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Complete Loss of Competition:

Uncontested Elections and Political Rents

September 17, 2025

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Complete Loss of Competition: Uncontested Elections and Political Rents

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Abstract

This study examines how the complete absence of electoral competition shapes politicians' behavior. To explore this, we focus on mayoral elections in Japanese municipalities, a setting where uncontested elections are a common and politically important phenomenon. Using the variation in uncontested elections across municipalities at each election year, we examine subsequent changes in mayors' salaries. We find that mayors who win office without a contest subsequently increase their salaries. The pattern also extends to pivotal stakeholders. These findings suggest that when public conflicts, such as the existence of other candidates, do not exist, politicians are more likely to seek personal gain, highlighting the fundamental role of elections in disciplining officeholders.

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1 Introduction

Politicians face persistent temptations to use their positions for private gain, and competition through election is widely regarded as the primary mechanism for disciplining politicians' behavior (Bernecker, 2014; Jones, 2013; Svaleryd and Vlachos, 2009). Standard principal–agent models emphasize that contested elections raise the expected cost of shirking and rent extraction, thereby deterring actions by politicians against residents' expectations (Barro, 1973; Ferejohn, 1986; Persson and Tabellini, 2002; Besley, 2006; Ashworth, 2012). Consistent with this view, a large empirical literature finds that competition in elections constrains politicians' behavior (Trounstine, 2006; Becker, Peichl, and Rincke, 2009; Galasso and Nannicini, 2011; Gavoille and Verschelde, 2017; Afridi, Bhattacharya, Dhillon, and Solan, 2024). However, many democracies routinely feature uncontested elections, in which a single candidate is elected without a vote. They are widely observed in the United States and Europe.[1] In Japan, roughly 30 to 40 percent of mayoral elections have been uncontested in recent decades. Despite their prevalence, we know relatively little about how this complete absence of electoral competition at the moment of selection shapes politicians' incentives for private gain.

To address this gap, we study mayoral elections in Japan, a setting with frequent uncontested races. In particular, we focus on the mayor's salary to show how the absence of competition reshapes incentives for private gain. In non-corrupt democracies, politicians' pay is a standard measure of political rents, and prior work shows these move systematically with competitive pressure (Svaleryd and Vlachos, 2009; Persson and Tabellini, 2002).

Our starting point is a simple dynamic model in which uncontested election outcomes serve as informational signals about the competitive environment. An uncontested win updates the incumbent's beliefs toward higher entry costs for potential challengers and a weaker threat of future competition. The belief updates relax electoral discipline and increase the incumbent's optimal level of rent extraction. The model yields two key predictions. First, mayors extract higher rents following uncontested elections than after contested ones. Second, under repeated uncontested elections, the marginal increase in rent extraction declines because politicians' beliefs about high entry costs converge, and as additional uncontested wins provide progressively less new information about the competitive environment.

Guided by this framework, we examine how uncontested mayoral elections in Japan influence the subsequent mayor's salary decisions. The predictions are straightforward. When no electoral challenge arises, incumbents infer that future competition is unlikely and raise their personal compensation to levels sufficient to deter potential challengers. The highest salary increases

[1] For example, 53 percent of nearly 8,000 U.S. mayoral contests (six states, 2000–2016) were uncontested (Marschall, Lappie, and Williams, 2017; Wrighton and Squire, 1997; Lappie and Marschall, 2018); in Bavaria (Germany), over 45 percent of 25,180 mayoral elections since 1945 featured a single candidate (Freier, 2015); in Italy, 19.2 percent of mayoral contests were uncontested in 2019 (Kouba and Lysek, 2023); in Japan, 47.2 percent of municipal mayoral elections between 2011 and 2014 were uncontested (Sumi, 2017).

follow the first uncontested victory.

The Japanese cases of uncontested elections have several advantages for investigating the effects of uncontested elections. First, Japanese mayors are directly elected under uniform national electoral rules, minimizing institutional heterogeneity.[2] Second, most mayoral candidates are formally nonpartisan, which reduces confounding from party strategy. Third, the incidence of uncontested races is high and stable, about 40 percent in recent decades, which provides rich variation within municipalities and over time for studying dynamic responses to uncontested elections.

Using the setting of this uncontested election, we construct a stacked event-time panel of municipalities centered on each mayoral election. We stack balanced windows spanning three years before to three years after the election.[3] In this stacked panel, we estimate difference-in-differences and an event-study specification with municipality-by-election fixed effects and event-time-by-election fixed effects, which absorb time-invariant factors within each election window and common shocks at each relative year across all windows, respectively. In addition, to address concerns about time-varying unobservables correlated with whether an election is uncontested, we also implement a difference-in-difference-in-differences (DDD) that introduces additional variation in whether a municipality has ever experienced an uncontested election.

We find that a mayor who wins unopposed raises their own salary by about 3.3 percent, with flat pre-trends and a sharp, persistent step-up beginning in the election year. The results are robust to alternative outcomes, additional covariate controls, and alternative comparison points. Furthermore, we conduct a subsample analysis and show that the effects of salary increases diminish as the number of consecutive uncontested wins grows. These findings are consistent with our theoretical prediction and suggest that mayors perceive residual slack for further increases beyond the second term, yet they are approaching the feasible upper bound. We also find that deputy-mayor salaries and council remuneration increase by approximately 1.2 percent and 2.0 percent, respectively. These results align with the institutional requirement that ordinances on salary increases pass the council and the mayor's appointment power over deputies, and they indicate that mayors who returned unopposed prioritize pecuniary gains and secure them by compensating key stakeholders.

This study contributes to several strands of the literature. First, it relates to the research on politicians' motivations for political rent-seeking (Curto-Grau, Solé-Ollé, and Sorribas-Navarro, 2018; Folke, Persson, and Rickne, 2017; Besley, Persson, and Sturm, 2010; Di Tella and Fisman, 2004; Besley and Case, 2003). Many recent studies find the effectiveness of the electoral

[2]Countries such as the United States, the United Kingdom, and Australia adopt regionally varied electoral systems in which both direct elections and parliamentary selections coexist. Additionally, in countries like Spain and France, mayors are elected within municipal councils.

[3]Since Japanese elections are held on a four-year cycle, no elections are held for the three years preceding and following an election.

competition on disciplining politicians (Broms, Dahlström, and Fazekas, 2019; Coviello and Gagliarducci, 2017; Bernecker, 2014; Galasso and Nannicini, 2011; Ferraz and Finan, 2011; Galasso and Nannicini, 2011; Becker et al., 2009; Trounstine, 2006) and such discipline may also be effective in addressing legal rent extraction, such as politicians' salaries (Benito, Bastida, Ríos, and Vicente, 2014; Svaleryd and Vlachos, 2009). By contrast, we examine the case of zero electoral competition in uncontested elections and show that the disappearance of competition reshapes private incentives after selection.

Second, we contribute to the literature on learning in electoral settings. Existing work emphasizes how voters learn about politician quality through electoral competition (Alexander, 2021; Gordon, Huber, and Landa, 2007; Ashworth, 2012; Morrier, 2024) and how past competition shapes party policy positions (Somer-Topcu, 2009; Abou-Chadi and Orlowski, 2016). We instead focus on politicians' own learning about the competitive environment from election results. We incorporate a dynamic learning component into models of political rent seeking and show that uncontested elections function as informative signals that update incumbent beliefs and reshape rent extraction behavior.

Finally, this paper contributes to the extensive literature on uncontested elections (Hidayat, 2024). Recent studies have primarily examined the effects of uncontested elections on the subsequent performance of elected officials, with a focus on legislative activities such as the number of parliamentary speeches and rates of absenteeism (Wrighton and Squire, 1997; Bowler, 2010; Poyet and Raunio, 2021; Bełdowski, Dąbroś, and Kantorowicz, 2024). However, only a limited number of cases allow for the strict application of the no-vote election definition, and the findings regarding their consequences have often been mixed (Nordström, 2024). In particular, the extent to which political leaders who win without an election engage in personal gain remains unclear. By shedding light to those, we contribute new evidence to this growing body of work.

2 Institutional Background

2.1 Local government

Japan has a three-tier system comprising the national government, 47 prefectures, and municipalities. Prefectures and municipalities constitute the two layers of local self-government. As of 2018, there were 1718 municipalities, consisting of 792 cities, 743 towns, and 183 villages. Municipalities are responsible for a broad portfolio that includes local infrastructure, primary and lower-secondary education, welfare and public health, disaster preparedness, and community services. These responsibilities are financed by local taxes, intergovernmental transfers, and user fees. The fiscal year runs from April to March.

2.2 Electoral institutions

Municipal mayors are directly elected in a single-winner by popular vote. Terms are four years, and there are no term limits. Election calendars are set municipality by municipality, so mayoral elections occur every year in Japan. [4] Candidates must be Japanese citizens aged 25 or older, and they need not reside in the municipality. Nearly all candidates run as independents rather than as nominees of official parties. Deputy mayors are appointed by the mayor and must be approved by the municipal assembly.

Municipal councils are elected separately from the mayor. Councillors serve four-year terms. Council elections use multi-member districts and are decided by popular vote. Although legally independent from mayoral contests.

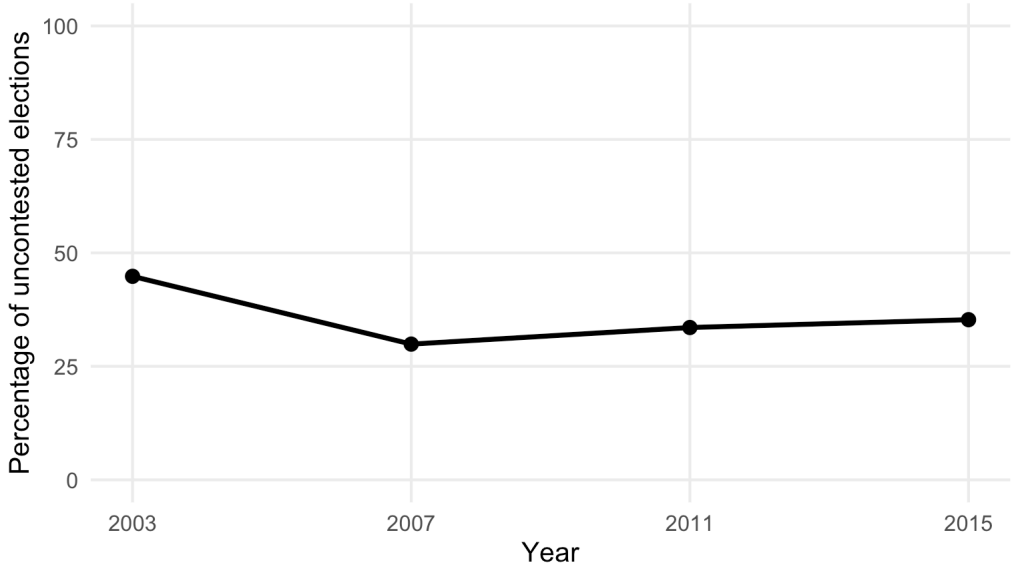
2.3 Mayor's salary

The salary (monthly pay and bonuses/allowances) of the mayor, deputy mayor, and municipal councillors is determined by a municipal salary ordinance. Any change to mayoral pay requires passage of an ordinance by the municipal councils. The mayor has the right of ordinance submission and budget initiative, but cannot unilaterally change compensation and must approve it by the assembly. Deputy mayors' and councillors' remuneration is likewise set in the same salary ordinance.

