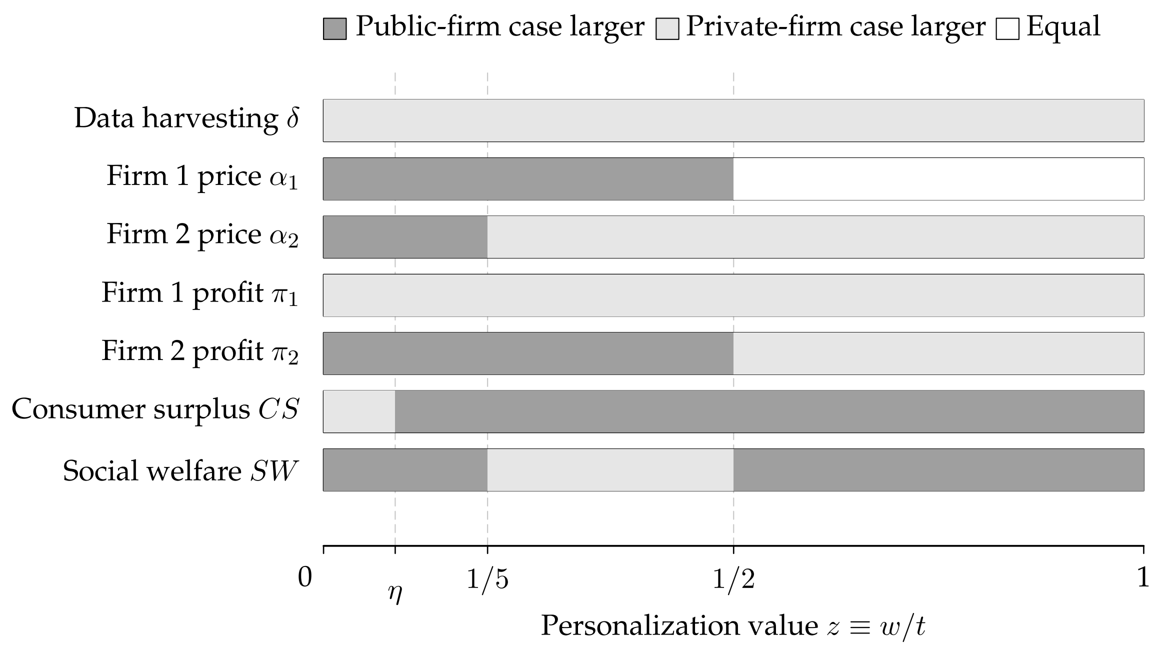
Note: Here $z \equiv w/t$ and $s(z) \equiv \sqrt{z(2-z)}$, as defined in Appendix A.

4 Comparison of outcomes

This section compares the outcomes in Sections 3.1 and 3.2. It is convenient to work with the normalized parameter $z \equiv w/t$. We continue to use $\kappa \approx 0.0183132$ from Proposition 2, and let $\eta \approx 0.0873780$ denote the consumer-surplus threshold characterized formally in Appendix A. Note that

$$0 < \kappa \approx 0.0183 < \eta \approx 0.0874 < \frac{1}{5} < \frac{1}{2}.$$

Thus, the consumer-surplus advantage of private competition is confined to a relatively small range of z, whereas the social-welfare advantage of private competition arises in the intermediate range $1/5 < z \leq 1/2$. Figure 3 provides a roadmap for the comparisons below.



Note: Dark gray indicates that the public-firm case yields the larger outcome, light gray indicates that the private-firm case yields the larger outcome, and white indicates equality over an interval. Equalities that arise only at isolated threshold values are omitted visually for clarity.

Figure 3: Comparison map of the public-rival and private-rival cases.