[4] Roughly one-third of municipalities coincide their elections with the nationwide Unified Local Elections held every four years in April.

2.4 Uncontested elections

Figure 1: Trends in the percentage of uncontested elections



Note: The figure plots the share of uncontested elections among all local elections in 2003, 2007, 2011, and 2015. Japan has a four-year election cycle called the Unified Local Elections, and roughly 30 percent of municipalities align their election timing with this cycle. Therefore, we focus on these cycle years, which offer relatively large samples.

An uncontested election refers to a situation where the number of candidates is equal to or fewer than the number of seats available in an electoral district, resulting in a selection without an election. In the case of mayoral elections, there is only one seat available, so an uncontested election for mayor refers to an election with only one candidate. Figure 1 plots the share of uncontested elections. The rates remained at around 30-40 percent between 2003 and 2015. [5] Prior work shows a correlation between uncontested elections and demographic and political conditions. Municipalities with smaller populations and lower tertiary-sector employment are more likely to experience uncontested races (Tsukiyama, 2019; Kobayashi, 2015). The prevalence of uncontested races is also associated with incumbents' entry decisions (Ibaraki, 2007; Sumi, 2017; Ishigami, 2020).

[5]The election cycles in 2003, 2007, 2013, and 2015 are "unified local elections," with roughly 30 percent of municipalities participating in this cycle. Consequently, the largest number of elections is typically held in these years.

3 Theoretical Framework

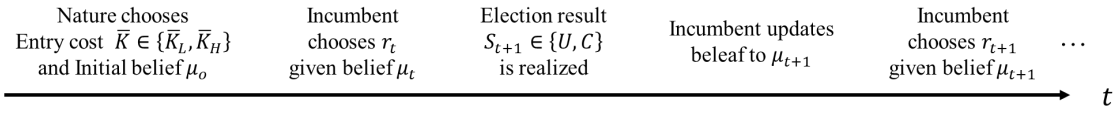
In this section, we present a theoretical model that delivers a central hypothesis. Uncontested elections lead to higher rent extraction by incumbent politicians through a Bayesian learning mechanism about future electoral competition. The reason our model makes this prediction is that electoral outcomes serve as informational signals about the underlying cost structure facing potential challengers. When incumbents win uncontested, they update their beliefs about entry costs through Bayesian inference, concluding that barriers to challenger entry are likely high. This belief revision reduces the perceived threat of future competition, thereby relaxing electoral discipline and creating incentives for greater rent extraction.

3.1 Model layout

We study a single municipality with three actors: a unit mass of voters, a potential challenger, and an incumbent mayor. Time is discrete, $t = 1, 2, 3, \ldots$. In each period, the incumbent chooses a rent level r_t representing the mayor's salary, which constitutes the per-period payoff when in office, with future payoffs discounted by $\beta \in (0, 1)$.

The incumbent does not observe the true entry cost environment but maintains a belief μ_t about it, which is updated based on election outcomes. The incumbent's objective is to maximize current rents plus the discounted continuation value. The timing of the game is shown in figure 2. We focus on stationary behavior in which the current decision depends only on payoff-relevant state variables.[6]

Figure 2: Timing of the Game



Note: The figure illustrates the sequence and timing of moves in the game.

3.2 Voter behavior

Following the probabilistic voting model of Lindbeck and Weibull (1987), voters evaluate the incumbent based on the rent level r_t and an idiosyncratic ideological preference term $\epsilon_{i,t}$. Specifically, voter i compares the incumbent's utility $U^{\text{inc}}_{i,t+1} = -r_t + \epsilon_{i,t}$ with the challenger's expected utility $U^{\text{chal}}_{i,t+1} = -\bar{r}_e$, where $\epsilon_{i,t}$ is uniformly distributed on $[-\frac{1}{2}, \frac{1}{2}]$ and captures voter-specific ideological preferences.

[6] See Appendix B for details on the model.

In the interior case where $-\frac{1}{2} \leq r_t - \bar{r}_e \leq \frac{1}{2}$, the incumbent's winning probability in a contested election becomes

$$\phi_{t+1}(r_t) = \tfrac{1}{2} + \bar{r}_e - r_t,$$

which is strictly decreasing in r_t since $\phi'_{t+1}(r_t) = -1 < 0$, confirming that voters punish rent extraction. When $r_t - \bar{r}_e$ falls outside this range, the winning probability reaches its boundaries. If $r_t - \bar{r}_e < -\frac{1}{2}$, all voters prefer the incumbent and thus $\phi_{t+1}(r_t) = 1$. Conversely, if $r_t - \bar{r}_e > \frac{1}{2}$, no voters prefer the incumbent and $\phi_{t+1}(r_t) = 0$.

3.3 Challenger entry

Before the election at $t + 1$, potential challengers observe the incumbent's past rent r_t and decide whether to enter. Each challenger privately knows her entry cost K, which is uniformly distributed on $[0, \bar{K}]$. The true upper bound $\bar{K} \in \{\bar{K}_L, \bar{K}_H\}$ with $\bar{K}_L < \bar{K}_H$ is unknown to the incumbent. Let $V^P_{C,t+1}$ denote the value of holding office following a contested election, which represents the present discounted value of future rents from being in office. Entry occurs whenever the expected benefit $[1 - \phi_{t+1}(r_t)]V^P_{C,t+1}$ exceeds the privately observed cost K.

Given $\bar{K} = \bar{K}_j$ for $j \in \{L, H\}$, the objective probability of entry is

$$p_{t+1,j}(r_t) = \frac{(\frac{1}{2} - \bar{r}_e + r_t)V^P_{C,t+1}}{\bar{K}_j},$$

which is strictly increasing in r_t and larger under the low cost regime since $p_{t+1,L}(r_t) > p_{t+1,H}(r_t)$. This differential response to rent extraction across cost regimes drives the incumbent's learning process about the competitive environment.

3.4 Belief updating

The incumbent does not observe the true cost distribution parameter $\bar{K} \in \{\bar{K}_L, \bar{K}_H\}$ but maintains a belief $\mu_t = \Pr(\bar{K} = \bar{K}_L)$ about the entry cost regime. Following the election at $t + 1$, this belief is updated through Bayesian learning based on the observed election outcome.

After an uncontested election at $t + 1$, the incumbent updates the belief to $\mu_{U,t+1} < \mu_t$. Conversely, after a contested election, the belief increases to $\mu_{C,t+1} > \mu_t$, suggesting lower entry costs. The specific updating formulas follow standard Bayesian rules and are provided in Appendix B.

3.5 Incumbent's optimization problem

Given belief $\mu_{s,t}$ in state $s \in \{U, C\}$, the incumbent forms a subjective probability of challenger entry by weighting the objective probabilities under each cost regime,

$$p^{\text{subj}}_{s,t+1}(r_t) = \mu_{s,t} \cdot p_{t+1,L}(r_t) + (1 - \mu_{s,t}) \cdot p_{t+1,H}(r_t).$$

Substituting the expressions for $p_{t+1,j}(r_t)$ yields:

$$p^{\text{subj}}_{s,t+1}(r_t) = \left(\tfrac{1}{2} - \bar{r}_e + r_t\right) V^P_{C,t+1} \cdot \Theta_{s,t},$$

where $\Theta_{s,t} = \mu_{s,t}/\bar{K}_L + (1 - \mu_{s,t})/\bar{K}_H$ captures the incumbent's perception of competition intensity.

The incumbent chooses rent r_t to maximize,

$$V^P_{s,t} = \max_{r_t} \left\{ r_t + \beta \mathbb{E}[V^P_{s',t+1} | s, r_t] \right\},$$

where the expected continuation value is

$$\mathbb{E}[V^P_{s',t+1} | s, r_t] = [1 - p^{\text{subj}}_{s,t+1}(r_t)] V^P_{U,t+1} + p^{\text{subj}}_{s,t+1}(r_t) \phi_{t+1}(r_t) V^P_{C,t+1}.$$

The first term represents the continuation value when no entry occurs and the election is uncontested, while the second term captures the case with entry, where the incumbent retains office with probability $\phi_{t+1}(r_t)$. We focus on stationary equilibria where the optimal rent depends only on the current state s, yielding policy functions r^*_U and r^*_C for uncontested and contested states, respectively. The first-order condition characterizes these optimal choices, balancing current rent extraction against future electoral consequences through both entry probability and reelection probability channels.

3.6 Results

The model yields two main predictions that guide our empirical analysis. The formal statements and complete proofs of these results are provided in Appendix B.

Result 1 (Electoral competition and rent extraction)**.** *Politicians extract higher rents following uncontested elections than following contested elections in the stationary equilibrium.*

$$r^*_U > r^*_C$$

This result emerges from the Bayesian learning mechanism. When an election is uncontested, the incumbent rationally infers that challenger entry costs are likely high ($\mu_U < \mu_C$),

which reduces the perceived competition intensity ($\Theta_U < \Theta_C$). The first-order condition implies that politicians facing lower perceived competition extract higher rents. Conversely, contested elections signal lower entry costs, leading to more restrained rent extraction.

The mechanism operates through the informational content of electoral outcomes. An uncontested victory serves as a credible signal that potential challengers face prohibitive entry barriers, effectively relaxing the disciplinary constraint of future electoral competition. This allows incumbents to increase their compensation without fear of attracting challengers. The magnitude of this effect depends on the difference in perceived competition intensity between the two states, with larger belief differentials leading to more substantial rent extraction. See Proposition 1 in Appendix B for the formal statement and proof.

Result 2 (Dynamic learning effects)**.** *Under sequential uncontested elections, the optimal rent path* $\{r^*_{U,n}\}$ *converges monotonically and the marginal increase in rent extraction diminishes over time, with*

$$\lim_{n\to\infty} |r^*_{U,n+1} - r^*_{U,n}| = 0$$

Moreover, the rent increments decrease monotonically, satisfying $r^*_{U,n+2} - r^*_{U,n+1} < r^*_{U,n+1} - r^*_{U,n}$ *for all n.*

This result follows from the convergence of beliefs under repeated uncontested elections. As politicians experience consecutive uncontested elections, their beliefs about high entry costs strengthen but at a decreasing rate. The sequential Bayesian updating leads to belief convergence, with $\mu_{U,n}$ approaching 0 as n increases, which means the incumbent becomes increasingly certain that entry costs are high and thus $\bar{K} = \bar{K}_H$. This convergence in beliefs causes the rent extraction response to flatten over time.