4.1 Comparison of the data-harvesting range

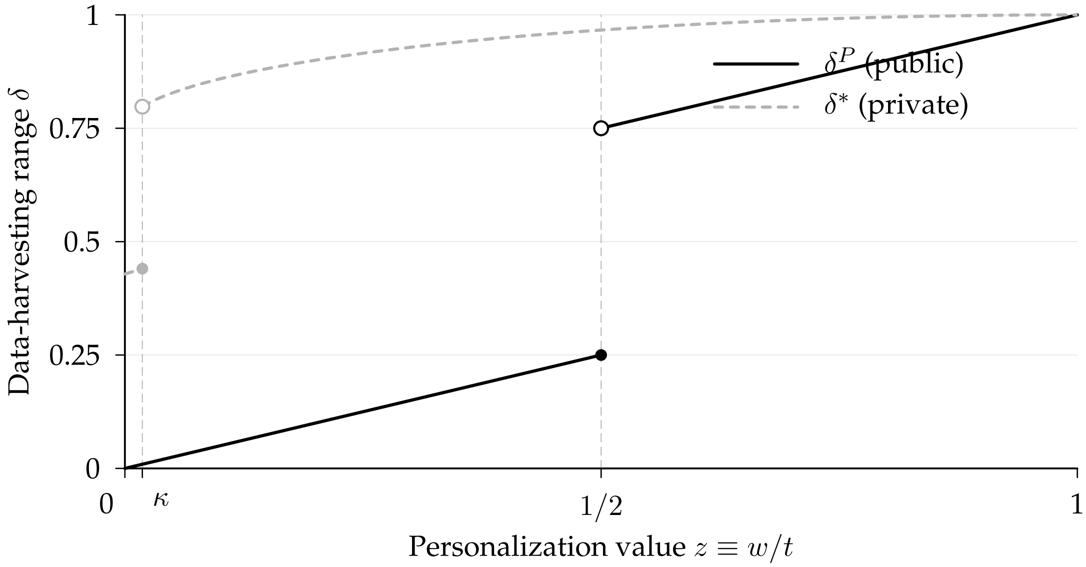
First, we compare the extent of data collection by firm 1 in Sections 3.1 and 3.2, i.e., δ^P and δ^*. We obtain Proposition 3:

Proposition 3 *Firm 1 always collects consumer information over a wider range when firm 2 is a private firm than when it is a public firm, i.e.,* $\delta^* > \delta^P$.

Proof See Appendix A. □

Consider first $w \in [0, \kappa t]$. As δ increases, all uniform prices and the personalized price fall in both cases. However, the decline is larger when firm 2 is public. As a result, firm 1 chooses a wider data-harvesting range when firm 2 is private.

Next, consider $w \in [\kappa t, t/2]$. When firm 2 is public, an increase in δ lowers all equilibrium prices, as noted above. When firm 2 is private, demand for firm 1's standard product disappears, and only demand for firm 1's personalized product remains. In



Note: The solid black line shows the public-rival case, δ^P, and the dashed gray line shows the private-rival case, δ^*. At the threshold values $z = \kappa$ and $z = 1/2$, the selected equilibrium is indicated by the closed point, while the open point marks the value just to the right of the threshold.

Figure 4: Comparison of equilibrium data-harvesting ranges.

that case, firm 2 raises its uniform price, and firm 1 also raises its personalized prices. As δ increases, firm 1 can therefore increase its profit. As a result, firm 1 chooses a wider data-harvesting range when firm 2 is private.

Finally, consider $w \in [t/2, t]$. When firm 2 is public, even if firm 1 increases δ, firm 2 chooses the uniform price α_2 to maximize social welfare, so the indifferent consumer between firm 1's personalized product and firm 2's standard product does not change. Given our assumption on firm 1's data-harvesting strategy, firm 1 then collects data only up to the margin. When firm 2 is private, by contrast, an increase in δ raises the personalized price, so firm 1 has an incentive to increase δ. Hence, firm 1 collects data over a wider range when firm 2 is private than when it is public.

4.2 Comparison of uniform prices

Next, we compare each firm's uniform price in Sections 3.1 and 3.2, that is, α_i^P and α_i^*. We obtain Proposition 4:

Proposition 4 *(i) When $w \leq t/2$, firm 1 sets a higher uniform price when firm 2 is public* *than when it is private; when $w > t/2$, it sets a zero uniform price in both cases. (ii) When $w < t/5$, firm 2 sets a higher uniform price when it is public than when it is private; when $w = t/5$, its price when it is public is the same as when it is private; when $w > t/5$, it sets a lower uniform price when it is public than when it is private.*

Proof See Appendix A. □

Consider firm 1's uniform price. When firm 2 is public and $w \leq t/2$, the extra benefit from firm 1's personalized product is limited, so it is socially desirable to minimize consumers' mismatch costs. To do that, firm 2 sets the same uniform price as firm 1. This softens competition and allows firm 1 to set a slightly higher uniform price. When $w > t/2$, the extra benefit is large enough that it becomes socially desirable for more consumers to buy firm 1's personalized product. To achieve that outcome, firm 2 chooses a uniform price that eliminates demand for firm 1's standard product. Firm 1 then sets its uniform price to zero.

When firm 2 is private, the two firms compete more aggressively for demand. If $w \leq \kappa t$, it is profitable for firm 1 to keep demand for its standard product, so it sets a low uniform price to attract those consumers. If $w > \kappa t$, firm 1 gives up demand for its standard product and serves consumers only through personalized offers, so it sets its uniform price to zero. This proves Proposition 4(i).

Now consider firm 2's uniform price. When firm 2 is public and $w \leq t/2$, as noted above, it is socially desirable to reduce consumers' mismatch costs, so firm 2 sets a relatively high uniform price. When $w > t/2$, firm 2 sets its uniform price to zero to induce firm 1 to give up demand for the standard product.

When firm 2 is private and $w \leq \kappa t$, it faces intense competition from firm 1 and therefore sets a low uniform price. When $w > \kappa t$, firm 1 gives up demand for its standard product. Firm 2 can then set a higher uniform price for consumers whose information firm 1 does not collect. In addition, as w increases, firm 1 can raise its personalized price, so firm 2 can also raise its uniform price without losing demand. As a result, when $w > t/5$, firm 2's uniform price is higher in the private-firm case than in the public-firm case. This proves Proposition 4(ii).

4.3 Comparison of profits

Next, we compare each firm's profit in Sections 3.1 and 3.2, that is, π_i^P and π_i^*. We obtain Proposition 5.

Proposition 5 *(i) Firm 1 earns higher profit when firm 2 is private than when it is public for any w. (ii) Firm 2 earns higher profit when it is public if $w \leq t/2$, and higher profit when it is private if $w > t/2$.*

Proof See Appendix A. □

Consider first firm 1's profit. From Proposition 4, firm 1 sets a higher uniform price and personalized prices when firm 2 is public than when it is private if $w \leq t/5$. Even so, firm 1 offers more personalized products when firm 2 is private. That larger personalized segment more than offsets the lower prices, so firm 1 earns a higher profit when firm 2 is private. When $w > t/5$, personalized prices are also higher when firm 2 is private than when it is public. Together with the wider personalized segment, this again implies that firm 1 earns a higher profit when firm 2 is private.

Now consider firm 2's profit. When $w > t/2$, firm 2 earns more when it is private than when it is public, because its uniform price is zero in the public-firm case but positive in the private-firm case. When $w \leq t/2$, firm 2 serves a much larger demand when it is public. As a result, even when its uniform price is higher in the private-firm case for $w > t/5$, the larger demand in the public-firm case gives firm 2 higher profit there. When $w < t/5$, firm 2's uniform price is also higher in the public-firm case, which reinforces the same conclusion.

4.4 Comparison of consumer surplus

Next, we compare consumer surplus in Sections 3.1 and 3.2, that is, CS^P and CS^*. We obtain Proposition 6:

Proposition 6 *Consumer surplus is strictly larger when firm 2 is private if $w < \eta t$, equal at $w = \eta t$, and larger when firm 2 is public if $w > \eta t$.*

Proof See Appendix A. □

Consider first the case in which w is very small, namely $0 \leq w < \eta t$. In this range, Proposition 4 implies that equilibrium prices are lower when firm 2 is private than when it is public. Relative to the public-firm case, many consumers benefit from these lower prices, although some consumers lose utility because they buy a somewhat more expensive personalized product in the private-firm case. On net, consumer surplus is higher when firm 2 is private.

Now consider the intermediate range $\eta t < w < t/5$. In this range, equilibrium prices are still lower when firm 2 is private than when it is public. However, the gain from lower prices is no longer strong enough to offset the allocation effect. When firm 2 is public, many consumers buy firm 1's low-priced standard product, and that standard product gives them higher utility than the more expensive personalized product supplied under private competition. Because of this effect, consumer surplus is higher when firm 2 is public.