The diminishing effect reflects the decreasing informational value of additional uncontested elections. The first uncontested election provides substantial information about the competitive environment, while subsequent ones add progressively less new information. This pattern is intuitive. As mayors become increasingly confident about the absence of future competition through repeated uncontested victories, they approach what they perceive as the maximum feasible salary level. The convexity of the rent function with respect to beliefs ensures that each successive uncontested victory leads to a smaller salary increase than the previous one, creating a monotone convergence pattern. This adjustment path reflects the systematic Bayesian learning process, where uncertainty gradually resolves and rent extraction stabilizes at its long-run equilibrium.[7]

These theoretical predictions provide testable implications. The first suggests politicians extract higher rents after uncontested elections than after contested elections. The second implies

[7] See Propositions 2 and 3 in Appendix B for the formal statements, which establish both the belief convergence and the resulting monotone rent dynamics.

that the rent-extraction response to consecutive uncontested elections should exhibit monotone convergence over time. Specifically, while the first uncontested victory generates substantial salary increases due to significant belief updating, each subsequent uncontested win produces progressively smaller increases as politicians approach their perceived upper bound. This monotone diminishing pattern reflects the smooth Bayesian learning process, where each additional uncontested election provides less informational value than the previous one. We test these predictions using Japan's municipal elections, where uncontested races are sufficiently common to observe both the immediate salary responses and the dynamic learning patterns.

4 Data

We assemble a stacked panel of municipalities built from seven-year windows centered on each municipal mayoral election. In Japan, mayoral elections occur on a fixed four-year cycle, so there is exactly one election year within each seven-year window. Therefore, as illustrated in Figure 3, each seven-year window comprises the election year, the three preceding years, and the three subsequent years.[8] Because election calendars are set by each municipality, election years vary across places. We stack all seven-year windows whose focal election falls between 2009 and 2013, creating a dataset that spans calendar years 2006–2016.[9] Summary statistics for the main variables are presented in Table A.1.

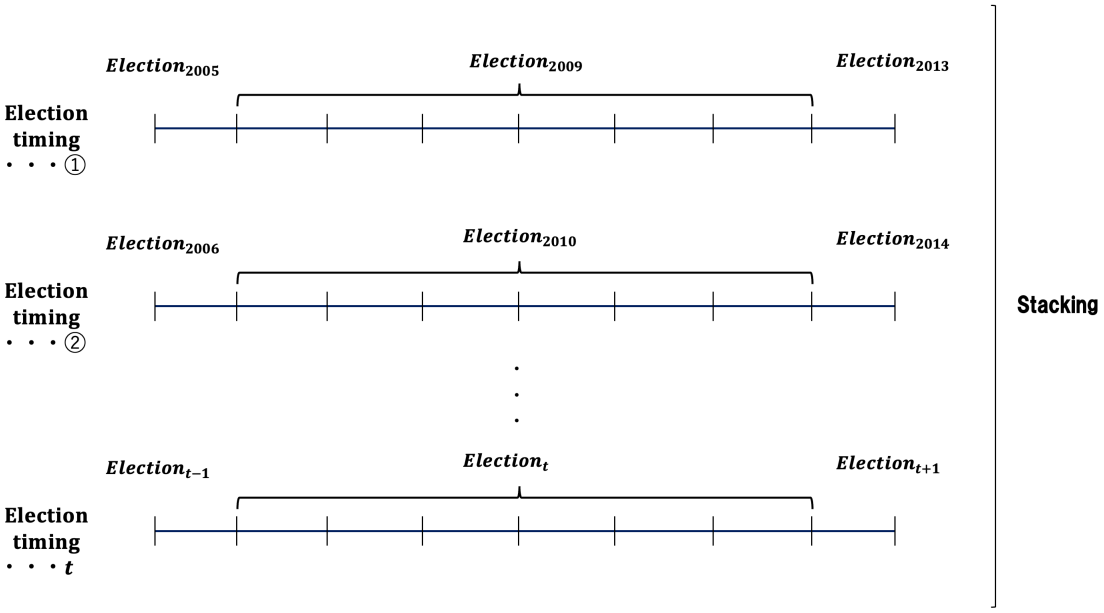
These data constructions indicate that the unit of observation is the municipality, election, and year relative to the election. The stacked panel contains each municipality multiple times, and the years relative to the election can span different calendar years. Our fixed effects specification absorbs any overlap across units and includes calendar-year indicators (see Section 5). Furthermore, to avoid confounding from institutional restructuring, we exclude municipalities that merged during the sample period.[10]

[8] If a mayor resigns, a municipality may hold more than two elections within the seven-year window. We exclude such municipalities from the sample.

[9] We begin in FY2006 because mayoral salary data are first available in that year. Election outcomes are compiled from the Local Election Results Survey by the Japan Research Institute for Local Government, which covers up to FY2016.

[10] The number of municipalities declined from 1,821 in 2006 to 1,718 in 2016 due to mergers. Because mergers often entail concurrent changes in administrative structure and fiscal arrangements, we drop municipalities that undergo a merger within our sample years.

Figure 3: Data structure



Note: The figure shows the process of dataset creation.

4.1 Treatment variables

We define the treatment group as municipalities with uncontested elections. Accordingly, we construct a treatment indicator that takes the value of 1 if the election is uncontested in a given seven-year window, and 0 otherwise. In the baseline analysis, we exclude from the sample any constituencies with two consecutive uncontested elections. Figure A.1 shows a map of cities, towns, and villages where uncontested elections occurred at least once between 2006 and 2016, illustrating that uncontested elections were distributed sporadically across Japan. Although cities, towns, and villages typically tend to have larger populations in that order, the distribution of uncontested elections shows no clear pattern once the relative shares of each municipal type are taken into account.

4.2 Outcome variables

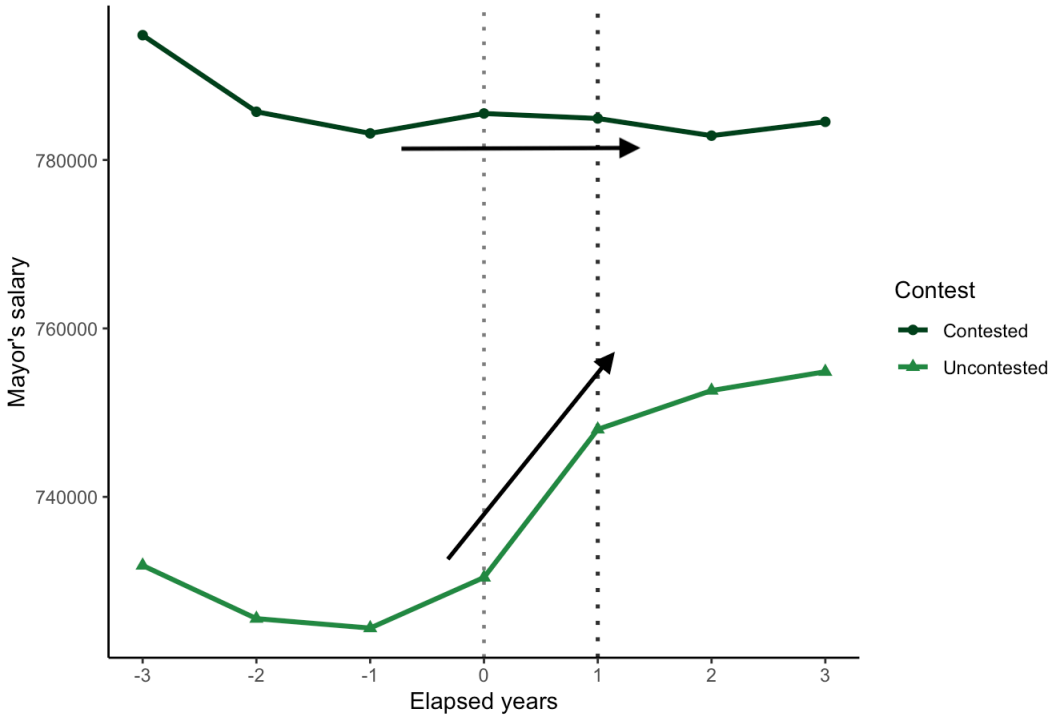
The outcomes of interest are the salary of the municipal mayor. We focus on the average monthly salary as of April 1 of each year, which is set by municipal ordinance.[11] Any change in the mayor's pay requires that the mayor enact ordinances to raise their salaries, and the passage of a salary ordinance by the municipal council.

[11] Japan's fiscal year runs from April 1 to March 31

Figure 4 illustrates event–time means of the average monthly mayoral salary from the stacked panel. The light green line with triangles indicates municipalities whose focal election was uncontested (treatment group), while the dark green line with circles indicates municipalities whose election was contested (control group). The horizontal axis is event time, with $t = 0$ denoting the election fiscal year.

During the pre-treatment periods (i.e., $-3, -2, -1$), the two groups' mean outcomes evolve in parallel. In the post-treatment periods (i.e., 3, 2, 1), average salaries rise sharply only in municipalities with uncontested elections, while salaries remain essentially flat in municipalities with contested elections. The absence of differential pre-treatment trends is consistent with the parallel trend assumption underlying our empirical strategy.

Figure 4: Mayoral salary trends in municipalities with contested and uncontested elections



Note: This figure plots average monthly mayoral salaries by event time, computed using the stacked panel. Light green triangles indicate municipalities with uncontested elections, and dark green circles indicate municipalities with contested elections.

Furthermore, we focus on the average monthly salary of the deputy mayor and expenditure on the municipal council's salary. Because municipality-level data on councilors' monthly salaries are unavailable, we use the annual expenditure on municipal council salaries as a proxy for council compensation. In paticular, given that a mayoral salary raise requires council approval, mayors may try to increase their salary by parallel increasing council salaries.

We also use the annual expenditure on executive salaries. This expenditure encompasses the

total amount spent by municipalities on the salaries of key officials, including mayors, deputy mayors, superintendents of education, and members of personnel and audit committees. Because those variables are available for a longer time span than the average mayoral salary, we use them to extend the observation window of our stacked panel (see Section 5 for details).

5 Empirical Strategy

Uncontested elections occur across Japan, but specific municipal factors may lead to uncontested elections, as noted in Section 2. If there are unobserved differences between municipalities with uncontested elections and those with contested elections, and if macro shocks coincide with election timing, a simple comparison between those two groups and before and after contested timing could be misleading to understanding the results of uncontested elections. To address this concern, we estimate the following difference-in-differences specification.

$$Y_{i,e,t} = \beta(Uncontested_{i,e} \times I_t) + \mu_{i,e} + \rho_{e,t} + \varepsilon_{i,e,t} \tag{1}$$

We use normalized time t, which represents the year relative to the election within each seven-year panel (i.e., t ranges from -3 to 3). The variable $Y_{i,e,t}$ is the outcome of interest for municipality i in a seven-year window in election timing e at normalized time t. All outcome variables are converted into their logarithmic form. $Uncontested_{i,e}$ is a binary variable equal to 1 if municipality i experienced an uncontested election during the seven-year panel, and 0 otherwise. I_t is an indicator variable equal to 1 for post-election years (i.e., when t is equal to 0 or more) and 0 otherwise. The main parameter of interest is β. $\mu_{i,e}$ represents municipality-by-election-window fixed effects, which control for time-invariant differences across municipalities within each window. $\rho_{e,t}$ denotes normalized-time-by-election-window fixed effects, which control for macro-level shocks that vary over time but are common across municipalities. In particular, although the treatment timing is aligned across municipalities, treatment effects are estimated across different calendar years. As a result, this specification ensures robustness to potential confounding from national-level macroeconomic shocks. $\varepsilon_{i,e,t}$ is the error term.