Finally, consider the case in which $w \geq t/5$. In this range, at least firm 2's uniform price is higher when firm 2 is private than when it is public, which strengthens the consumer-surplus advantage of the public-firm case. It then follows that consumer surplus is higher when firm 2 is public. At the boundary $w = \eta t$, the two regimes yield the same consumer surplus.

4.5 Comparison of social welfare

Finally, we compare social welfare in Sections 3.1 and 3.2, that is, SW^P and SW^*. We obtain Proposition 7:

Proposition 7 *Social welfare is larger when firm 2 is private if $t/5 < w \leq t/2$, equal at $w = t/5$, and larger when firm 2 is public otherwise.*

Proof See Appendix A. □

Social welfare involves a trade-off in this model. When firm 2 is private, firm 1 collects consumer information over a wider range and therefore supplies personalized products to more consumers. Because the personalized product creates an extra benefit, expanding its use can raise welfare. But as more consumers buy from firm 1, total mismatch costs also rise. More personalization is therefore not always welfare-improving.

First, consider the case in which w is small ($w \leq t/5$). In this range, the extra benefit from the personalized product is limited, so it is better to reduce mismatch costs. When firm 2 is private, more than half of consumers buy from firm 1, which raises total mismatch costs. Hence social welfare is higher when firm 2 is public.

Next, consider the intermediate range $t/5 < w \leq t/2$. Here the extra benefit from the personalized product becomes important. It is therefore desirable to let more consumers buy the personalized product, even though mismatch costs rise. This outcome is achieved when firm 2 is private, so social welfare is higher in that case.

Finally, consider the case in which $w > t/2$. The extra benefit remains large, but the comparison changes. Even when firm 2 is public, many consumers buy the personalized product. At the same time, a public firm sets its uniform price with total mismatch costs in mind. A private firm does not. As a result, when firm 2 is private, too many consumers buy the personalized product and total mismatch costs become too high. Social welfare is therefore again higher when firm 2 is public. At the boundary $w = t/2$, the ranking in Proposition 7 reflects Assumption 2, which selects the smaller data-harvesting range in the public-firm case when two values of δ yield the same profit.

4.6 Implications

Our analysis provides a cautionary note for debates on privatization that are motivated primarily by the low profitability of public entities. In our model, the public firm sets its uniform price to maximize social welfare rather than profit. As a consequence, when the additional gain from personalization is sufficiently large (in particular, when $w > t/2$), the public firm earns zero profit in equilibrium. Importantly, however, this outcome should not be interpreted as evidence that the public firm is "inefficient" or irrelevant. Instead, zero profit arises because the public firm acts as a disciplinary device: by pricing aggressively, it limits the market power of the data-rich private firm and reduces inefficient taste mismatches. Consistent with this mechanism, privatization can lower consumer surplus and social welfare in this region (Propositions 6 and 7).

This implication speaks directly to real-world policy discussions in which the performance of public entities is often judged by financial indicators. Our results suggest that low profitability—possibly even zero profit—may reflect welfare-oriented pricing that delivers a substantial non-pecuniary benefit: disciplining the market and mitigating mismatch costs. Therefore, policy conclusions based solely on the profit performance of public entities may be misleading. A more relevant criterion is whether there remains sufficient demand for the public entity's service (or, more broadly, for its market-disciplining role). When such demand persists, privatization decisions should be made with greater caution, even if the public entity appears financially unprofitable.

A limitation of our argument is that the additional gain from personalization, the level of w, is treated as exogenous. In reality, the level of w may increase over time as the data-rich firm accumulates data and improves its analytics ("learning-by-data"). At an early stage, the level of w may be small, while prolonged data accumulation may gradually raise w. This dynamic consideration matters for policy: if the objective is consumer welfare, the case against privatization becomes stronger as the level of w grows. By contrast, if policymakers evaluate social welfare, privatization may increase social welfare in an intermediate range of w even though it harms consumer surplus ($t/5 < w \leq t/2$) (Propositions 6 and 7). This creates a genuine policy trade-off and helps explain why privatization can remain controversial even when it improves aggregate welfare.