Furthermore, to examine dynamics and the parallel-trends assumption, we estimate the following event-study specification.

$$Y_{i,e,t} = \sum_{\tau=-3,\tau\neq-1}^{3} \beta_\tau(Uncontested_{i,e} \times I_t^\tau) + \mu_{i,e} + \rho_{e,t} + \varepsilon_{i,e,t} \tag{2}$$

I_t^τ is the lead and lag indicators that take a value of 1 if t is equal to τ.[12] β_τ are the coefficients of interest, representing dynamic treatment effects for the 3 periods before and the 3 periods after

[12] we exclude τ = -1 from the equation as the reference period.

the election timing. Event study analysis offers two advantages. First, the DiD approach relies on the parallel trends assumption, which requires that the outcome variables for the treatment and control groups would have followed similar trends in the absence of treatment. By examining the coefficients on the lead indicators, we can check the validity of the parallel trend assumption. Second, the event study design enables us to explore how the effects of uncontested elections evolve over time following the election. For instance, if a salary increase ordinance is enacted and remains unchanged, we expect sustained or rising post-election coefficients.

While the two specifications control for unobservable time-invariant municipal characteristics and macro-level shocks, they do not account for time-varying confounders across municipalities. As a robustness check, we implement the following difference-in-difference-in-differences (DDD) design.

$$\begin{aligned} Y_{i,e,d,t} =& \beta_1(Uncontested_i \times I_t) + \beta_2(Uncontested_i \times D_e \times I_t) \\ &+ \mu_{i,e} + \rho_{e,t} + \varepsilon_{i,e,t} \end{aligned} \tag{3}$$

where *Uncontestedi* equals one if municipality i experiences an uncontested mayoral election at any point in our sample, and $D_{i,e}$ equals one if election timing e is the window in which municipality i actually has an uncontested race and zero in its other windows. The coefficient β_2 is our object of interest, capturing the within-municipality treatment effect by contrasting a municipality's treated window with its own untreated windows. By contrast, β_1 captures any post-election divergence common to municipalities that ever experience an uncontested race, which is a time-varying confounder on the selection of uncontested elections. Therefore, β_2 can be divided from over time confounders based on geographical, sociodemographic.[13]

Mayoral salary data are available from 2006 onward, which limits within-municipality repetition in the stacked panel. To increase temporal coverage and ensure multiple elections per municipality, we construct an alternative outcome using annual expenditure on executive salaries, including the mayor, as a proxy for mayoral salary. These variables are available from FY2000, allowing us to extend our panel dataset to focal elections from 2003 to 2013. In this extension, each municipality contributes at least two mayoral elections within the sample. We examine the effects on this proxy outcome as a robustness check.

6 Results

6.1 Main Results

Columns (1) and (2) of Table 1 present the estimation results in Equation (1) using the average monthly mayoral salary and the annual expenditure on executive salaries as outcomes. The

[13] For more details, see Olden and Møen, 2022, for example.

results indicate that an uncontested mayoral election leads to increases of approximately 3.3 percent and 3.9 percent in these outcomes, respectively, both statistically significant at the 1 percent level. At the sample mean, the 3.3 percent effect corresponds to an increase of about 25000 yen (roughly $170). Column (3) reports estimates of Equation (3) for the annual expenditure on executive salaries and likewise indicates a post-election increase in uncontested municipalities. The magnitude closely matches that in Column (2), reinforcing that the baseline results are robust to time-varying confounders.[14]

Table 1: DiD estimate of the effect of uncontested elections on mayoral salaries

	(1) Monthly salaries of mayors	(2) Expenditure on salaries	(3) Expenditure on salaries
Uncontested	0.033***	0.030***	
	(0.005)	(0.010)	
Uncontested Experience × Indicator			0.032**
			(0.014)
R-squared	0.827	0.882	0.889
Observations	9296	9296	13447
Municipality by Election FE	Yes	Yes	Yes
Year by Election FE	Yes	Yes	Yes

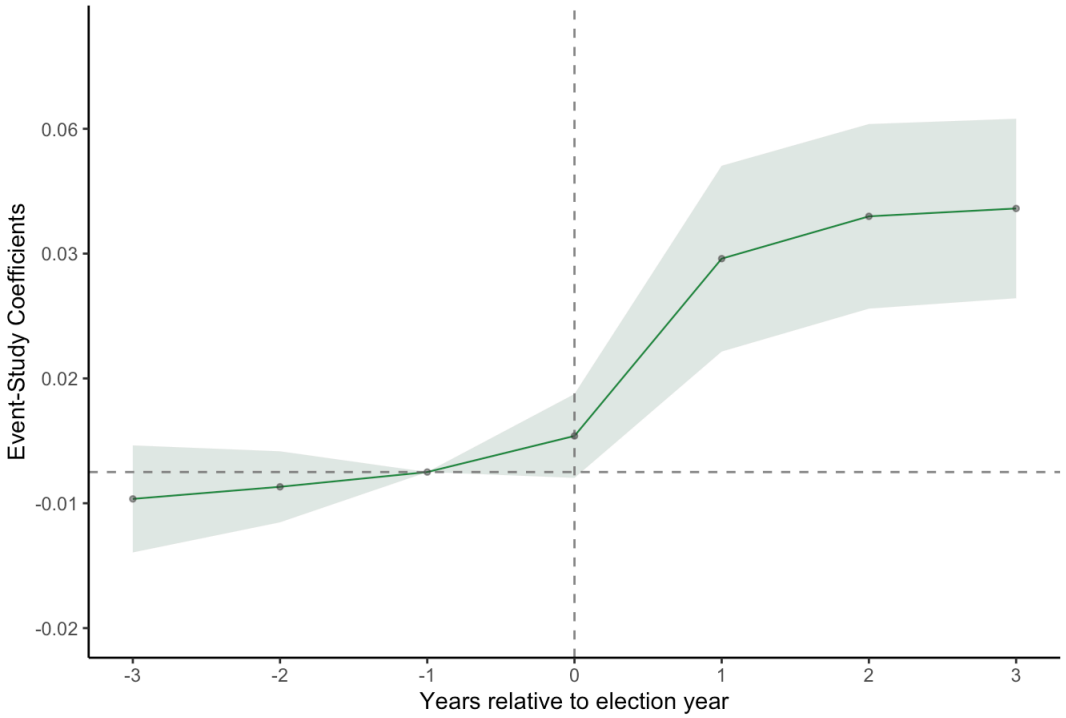
Note: Columns (1) and (2) report the estimation results from Equation 1 on the average monthly salary of the mayor and the annual expenditure on executive salaries, respectively. Columns (3) show the estimation results from Equation 3 on the annual expenditure on executive salaries. All specifications include municipality-by-election and year-by-election fixed effects. Standard errors clustered at the municipality level are shown in brackets. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Figure 5 presents the point estimates of β_τ from Equation (2) for the average monthly mayoral salary, with 95% confidence intervals. The post-election coefficients are positive and statistically significant, whereas the pre-election coefficients are close to zero and statistically indistinguishable from zero. This pattern supports the parallel-trends assumption and indicates no pretreatment differences between treated and control municipalities. The results are also consistent with the visual evidence in Figure 5.[15] Taken together, the estimate results suggest that mayors elected without opposition subsequently raise their own salaries to maximize personal gain.

[14] Because the expenditure on executive salaries aggregates one year's compensation for multiple senior officials in addition to the mayor's salary, the coefficient is not directly comparable to the coefficient on mayoral salary.

[15] The estimated effects are larger from the second post-election period. This pattern likely reflects the timing of the salary records. Data are measured on April 1 and capture information available only through March of that year. As a result, the treatment is reliably incorporated into the outcome variables in the following year.

Figure 5: Event study estimates of the effect of uncontested elections on mayoral salaries



Note: The figure presents the estimation results from the event study analysis based on Equation 2, along with 95% confidence intervals shaded in green. The horizontal axis represents years relative to the election year. Standard errors are clustered at the municipality level.

6.1.1 Robustness checks and placebo test

In this section, we examine the robustness of the main results and the placebo exercise. First, we estimate Equation (1) with a vector of time-varying covariates capturing demographics, industrial structure, and fiscal capacity. These covariates include (i) population size, (ii) the shares of residents under 15 years old and over 65 years old, (iii) the employment shares in secondary and tertiary industries, and (iv) a fiscal indicator that measures the gap between expenditure needs and revenue from taxes and other sources. Figure A.2 reports covariate balance regressions of the treatment indicator on these controls. Although coefficients are generally small, the share of residents over 65 and the employment shares in the primary and secondary sectors are positively associated with treatment, whereas the share under 15 and the fiscal indicator are negatively associated. Figure A.3 then presents event-study estimates from Equation (2) including this covariate set. The post-election response exhibits the same sharp, positive jump as in Figure 5, while the pre-treatment coefficients remain close to zero. Thus, conditioning on observed time-varying municipal characteristics leaves the substantive conclusions unchanged.

Second, to account for electoral-cycle dynamics, we consider the possibility that incumbents

strategically depress salaries immediately before elections to enhance re-election prospects. In this case, using the period just before the election as the reference period in Equation (2) would tend to overestimate the treatment effect, because $t = -1$ would be a lower baseline. To address this concern, Figure A.4 reports event-study estimates that instead use the election two and three terms prior as the omitted category, respectively. The resulting estimates closely mirror those in Figure 5. We find no evidence of differential pre-election manipulation by incumbents between treated and control municipalities.

Third, as a placebo test, we estimate Equation (1) using expenditure on public-employee salaries as the outcome variable. This expenditure refers to public sector wages that are not directly linked to the mayor's or the executive's salary. Since Japanese public employee salaries follow a seniority-based system, and estimates may be influenced by factors such as the correlation between uncontested elections and an aging population, we estimate Equation (3) to address those factors. Table A.2 shows no statistically significant effect of uncontested elections on public-employee salaries, and the coefficient is close to zero.

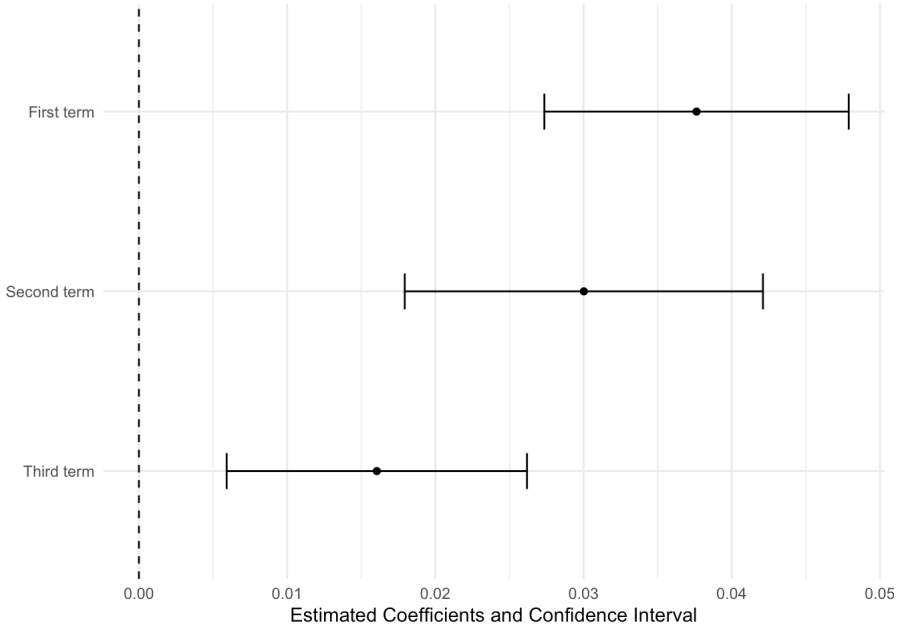
6.2 Consecutive uncontested elections

The model in Section 3 predicts that the mayor adjusts the post-election salary increase in response to experience with uncontested elections, and that the magnitude declines as the number of such experiences grows. To explore the potential mechanisms, we examine post-election effects in subsamples defined by the incumbent's number of consecutive uncontested victories.