Overall, our results suggest that privatization should not be viewed as an automatic remedy for low public-firm profitability. When personalization gains are sufficiently high, public ownership can serve a valuable function by restraining market power and reducing mismatch costs, thereby improving welfare outcomes despite low or zero profits.

5 Conclusion

We study a mixed Hotelling market in which a data-rich private firm chooses the scope of consumer data collection and then offers a personalized product to those consumers, while its rival is either a welfare-maximizing public firm or a profit-maximizing private firm. The central ownership result is clear: competition against a privatized rival induces more aggressive data harvesting than competition against a public rival. A public rival disciplines data collection because its pricing policy internalizes the welfare cost of drawing additional consumers away from their preferred match.

The welfare implications are non-monotonic in the value of the personalized product. When the benefit from the personalized product is small or large, the public-rival regime yields higher social welfare; when the benefit is intermediate, competition against a private rival can improve welfare by expanding access to personalized products. More data collection and more personalization are therefore not synonymous with better market performance.

For health policy, the main implication is that the ownership structure of incumbent organizations matters for digital market design. Public or mission-oriented benchmark institutions can curb excessive data expansion, but they may also slow the diffusion of valuable personalization. A natural next step of this research is to introduce explicit data-collection costs, privacy disutility, regulated prices, or richer institutional features of insurance and provider payment systems.

We close with a remark on the costs of collecting data. These costs can be incorporated by assuming that firm 1 incurs a data-acquisition cost of $c\delta$, where $c > 0$. Under this assumption, equilibrium data acquisition shrinks in both the public-rival and private-rival cases as c increases. Moreover, since the private-rival case generally involves a larger data scope than the public-rival case, the resulting total data-acquisition cost $c\delta$ is also larger in the private-rival case. These two effects jointly affect the comparison between the two cases. Even so, we expect the introduction of data-acquisition costs not to alter our main qualitative results.

Declaration of generative AI and AI-assisted technologies in the manuscript preparation process. Statement: During the preparation of this work, the authors used ChatGPT for language editing and proofreading. After using this tool, the authors carefully reviewed and edited the content as needed and takes full responsibility for the content of the published article.

A Proofs of propositions

Throughout this appendix, let $z \equiv w/t \in [0, 1)$ and $s(z) \equiv \sqrt{z(2-z)}$. Let $\kappa \approx 0.0183132$ denote the unique solution on $(0, 1/2)$ to

$$\frac{3(6+4z+3z^2)}{28} = \frac{(s(z)+3)(s(z)+3+4z)}{16}, \tag{22}$$

and let $\eta \approx 0.0873780$ denote the unique solution on $(0, 1/2)$ to

$$z^3 + 5z^2 + 11z - 1 = 0. \tag{23}$$

For later reference, (22) can be rewritten as

$$43z^2 - 50z + 9 = 14(2z+3)s(z).$$

Squaring yields

$$2633z^4 - 3516z^3 + 334z^2 - 4428z + 81 = 0,$$

whose real roots are approximately 0.0183132 and 1.7861011. Hence only one root lies in $(0, 1/2)$, which justifies the uniqueness of κ. We report the numerical value of κ only for expositional convenience.

A.1 Proof of Proposition 1

From Lemma 1, firm 1's profit is

$$\pi_1^P(\delta) = \begin{cases} w\delta + t\left(\frac{1}{2} - \delta^2\right), & 0 \le \delta \le \frac{1}{2}, \\ t\delta^2, & \frac{1}{2} < \delta \le \frac{t+w}{2t}, \\ \frac{(t+w)^2}{4t}, & \frac{t+w}{2t} < \delta \le 1. \end{cases}$$

On $[0, 1/2]$, $\pi_1^{P\prime}(\delta) = w - 2t\delta$ and $\pi_1^{P\prime\prime}(\delta) = -2t < 0$, so the unique maximizer on this interval is

$$\delta_{(i)}^P = \frac{w}{2t}.$$

On $(1/2, (t+w)/(2t)]$, the profit $t\delta^2$ is strictly increasing in δ, so the maximizer on this interval is

$$\delta_{(ii)}^P = \frac{t+w}{2t}.$$

On $((t+w)/(2t), 1]$, the profit is constant and equal to its value at $\delta_{(ii)}^P$.

Evaluating profits at the two candidate maximizers yields

$$\pi_1^P(\delta_{(i)}^P) = \frac{t}{2} + \frac{w^2}{4t}, \qquad \pi_1^P(\delta_{(ii)}^P) = \frac{(t+w)^2}{4t}.$$

Hence

$$\pi_1^P(\delta_{(i)}^P) - \pi_1^P(\delta_{(ii)}^P) = \frac{t-2w}{4}.$$

Therefore, if $w < t/2$, the global maximizer is $\delta_{(i)}^P$; if $w > t/2$, the global maximizer is $\delta_{(ii)}^P$; and if $w = t/2$, the two values yield the same profit, so Assumption 2 selects the lower one, namely $\delta_{(i)}^P$. Substituting the selected δ into the equilibrium expressions in cases (i) and (ii) gives the stated allocations and prices. □

A.2 Proof of Proposition 2

From Lemma 2, firm 1's profit is

$$\pi_1^*(\delta) = \begin{cases} \frac{t}{2} + \left(\frac{2}{3}t + w\right)\delta - \frac{7}{9}t\delta^2, & 0 \le \delta < \frac{3}{4}, \\ \delta(w + t\delta), & \frac{3}{4} \le \delta \le \frac{3}{4} + \frac{\sqrt{w(2t-w)}}{4t}, \\ \frac{(3t+w)^2}{16t}, & \frac{3}{4} + \frac{\sqrt{w(2t-w)}}{4t} < \delta \le 1. \end{cases}$$

On $[0, 3/4)$, the profit is strictly concave. Its first-order condition is

$$(\pi_1^*)'(\delta) = \frac{2}{3}t + w - \frac{14}{9}t\delta = 0,$$

which gives

$$\delta_{(i)}^* = \frac{3(2t+3w)}{14t}.$$

This interior point belongs to $[0, 3/4)$ when $w < t/2$; otherwise the maximizer on the first interval is the boundary point $3/4$.

On $[3/4,\ 3/4 + \sqrt{w(2t-w)}/(4t)]$, the derivative is

$$(\delta(w + t\delta))' = w + 2t\delta > 0,$$

so the maximizer on the second interval is its upper bound,

$$\delta_{(ii)}^* = \frac{3}{4} + \frac{\sqrt{w(2t-w)}}{4t}.$$

On the third interval, the profit is constant and equal to its value at $\delta_{(ii)}^*$.

Thus the only candidates for the global maximizer are $\delta^*_{(i)}$ and $\delta^*_{(ii)}$. Their corresponding profits are

$$\Pi^*_{(i)} = \frac{3(6t^2+4tw+3w^2)}{28t}, \qquad \Pi^*_{(ii)} = \frac{(\sqrt{w(2t-w)}+3t)(\sqrt{w(2t-w)}+3t+4w)}{16t}.$$

Writing $w = zt$, the equality $\Pi^*_{(i)} = \Pi^*_{(ii)}$ is exactly (22), which has the unique solution $z = \kappa \approx 0.0183132$ on $(0, 1/2)$. Therefore, if $w < \kappa t$, the global maximizer is $\delta^*_{(i)}$; if $w > \kappa t$, it is $\delta^*_{(ii)}$; and if $w = \kappa t$, the two values yield the same profit and Assumption 2 selects the lower one, $\delta^*_{(i)}$. Substituting the selected δ into cases (i) and (ii) gives the stated market allocations and prices. □

A.3 Proof of Proposition 3

Using Tables 1 and 2, the equilibrium data-harvesting ranges are

$$\delta^P = \begin{cases} z/2, & 0 \le z \le 1/2, \\ (1+z)/2, & 1/2 < z < 1, \end{cases} \qquad \delta^* = \begin{cases} 3(2+3z)/14, & 0 \le z \le \kappa, \\ 3/4 + s(z)/4, & \kappa < z < 1. \end{cases}$$