Figure 6 shows the point estimates with 95 percent confidence intervals. We restrict the treatment group to the mayor's first uncontested win, the same mayor's second consecutive uncontested win, and the same mayor's third consecutive uncontested win, ordered from top to bottom. The first uncontested victory yields the largest salary increase, which is statistically significant at the 1 percent level. The effect falls for the second consecutive uncontested win and further declines for the third. This monotonic decline in the magnitude of salary increases supports the model's predictions.

These findings are consistent with the theoretical mechanism of Bayesian learning about the competitive environment. A first uncontested election provides new information about high entry costs, leading mayors to increase their compensation. Subsequent uncontested victories add less information as beliefs about the absence of future competition converge. The declining pattern suggests that mayors approach what they perceive as the maximum feasible salary level through a gradual adjustment process rather than immediate extraction. This is the convergence pattern implied by the theoretical framework in which politicians learn about their political environment through repeated electoral outcomes.

Figure 6: Heterogeneous effects on mayoral salaries by the number of consecutive uncontested elections



Note: The figure presents point estimates from Equation 1 with 95% confidence intervals. In the results, the treatment group is restricted to uncontested victories by the same mayor at the first, second, and third consecutive elections, shown from top to bottom.

6.3 Other executive compensation

We also examine two proximate pecuniary outcomes, the deputy mayor's salary and expenditure on municipal council salaries. In Japan, deputy mayors are appointed by the mayor, and any change to the mayor's salary requires a salary ordinance passed by the municipal council. These institutional features create incentives for mayors to raise the compensation of these stakeholders to secure support for increases in their own pay.

Columns (1) and (2) of Table 2 report estimates of Equation (1) with the average monthly deputy mayor salary and the annual expenditure on councilor salaries. Those results show that following the elections, the monthly deputy mayor salary and council salary expenditure rise by approximately 1.2 percent and 2.0 percent in municipalities with uncontested races relative to those with contested races. These patterns suggest that unopposed mayors compensate key stakeholders to facilitate the passage of ordinances that raise their own salaries.

Table 2: DiD estimate of the effect of uncontested elections on stakeholders' salaries

	(1) Monthly salaries of deputy mayors	(2) Expenditure on municipal council's salary
Uncontested	0.012***	0.011***
	(0.003)	(0.006)
R-squared	0.921	0.991
Observations	9296	9296
Municipality by Election FE	Yes	Yes
Year by Election FE	Yes	Yes

Note: Columns (1) and (2) report the estimation results from Equation 1 on the deputy mayor's salary and expenditure on the municipal council's salary, respectively. All specifications include municipality-by-election and year-by-election fixed effects. Standard errors clustered at the municipality level are shown in brackets. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

7 Conclusions

We study how the complete absence of electoral competition shapes politicians' behavior. Japan's mayoral system provides a setting with uniform institutions, direct executive elections, and a persistently large share of uncontested races. We use the setting to estimate the effect of uncontested races on mayoral salary.

Using a stacked panel with difference-in-differences and event-study estimators, we find that mayors who win unopposed increase their own monthly salaries by about 3.3 percent. The increase appears as a jump after the election, with flat pre-trends. Estimates are stable across extensive robustness checks. Moreover, the magnitude of the rise declines with additional consecutive uncontested wins.

The pattern extends to pivotal stakeholders. Deputy mayor salaries and expenditure on councilor salaries also rise following an uncontested mayoral victory. Because any change in the mayor's pay requires a council-passed salary ordinance and deputy mayors are appointed by the mayor, these findings suggest that mayors increase stakeholder compensation to secure support for their own pay increases.

We formalize a dynamic learning mechanism. In the model, an uncontested win reveals high entry costs and a low probability of future competition. The signal relaxes electoral discipline and increases the optimal level of rent extraction. Our main findings match these predictions. The model also implies diminishing increments across consecutive uncontested wins as mayors' beliefs about the competitive environment converge. Consistent with this channel, the first uncontested win generates the largest raise, with progressively smaller increments after the second and third wins. Our evidence shows that without public conflicts and competition, incumbents shift toward private gain, highlighting elections as a core mechanism of discipline.

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A Appendix

Figure A.1: Geographic distribution of constituencies with uncontested elections

Note: This figure displays the geographic distribution of municipalities that experienced at least one uncontested mayoral election between 2006 and 2016. Darker shades indicate villages with uncontested elections, medium shades represent towns, and lighter shades represent cities.

Table A.1: Summary statistics

Variables	Mean	SD
Outcome variables		
Mayor's salary (per month)	770054	140076
Deputy mayor's salary (per month)	643844	106436
Expenditure on executive salaries (thousand JPY)	39270	18121
Expenditure on council salaries (thousand JPY)	35094	15654
Covariates		
Population	65590	170314
Pop. 15 (%)	0.13	0.022
Pop. 65 (%)	0.26	0.068
Primary ind. (%)	0.13	0.11
Secondary ind. (%)	0.28	0.083
Financial indicator	0.53	0.32

Note: The mayor's salary and the deputy mayor's salary are the average monthly salaries as of April 1. The expenditure unit is 1000 yen, which equals approximately 0.67 dollars at an exchange rate of 150 yen to 1 U.S. dollar. The first column shows the average of the outcome variables. The second column shows the standard deviation of the outcome variables.

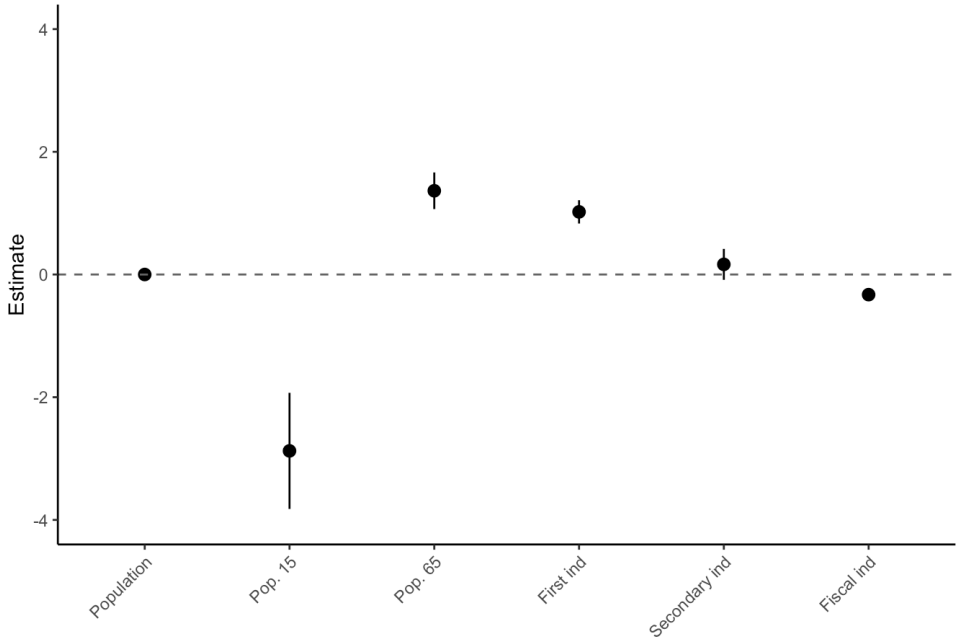
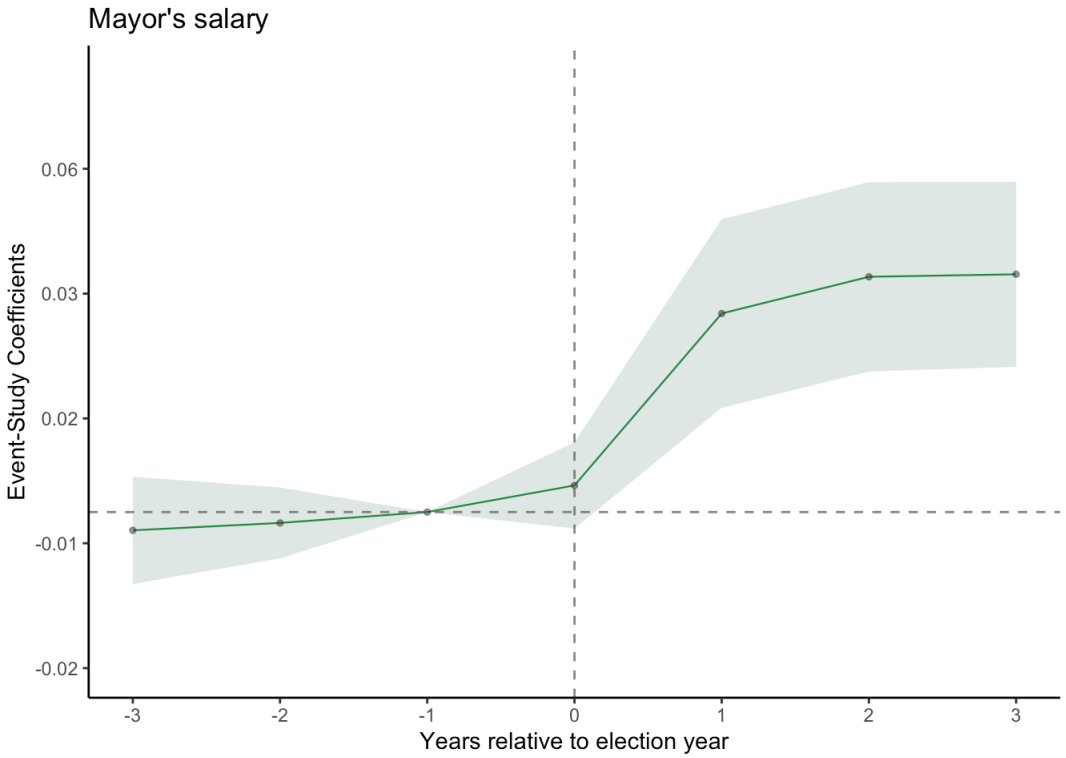


Figure A.2: Covariate balance

Note: This figure presents the results of covariate balance tests across election timing windows in 2011. The treatment indicator, which equals 1 for municipalities that experienced uncontested elections, is regressed on the full set of covariates.