If $0 \le z \le \kappa$,

$$\delta^* - \delta^P = \frac{3(2+3z)}{14} - \frac{z}{2} = \frac{3+z}{7} > 0.$$

If $\kappa < z \le 1/2$,

$$\delta^* - \delta^P = \frac{3+s(z)-2z}{4} > 0.$$

If $1/2 < z < 1$,

$$\delta^* - \delta^P = \frac{1-2z+s(z)}{4}.$$

Because $z > 1/2$, the inequality $s(z) > 2z-1$ is equivalent to

$$z(2-z) > (2z-1)^2 \iff (1-z)(5z-1) > 0,$$

which holds on $(1/2, 1)$. Hence $\delta^* - \delta^P > 0$ in every case. □

A.4 Proof of Proposition 4

From Tables 1 and 2,

$$\alpha_1^P = \begin{cases} t(1-z), & 0 \le z \le 1/2, \\ 0, & 1/2 < z < 1, \end{cases} \qquad \alpha_1^* = \begin{cases} 3t(1-2z)/7, & 0 \le z \le \kappa, \\ 0, & \kappa < z < 1, \end{cases}$$

and

$$\alpha_2^P = \begin{cases} t(1-z), & 0 \le z \le 1/2, \\ 0, & 1/2 < z < 1, \end{cases} \qquad \alpha_2^* = \begin{cases} t(5-3z)/7, & 0 \le z \le \kappa, \\ t(1+s(z))/2, & \kappa < z < 1. \end{cases}$$

For firm 1, if $0 \le z \le \kappa$,

$$\alpha_1^P - \alpha_1^* = t\left(1 - z - \frac{3(1-2z)}{7}\right) = \frac{t(4-z)}{7} > 0.$$

If $\kappa < z \le 1/2$, then $\alpha_1^* = 0 < \alpha_1^P = t(1-z)$. If $z > 1/2$, both prices are zero. This proves part (i).

For firm 2, if $0 \le z \le \kappa$,

$$\alpha_2^P - \alpha_2^* = t\left(1 - z - \frac{5-3z}{7}\right) = \frac{2t(1-2z)}{7} > 0.$$

If $\kappa < z \le 1/2$,

$$\alpha_2^P - \alpha_2^* = \frac{t(1-2z-s(z))}{2}.$$

Since $1 - 2z \ge 0$ on this interval, equality holds iff $1 - 2z = s(z)$, which is equivalent to

$$(1-2z)^2 = z(2-z) \iff 5z^2 - 6z + 1 = 0.$$

The relevant root is $z = 1/5$. Hence $\alpha_2^P > \alpha_2^*$ for $z < 1/5$, $\alpha_2^P = \alpha_2^*$ at $z = 1/5$, and $\alpha_2^P < \alpha_2^*$ for $1/5 < z \le 1/2$. Finally, if $z > 1/2$, then $\alpha_2^P = 0 < \alpha_2^*$. This proves part (ii). □

A.5 Proof of Proposition 5

Using Tables 1 and 2, firm 1's equilibrium profits satisfy

$$\pi_1^* - \pi_1^P = \begin{cases} \dfrac{t(z^2+6z+2)}{14}, & 0 \le z \le \kappa, \\ \dfrac{t\left(4zs(z) + 6s(z) - 5z^2 + 14z + 1\right)}{16}, & \kappa < z \le 1/2, \\ \dfrac{t\left(4zs(z) + 6s(z) - 5z^2 + 6z + 5\right)}{16}, & 1/2 < z < 1. \end{cases}$$

The first expression is clearly positive. In the second case, $s(z) > 0$ and $-5z^2+14z+1 > 0$ for $z \in (0, 1/2]$, so the second expression is also positive. In the third case, $s(z) > 0$

and $-5z^2 + 6z + 5 > 0$ for $z \in (1/2, 1)$, so the third expression is positive as well. Therefore $\pi_1^* > \pi_1^P$ for every w.