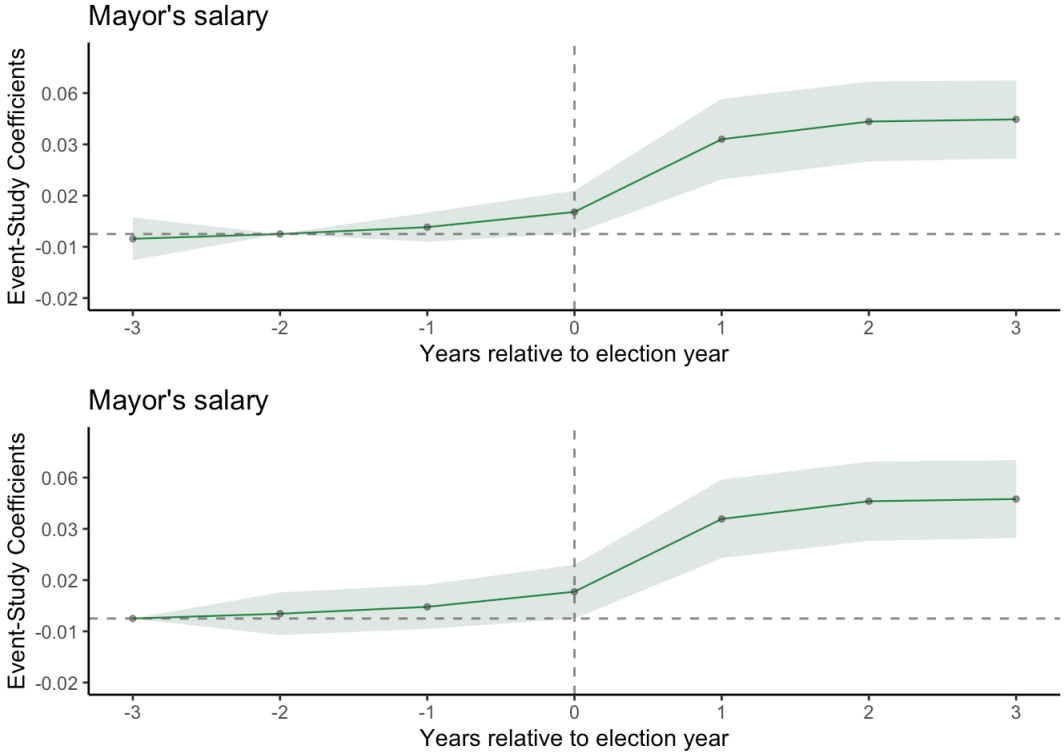
Figure A.3: Event study estimates with covariate



Note: The figure presents event-study estimates from Equation 2 with the full set of covariates included. The

shaded area represents 95% confidence intervals. The horizontal axis represents years relative to the election year. Standard errors are clustered at the municipality level.

Figure A.4: Event study estimates with alternative reference periods



Note: The figure presents event-study estimates from Equation 2. The shaded area represents 95% confidence intervals. The top panel uses the election two terms prior as the omitted category; the bottom panel uses the election three terms prior. The horizontal axis represents years relative to the election year. Standard errors are clustered at the municipality level.

Table A.2: DiD estimate of the effect of uncontested elections on general staff salary

	(1) Expenditure on general staff salary
Uncontested Experience × Indicator	-0.006
	(0.004)
R-squared	0.999
Observations	13447
Municipality by Election FE	Yes
Year by Election FE	Yes

Note: Columns (1) report the estimation results from Equation 1 on the expenditure on general staff salary. All specifications include municipality-by-election and year-by-election fixed effects. Standard errors clustered at the municipality level are shown in brackets. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

B Theoretical Framework: Detailed Calculations and Proofs

B.1 Voter behavior and winning probability

Consider a representative voter i in municipality at time $t+1$. The voter derives utility from the incumbent politician's rent extraction level r_t and has an idiosyncratic ideological preference $\epsilon_{i,t}$ for the incumbent.

In a contested election, voter i compares utilities from the incumbent and challenger:

$$\begin{aligned} U_{i,t+1}^{\text{inc}} &= -r_t + \epsilon_{i,t} \quad \text{(utility from incumbent)} \\ U_{i,t+1}^{\text{chal}} &= -\bar{r}_e \quad \text{(utility from challenger)} \end{aligned}$$

where r_t is the rent extracted by the incumbent in period t, $\bar{r}_e$ is the expected rent from the challenger, and $\epsilon_{i,t} \sim \text{Unif}[-\frac{1}{2}, \frac{1}{2}]$ is voter i's idiosyncratic preference for the incumbent. Voter i votes for the incumbent if and only if $U_{i,t+1}^{\text{inc}} \geq U_{i,t+1}^{\text{chal}}$.

Voter i votes for the incumbent if and only if

$$U_{i,t+1}^{\text{inc}} \geq U_{i,t+1}^{\text{chal}}$$

This condition is equivalent to

$$\begin{aligned} -r_t + \epsilon_{i,t} &\geq -\bar{r}_e \\ \epsilon_{i,t} &\geq r_t - \bar{r}_e \end{aligned}$$

Since $\epsilon_{i,t} \sim \text{Unif}[-\frac{1}{2}, \frac{1}{2}]$, the probability that a randomly selected voter supports the incumbent is

$$\begin{aligned} \phi_{t+1}(r_t) &= \Pr[\epsilon_{i,t} \geq r_t - \bar{r}_e] \\ &= \Pr[\epsilon_{i,t} > r_t - \bar{r}_e] \quad \text{(since } \epsilon_{i,t} \text{ is continuous)} \\ &= 1 - F_\epsilon(r_t - \bar{r}_e) \end{aligned}$$

where $F_\epsilon(\cdot)$ is the CDF of the uniform distribution on $[-\frac{1}{2}, \frac{1}{2}]$.

For $\epsilon \sim \text{Unif}[-\frac{1}{2}, \frac{1}{2}]$, the CDF is

$$F_\epsilon(x) = \begin{cases} 0 & \text{if } x < -\frac{1}{2} \\ x + \frac{1}{2} & \text{if } -\frac{1}{2} \leq x \leq \frac{1}{2} \\ 1 & \text{if } x > \frac{1}{2} \end{cases}$$

Therefore, when $-\frac{1}{2} \leq r_t - \bar{r}_e \leq \frac{1}{2}$ (the interior case)

$$\begin{aligned}\phi_{t+1}(r_t) &= 1 - F_\epsilon(r_t - \bar{r}_e) \\ &= 1 - (r_t - \bar{r}_e + \frac{1}{2}) \\ &= \frac{1}{2} + \bar{r}_e - r_t \end{aligned} \tag{4}$$

This expression reveals the fundamental trade-off facing the incumbent. The winning probability starts at one-half when the incumbent extracts the same rent as the challenger would be expected to extract, reflecting the symmetric distribution of voter preferences. Each additional unit of rent extraction directly reduces the winning probability by one unit, creating a linear relationship between rent-seeking and electoral prospects. The incumbent thus faces a choice between immediate pecuniary gains and future electoral success.

Taking the derivative with respect to r_t

$$\phi'_{t+1}(r_t) = \frac{\partial}{\partial r_t}\left(\frac{1}{2} + \bar{r}_e - r_t\right) = -1 < 0$$

This negative derivative confirms that higher rent extraction reduces the incumbent's winning probability.

Boundary cases and probability projection When $r_t - \bar{r}_e$ falls outside $[-\frac{1}{2}, \frac{1}{2}]$

- If $r_t - \bar{r}_e < -\frac{1}{2}$: All voters prefer the incumbent, so $\phi_{t+1}(r_t) = 1$
- If $r_t - \bar{r}_e > \frac{1}{2}$: No voters prefer the incumbent, so $\phi_{t+1}(r_t) = 0$

B.2 Challenger entry

A potential challenger at time $t + 1$ faces an entry decision. The challenger must pay a privately known entry cost K to enter the race. The potential challenger observes the incumbent's rent r_t and decides whether to enter. We assume that the value of holding office is the same for both the incumbent and the challenger, denoted by $V^P_{C,t+1}$. The challenger enters if and only if the expected benefit from entry exceeds the privately observed cost K, which gives us

$$[1 - \phi_{t+1}(r_t)]V^P_{C,t+1} \geq K$$

Substituting $\phi_{t+1}(r_t) = \frac{1}{2} + \bar{r}_e - r_t$ from the previous section yields

$$\left(\frac{1}{2} - \bar{r}_e + r_t\right) V^P_{C,t+1} \geq K$$

Therefore, entry occurs if

$$K \leq \left(\frac{1}{2} - \bar{r}_e + r_t\right) V_{C,t+1}^P$$

Distribution of entry costs The entry cost K is privately observed by each potential challenger and follows a uniform distribution.

$$K \sim \text{Unif}[0, \bar{K}_j]$$

where $\bar{K}_j \in \{\bar{K}_L, \bar{K}_H\}$ with $\bar{K}_L < \bar{K}_H$, and $j \in \{L, H\}$ denotes the low and high cost regimes, respectively.

Probability of entry Given the uniform distribution of K on $[0, \bar{K}_j]$, the probability that a challenger enters is

$$p_{t+1,j}(r_t) = \Pr\left[K \leq \left(\frac{1}{2} - \bar{r}_e + r_t\right) V_{C,t+1}^P\right]$$

When $0 \leq \left(\frac{1}{2} - \bar{r}_e + r_t\right) V_{C,t+1}^P \leq \bar{K}_j$ (interior case)

$$p_{t+1,j}(r_t) = \frac{\left(\frac{1}{2} - \bar{r}_e + r_t\right) V_{C,t+1}^P}{\bar{K}_j} \tag{5}$$

This expression captures the strategic interaction between incumbent rent extraction and challenger entry. Higher rent extraction by the incumbent creates electoral vulnerability, which increases the expected payoff from challenging. Specifically, each unit increase in rent extraction raises the entry probability proportionally, scaled by the value of holding office and inversely by the entry cost distribution. The mechanism operates through voter dissatisfaction—as the incumbent extracts more rents, the challenger's expected vote share increases, making entry more attractive even for challengers with higher entry costs. The parameter $\bar{K}_j$ represents the heterogeneity in entry barriers, capturing factors such as campaign costs, organizational requirements, and political networks. In environments with lower entry barriers (smaller $\bar{K}_j$), the incumbent faces a more elastic entry response to rent extraction, creating stronger disciplinary pressure.

Comparative statics Since $\bar{K}_L < \bar{K}_H$, we have

$$\frac{\left(\frac{1}{2} - \bar{r}_e + r_t\right) V_{C,t+1}^P}{\bar{K}_L} > \frac{\left(\frac{1}{2} - \bar{r}_e + r_t\right) V_{C,t+1}^P}{\bar{K}_H}$$

$$p_{t+1,L}(r_t) > p_{t+1,H}(r_t)$$

This inequality shows that entry is more likely under the low-cost regime.

B.3 Belief updating

The incumbent does not observe the true entry cost parameter $\bar{K} \in \{\bar{K}_L, \bar{K}_H\}$ but maintains a belief about it. Let μ_t denote the incumbent's belief at time t that the entry cost is low.

$$\mu_t = \Pr(\bar{K} = \bar{K}_L)$$

After observing the election outcome at $t+1$, the incumbent updates this belief using Bayes' rule.

Updating after an uncontested election If the election at $t+1$ is uncontested (no challenger enters), the incumbent updates the belief as follows.