For firm 2,

$$\pi_2^P - \pi_2^* = \begin{cases} \dfrac{t(8-9z)(z+3)}{98}, & 0 \leq z \leq \kappa, \\ \dfrac{t(1-z)(z+3)}{8}, & \kappa < z \leq 1/2, \\ -\dfrac{t(1-z)^2}{8}, & 1/2 < z < 1. \end{cases}$$

Hence $\pi_2^P > \pi_2^*$ when $z \leq 1/2$, whereas $\pi_2^P < \pi_2^*$ when $z > 1/2$. This proves Proposition 5. □

A.6 Proof of Proposition 6

Using Tables 1 and 2, consumer surplus satisfies

$$CS^* - CS^P = \begin{cases} \dfrac{t(16-50z-13z^2)}{196}, & 0 \leq z \leq \kappa, \\ -\dfrac{t\left(2s(z)+z^2+2z-1\right)}{4}, & \kappa < z \leq 1/2, \\ -\dfrac{t(1+s(z))}{2}, & 1/2 < z < 1. \end{cases}$$

For $0 \leq z \leq \kappa$, the first expression is positive because its positive root is approximately 0.2971, which is larger than κ. For $\kappa < z \leq 1/2$, define

$$h(z) \equiv 2s(z) + z^2 + 2z - 1.$$

Then

$$h'(z) = \frac{2(1-z)}{s(z)} + 2z + 2 > 0,$$

so h is strictly increasing on $(\kappa, 1/2]$. Moreover, $h(z) = 0$ is equivalent, after squaring, to (23); hence h has the unique root $\eta \approx 0.0873780$ on $(0, 1/2)$. Therefore $h(z) < 0$ for $z < \eta$ and $h(z) > 0$ for $z > \eta$. Thus $CS^* > CS^P$ for $z < \eta$ and $CS^* < CS^P$ for $z > \eta$ on $(\kappa, 1/2]$. Finally, if $z > 1/2$, the third expression is strictly negative. Consequently, consumer surplus is larger under private competition iff $w < \eta t$, and equal at $w = \eta t$. □

A.7 Proof of Proposition 7

Using Tables 1 and 2, social welfare satisfies

$$SW^* - SW^P = \begin{cases} \dfrac{t(19z^2 + 72z - 4)}{196}, & 0 \le z \le \kappa, \\ \dfrac{t\,G(z)}{16}, & \kappa < z \le 1/2, \\ \dfrac{t\left(4zs(z) - 2s(z) - 3z^2 + 2z - 1\right)}{16}, & 1/2 < z < 1, \end{cases}$$

where

$$G(z) \equiv 4zs(z) - 2s(z) - 7z^2 + 10z - 1.$$

For $0 \le z \le \kappa$, the first expression is negative because its positive root is approximately $0.0548 > \kappa$. Next, on $(\kappa, 1/2]$ we have $G(1/5) = 0$. Also,

$$G'(z) = \frac{2\left(s(z)(5 - 7z) - 4z^2 + 7z - 1\right)}{s(z)}.$$

If $z \ge (7 - \sqrt{33})/8 \approx 0.1569$, then $-4z^2 + 7z - 1 \ge 0$ and $s(z)(5 - 7z) > 0$, so $G'(z) > 0$. If $z \in [\kappa, (7 - \sqrt{33})/8]$, then both sides of

$$s(z)(5 - 7z) > 4z^2 - 7z + 1$$

are nonnegative, so we can square both sides. This yields

$$(5 - 7z)^2 z(2 - z) - (4z^2 - 7z + 1)^2 = (1 - z)\left(65z^3 - 159z^2 + 63z - 1\right).$$

The smallest positive root of $65z^3 - 159z^2 + 63z - 1$ is approximately 0.0165605, which is smaller than κ, so the right-hand side is positive on $[\kappa, (7-\sqrt{33})/8]$. Hence $G'(z) > 0$ on all of $(\kappa, 1/2]$. Because $G(1/5) = 0$, it follows that $G(z) < 0$ for $z < 1/5$ and $G(z) > 0$ for $z > 1/5$.

Finally, if $1/2 < z < 1$, then $s(z) \le 1$, so

$$4zs(z) - 2s(z) - 3z^2 + 2z - 1 \le 4z - 2 - 3z^2 + 2z - 1 = -3(z - 1)^2 < 0.$$

Therefore $SW^* < SW^P$ for $z > 1/2$.

Combining the three ranges proves that $SW^* > SW^P$ iff $1/5 < z \le 1/2$, that is, iff $t/5 < w \le t/2$. □

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