By Bayes' rule:

$$\mu_{U,t+1} = \Pr(\bar{K} = \bar{K}_L \mid \text{no entry at } t+1)$$

Using Bayes' theorem

$$\mu_{U,t+1} = \frac{\Pr(\text{no entry} \mid \bar{K} = \bar{K}_L) \times \Pr(\bar{K} = \bar{K}_L)}{\Pr(\text{no entry})}$$

The probability of no entry under each regime is

$$\Pr(\text{no entry} \mid \bar{K} = \bar{K}_L) = 1 - p_{t+1,L}(r_t)$$
$$\Pr(\text{no entry} \mid \bar{K} = \bar{K}_H) = 1 - p_{t+1,H}(r_t)$$

The unconditional probability of no entry is

$$\begin{aligned}\Pr(\text{no entry}) &= \Pr(\text{no entry} \mid \bar{K}_L) \times \Pr(\bar{K}_L) + \Pr(\text{no entry} \mid \bar{K}_H) \times \Pr(\bar{K}_H) \\ &= [1 - p_{t+1,L}(r_t)]\mu_t + [1 - p_{t+1,H}(r_t)](1 - \mu_t)\end{aligned}$$

Therefore,

$$\mu_{U,t+1} = \frac{\mu_t[1 - p_{t+1,L}(r_t)]}{\mu_t[1 - p_{t+1,L}(r_t)] + (1 - \mu_t)[1 - p_{t+1,H}(r_t)]} \tag{6}$$

Since $p_{t+1,L}(r_t) > p_{t+1,H}(r_t)$, we have $1 - p_{t+1,L}(r_t) < 1 - p_{t+1,H}(r_t)$. This implies

$$\frac{1 - p_{t+1,L}(r_t)}{1 - p_{t+1,H}(r_t)} < 1$$

Rewriting the updated belief

$$\mu_{U,t+1} = \frac{\mu_t[\frac{1-p_{t+1,L}(r_t)}{1-p_{t+1,H}(r_t)}]}{\mu_t[\frac{1-p_{t+1,L}(r_t)}{1-p_{t+1,H}(r_t)}] + (1-\mu_t)}$$

Since $\frac{1-p_{t+1,L}(r_t)}{1-p_{t+1,H}(r_t)} < 1$, the updated belief decreases

$$\mu_{U,t+1} < \mu_t \tag{7}$$

This downward revision in beliefs following an uncontested election represents a critical learning mechanism. The absence of a challenger serves as an informative signal about the underlying competitive environment. When no opponent emerges despite the incumbent's rent extraction, the incumbent rationally infers that potential challengers likely face prohibitive entry costs. This inference strengthens with the level of rent extraction, as high rents that fail to attract challengers indicate particularly severe entry barriers. The updated belief $\mu_{U,t+1} < \mu_t$ implies that the incumbent perceives reduced future competition risk, which relaxes the electoral discipline constraint. This learning effect creates a self-reinforcing dynamic where uncontested victories lead to higher rent extraction, which in turn may deter future entry, perpetuating the cycle of non-competition.

Updating after a contested election If the election at $t+1$ is contested (a challenger enters), the incumbent updates similarly

$$\mu_{C,t+1} = \Pr(\bar{K} = \bar{K}_L \mid \text{entry at } t+1)$$

Following the same Bayesian logic

$$\begin{aligned}\mu_{C,t+1} &= \frac{\Pr(\text{entry} \mid \bar{K} = \bar{K}_L) \times \Pr(\bar{K} = \bar{K}_L)}{\Pr(\text{entry})} \\ &= \frac{\mu_t p_{t+1,L}(r_t)}{\mu_t p_{t+1,L}(r_t) + (1-\mu_t) p_{t+1,H}(r_t)}\end{aligned} \tag{8}$$

Since $p_{t+1,L}(r_t) > p_{t+1,H}(r_t)$, we have $\frac{p_{t+1,L}(r_t)}{p_{t+1,H}(r_t)} > 1$, which implies

$$\mu_{C,t+1} > \mu_t \tag{9}$$

This upward revision following a contested election reveals the disciplinary role of electoral competition. The emergence of a challenger signals that entry costs are likely low enough to make challenging worthwhile. The incumbent learns that the political environment is more competitive than previously believed, with potential challengers able and willing to mount cam-

paigns. This updated belief $\mu_{C,t+1} > \mu_t$ heightens the perceived threat of future competition, inducing the incumbent to moderate rent extraction to avoid attracting challengers. The magnitude of the belief revision depends on the likelihood ratio between the two cost regimes. A larger difference between $p_{t+1,L}(r_t)$ and $p_{t+1,H}(r_t)$ makes the election outcome more informative, leading to sharper belief updates. This learning mechanism ensures that electoral competition disciplines incumbent behavior not just through immediate electoral consequences but also through its informational content about future competitive threats.

B.4 Incumbent's optimization problem

Lemma 1 (Optimal rent function)**.** *In the stationary equilibrium, given belief μ_s about the entry cost environment in state $s \in \{U, C\}$, the optimal rent is*

$$r_s^* = \bar{r}_e - \frac{V_U^P}{2V_C^P} + \frac{1}{2\beta\,\Theta_s\,(V_C^P)^2} \tag{10}$$

where $\Theta_s = \frac{\mu_s}{\bar{K}_L} + \frac{1-\mu_s}{\bar{K}_H}$ captures the perceived competition intensity in state s.

The optimal rent expression reveals three key forces shaping the incumbent's rent extraction decision in the stationary equilibrium. The first term $\bar{r}_e$ represents the baseline rent level expected from challengers. The second term captures the value differential between uncontested and contested elections. This negative adjustment reflects the incumbent's incentive to deter entry by keeping rents below levels that would attract challengers. The third term represents the rent premium the incumbent can extract when believing entry costs are high. This premium is inversely related to the perceived competition intensity Θ_s. When Θ_s is low, the incumbent believes that entry costs are high and thus faces reduced electoral discipline, leading to more aggressive rent extraction.[16]

Proof. We derive this result by first solving the general dynamic optimization problem with time indices and then imposing the stationary equilibrium conditions. This approach allows us to clearly identify how the time-invariant value functions emerge from the underlying dynamic structure.

Consider the general dynamic problem where the incumbent at time t in state s maximizes

$$V_{s,t}^P = \max_{r_t} \left\{ r_t + \beta \mathbb{E}[V_{s',t+1}^P \mid s, r_t] \right\} \tag{11}$$

This optimization problem embodies the fundamental trade-off between current rent extraction and future electoral prospects. Higher rent extraction increases immediate payoffs but raises the

[16] This stationary equilibrium characterization emerges as a special case of the general dynamic problem where value functions and policy functions are time-invariant. Specifically, when $V_{s,t}^P = V_s^P$ for all t, the general dynamic formula reduces to the stationary form presented above.

probability of attracting challengers and reduces the likelihood of winning if challenged. The incumbent must balance these competing forces, with the optimal choice depending critically on beliefs about the competitive environment. When the incumbent believes entry costs are high, the perceived threat of competition weakens, tilting the balance toward more aggressive rent extraction.

We solve this optimization problem for arbitrary time t and then specialize to the stationary equilibrium where $V^P_{s,t} = V^P_s$ for all t. This method ensures that our stationary equilibrium characterization is consistent with the full dynamic model.

Step 1: Expected continuation value The expected continuation value depends on whether a challenger enters

$$\begin{aligned}\mathbb{E}[V^P_{s',t+1} \mid s, r_t] = \Pr(\text{no entry}) \times V^P_{U,t+1} + \Pr(\text{entry}) \times \Pr(\text{win} \mid \text{entry}) \times V^P_{C,t+1} \\ + \Pr(\text{entry}) \times \Pr(\text{lose} \mid \text{entry}) \times 0 \end{aligned} \tag{12}$$

Substituting the probabilities

$$\mathbb{E}[V^P_{s',t+1} \mid s, r_t] = [1 - p^{\text{subj}}_{s,t+1}(r_t)]V^P_{U,t+1} + p^{\text{subj}}_{s,t+1}(r_t)\phi_{t+1}(r_t)V^P_{C,t+1}$$

where $p^{\text{subj}}_{s,t+1}(r_t)$ is the subjective probability of entry given belief $\mu_{s,t}$,

$$\begin{aligned} p^{\text{subj}}_{s,t+1}(r_t) &= \mu_{s,t}p_{t+1,L}(r_t) + (1-\mu_{s,t})p_{t+1,H}(r_t) \\ &= \mu_{s,t}\frac{(\frac{1}{2} - \bar{r}_e + r_t)V^P_{C,t+1}}{\bar{K}_L} + (1-\mu_{s,t})\frac{(\frac{1}{2} - \bar{r}_e + r_t)V^P_{C,t+1}}{\bar{K}_H} \\ &= (\tfrac{1}{2} - \bar{r}_e + r_t)V^P_{C,t+1}\left[\frac{\mu_{s,t}}{\bar{K}_L} + \frac{1-\mu_{s,t}}{\bar{K}_H}\right] \\ &= (\tfrac{1}{2} - \bar{r}_e + r_t)V^P_{C,t+1}\Theta_{s,t} \end{aligned} \tag{13}$$

Step 2: First-order condition The incumbent's objective function is

$$\begin{aligned}\mathcal{V}(r_t) &= r_t + \beta\mathbb{E}[V^P_{s',t+1} \mid s, r_t] \\ &= r_t + \beta\left\{[1 - p^{\text{subj}}_{s,t+1}(r_t)]V^P_{U,t+1} + p^{\text{subj}}_{s,t+1}(r_t)\phi_{t+1}(r_t)V^P_{C,t+1}\right\}\end{aligned} \tag{14}$$

Taking the derivative with respect to r_t

$$\frac{\partial \mathcal{V}}{\partial r_t} = 1 + \beta\left\{-\frac{\partial p^{\text{subj}}_{s,t+1}}{\partial r_t}V^P_{U,t+1} + \frac{\partial p^{\text{subj}}_{s,t+1}}{\partial r_t}\phi_{t+1}(r_t)V^P_{C,t+1} + p^{\text{subj}}_{s,t+1}(r_t)\frac{\partial \phi_{t+1}}{\partial r_t}V^P_{C,t+1}\right\}$$

Computing the derivatives

$$\frac{\partial p^{\text{subj}}_{s,t+1}}{\partial r_t} = V^P_{C,t+1} \times \Theta_{s,t}$$
$$\frac{\partial \phi_{t+1}}{\partial r_t} = -1$$

Substituting these derivatives yields

$$\frac{\partial \mathcal{V}}{\partial r_t} = 1 + \beta\Big\{V^P_{C,t+1}\Theta_{s,t}[\phi_{t+1}(r_t)V^P_{C,t+1} - V^P_{U,t+1}] - p^{\text{subj}}_{s,t+1}(r_t)V^P_{C,t+1}\Big\}$$

Step 3: Solving the first-order condition At the optimum, $\frac{\partial \mathcal{V}}{\partial r_t} = 0$. This yields

$$1 + \beta V^P_{C,t+1}\Theta_{s,t}[\phi_{t+1}(r_t)V^P_{C,t+1} - V^P_{U,t+1}] - \beta p^{\text{subj}}_{s,t+1}(r_t)V^P_{C,t+1} = 0$$

Substituting $\phi_{t+1}(r_t) = \frac{1}{2} + \bar{r}_e - r_t$ and $p^{\text{subj}}_{s,t+1}(r_t) = (\frac{1}{2} - \bar{r}_e + r_t)V^P_{C,t+1}\Theta_{s,t}$

$$1 + \beta V^P_{C,t+1}\Theta_{s,t}[(\tfrac{1}{2} + \bar{r}_e - r_t)V^P_{C,t+1} - V^P_{U,t+1}] - \beta(\tfrac{1}{2} - \bar{r}_e + r_t)V^P_{C,t+1}\Theta_{s,t}V^P_{C,t+1} = 0$$

Expanding and simplifying

$$1 + \beta V^P_{C,t+1}\Theta_{s,t}[(\tfrac{1}{2} + \bar{r}_e - r_t)V^P_{C,t+1} - V^P_{U,t+1}] - \beta(\tfrac{1}{2} - \bar{r}_e + r_t)(V^P_{C,t+1})^2\Theta_{s,t} = 0$$

The terms with $(\frac{1}{2} + \bar{r}_e - r_t)$ and $(\frac{1}{2} - \bar{r}_e + r_t)$ simplify to give

$$1 + \beta V^P_{C,t+1}\Theta_{s,t} \times (-2r_t V^P_{C,t+1} - V^P_{U,t+1} + 2\bar{r}_e V^P_{C,t+1}) = 0$$

Solving for r_t yields the general dynamic solution

$$1 = 2\beta(V^P_{C,t+1})^2\Theta_{s,t}r_t - 2\beta\bar{r}_e(V^P_{C,t+1})^2\Theta_{s,t} + \beta V^P_{C,t+1}V^P_{U,t+1}\Theta_{s,t}$$
$$r^*_{s,t} = \bar{r}_e - \frac{V^P_{U,t+1}}{2V^P_{C,t+1}} + \frac{1}{2\beta\Theta_{s,t}(V^P_{C,t+1})^2} \tag{15}$$

Step 4: Imposing stationarity In the stationary equilibrium, value functions are time-invariant. We have $V^P_{U,t+1} = V^P_U$ and $V^P_{C,t+1} = V^P_C$ for all t. Moreover, beliefs in state s are constant over time, implying $\mu_{s,t} = \mu_s$ and consequently $\Theta_{s,t} = \Theta_s$. Substituting these stationary values into the general solution yields

$$r^*_s = \bar{r}_e - \frac{V^P_U}{2V^P_C} + \frac{1}{2\beta\Theta_s(V^P_C)^2} \tag{16}$$

This establishes the stationary equilibrium rent formula presented in lemma1. The stationary equilibrium rent formula reveals three economically meaningful components. The baseline $\bar{r}_e$ anchors expectations. The negative adjustment $-\frac{V_U^P}{2V_C^P}$ reflects the incumbent's incentive to deter entry by keeping rents below levels that would attract challengers. The positive term $\frac{1}{2\beta\Theta_s(V_C^P)^2}$ represents the rent premium enabled by perceived high entry costs.

The parameter Θ_s serves as a sufficient statistic for the incumbent's perception of competition intensity in state s. When Θ_s is low, the incumbent believes that entry costs are high and thus perceives that even substantial rent extraction will not trigger entry. This effectively relaxes the electoral constraint. Crucially, this premium is inversely proportional to Θ_s, meaning that mayors who believe they face little competitive threat extract substantially higher rents. Since uncontested elections lead to lower beliefs about competition intensity ($\Theta_U < \Theta_C$), this mechanism drives the paper's main empirical prediction that uncontested elections lead to higher subsequent rent extraction. □

B.5 Main propositions

B.5.1 Result 1

Proposition 1 (Cross-state rent differential)**.** *In the stationary equilibrium, politicians extract higher rents after uncontested elections than after contested elections.*

$$r_U^* - r_C^* = \frac{1}{2\beta(V_C^P)^2}\left[\frac{1}{\Theta_U} - \frac{1}{\Theta_C}\right] > 0 \tag{17}$$

This rent differential arises from the informational content of electoral outcomes. An uncontested election signals high entry barriers, leading the incumbent to perceive weaker future competitive threats. The magnitude of the differential depends on the gap in perceived competition intensity between the two states, captured by the difference in $1/\Theta$. The term $(V_C^P)^2$ in the denominator shows that the rent differential increases with the square of office value, amplifying the stakes of electoral competition. The discount factor β moderates this effect, as more patient incumbents extract smaller rent premiums to preserve future electoral advantages.

Proof. From Lemma 1, in the stationary equilibrium we have

$$r_s^* = \bar{r}_e - \frac{V_U^P}{2V_C^P} + \frac{1}{2\beta\,\Theta_s\,(V_C^P)^2}$$

The first two terms are identical across states $s \in \{U, C\}$. The differential arises entirely from the third term, which depends on the state-specific belief parameter Θ_s.

The learning mechanism operates as follows. After an uncontested election, Bayesian updating yields $\mu_U < \mu_C$. Specifically, the absence of a challenger signals that entry costs are

likely high, leading to a lower belief that entry costs are low. Conversely, the presence of a challenger in a contested election signals that entry costs are sufficiently low to make challenging worthwhile, leading to a higher belief that entry costs are low.

Since $\Theta(\mu) = \mu/\bar{K}_L + (1-\mu)/\bar{K}_H$ is increasing in μ, we have $\Theta_U < \Theta_C$. Intuitively, when the incumbent believes entry costs are more likely to be low, the expected entry probability for any given rent level increases, captured by higher Θ.

The rent differential follows directly from the optimal rent formula

$$\begin{aligned} r_U^* - r_C^* &= \left[\bar{r}_e - \frac{V_U^P}{2V_C^P} + \frac{1}{2\beta\,\Theta_U\,(V_C^P)^2}\right] - \left[\bar{r}_e - \frac{V_U^P}{2V_C^P} + \frac{1}{2\beta\,\Theta_C\,(V_C^P)^2}\right] \\ &= \frac{1}{2\beta(V_C^P)^2}\left[\frac{1}{\Theta_U} - \frac{1}{\Theta_C}\right] \end{aligned} \tag{18}$$

Since $\Theta_U < \Theta_C$ implies $1/\Theta_U > 1/\Theta_C$, and with $V_C^P > 0$ and $\beta > 0$, we have $r_U^* - r_C^* > 0$. □

B.5.2 Result 2

Assumption 1 (Interior path)**.** For all $\mu \in (0, 1]$ the no-entry path satisfies

$$\tfrac{1}{2} - \bar{r}_e + r_U^*(\mu) \in (0, 1) \tag{19}$$

Equivalently, along the path one has $0 < p_H(r_U^*(\mu)) < p_L(r_U^*(\mu)) < 1$.

Proposition 2 (Belief convergence under repeated non-entry)**.** *Fix time-invariant primitives and Assumption 1. Let the incumbent set $r_t = r_U^*(\mu_t)$ each term and suppose that n consecutive elections are uncontested. Then the posterior sequence $\{\mu_{U,n}\}$, updated via $\mu_{U,n+1} = B(\mu_{U,n}, r_U^*(\mu_{U,n}))$, is strictly decreasing and $\mu_{U,n} \to 0$ as $n \to \infty$.*

Proof. Write $q_j(r) = 1 - p_j(r)$ for $j \in \{L, H\}$ and define the no-entry posterior

$$B(\mu, r) = \frac{\mu\, q_L(r)}{\mu\, q_L(r) + (1-\mu)\, q_H(r)} \tag{20}$$

Assumption 1 gives $p_L(r) > p_H(r)$ and thus $q_L(r) < q_H(r)$. Hence $B(\mu, r) < \mu$ for any $\mu \in (0, 1)$. Along the policy $r_t = r_U^*(\mu_t)$ one has $\mu_{t+2} = B(\mu_{t+1}, r_U^*(\mu_{t+1})) < \mu_{t+1}$, so $\{\mu_{U,n}\}$ is strictly decreasing and bounded below by 0. Let L be its limit. Continuity of B and of $r_U^*(\cdot)$ implies $L = B(L, r_U^*(L))$. Since $B(\mu, r) < \mu$ for every interior μ, no interior fixed point exists. The sequence is strictly decreasing whenever $\mu_0 < 1$, so $L \neq 1$. Therefore $L = 0$. □

Lemma 2 (Path properties under repeated non-entry)**.** *Under Assumption 1, the sequence $\{\mu_{U,n}\}_{n\geq 0}$ satisfies the following properties.*

(1) The sequence exhibits monotonicity, with $\mu_{U,n+1} < \mu_{U,n}$ for all n.

(2) The sequence converges to zero, that is, $\mu_{U,n} \to 0$ as $n \to \infty$.

(3) The updates diminish over time, satisfying $|\mu_{U,n+1} - \mu_{U,n}| \to 0$ as $n \to \infty$.

Proof. For (1), since $B(\mu, r) < \mu$ for all $\mu \in (0,1)$ as established in Proposition 2, and since $\mu_{U,n+1} = B(\mu_{U,n}, r^*_U(\mu_{U,n}))$, we have $\mu_{U,n+1} < \mu_{U,n}$ for all n. Property (2) follows directly from Proposition 2. For (3), the monotone convergence of a bounded sequence implies that the differences between consecutive terms must converge to zero, hence $|\mu_{U,n+1} - \mu_{U,n}| \to 0$. □

Proposition 3 (Monotone convergence of rent dynamics)**.** *Under Assumption 1 and along a run of uncontested elections, the optimal rent path $\{r^*_{U,n}\}$ exhibits monotone convergence. Specifically, the following properties hold.*

1. *The rent path converges to $r^*_{U,\infty}$ for some finite $r^*_{U,\infty}$.*
2. *The rent increments vanish, with $\lim_{n\to\infty} \left|r^*_{U,n+1} - r^*_{U,n}\right| = 0$.*
3. *The rent increments decrease monotonically, satisfying $r^*_{U,n+2} - r^*_{U,n+1} < r^*_{U,n+1} - r^*_{U,n}$ for all n.*

Proof. From Lemma 1 one has

$$r^*_{U,n} = \bar{r}_e - \frac{V^P_{U,t+1}}{2V^P_{C,t+1}} + \frac{1}{2\beta(V^P_{C,t+1})^2} \cdot \frac{1}{\Theta_{U,n}}$$
$$\Theta_{U,n} = \frac{\mu_{U,n}}{\bar{K}_L} + \frac{1-\mu_{U,n}}{\bar{K}_H}$$

By Lemma 2, we know that $\mu_{U,n} \to 0$, which implies $\Theta_{U,n} \to 1/\bar{K}_H$. Therefore, $r^*_{U,n}$ converges to a finite limit. The continuity of the rent function with respect to beliefs ensures that $\left|r^*_{U,n+1} - r^*_{U,n}\right| \to 0$.

To establish monotone convergence, we examine the convexity of the rent function. The mapping $\mu \mapsto \frac{1}{\Theta(\mu)}$ is strictly convex, as its second derivative

$$\frac{d^2}{d\mu^2}\left(\frac{1}{\Theta(\mu)}\right) = 2\left(\frac{1}{\bar{K}_L} - \frac{1}{\bar{K}_H}\right)^2 \cdot \frac{1}{\Theta(\mu)^3} > 0 \tag{21}$$

is positive for all $\mu \in (0,1)$. This convexity, combined with the monotone decrease of $\mu_{U,n}$ established in Lemma 2, guarantees that the rent increments $r^*_{U,n+1} - r^*_{U,n}$ decrease monotonically over time. Hence, the convergence occurs without oscillation, completing the proof. □

The monotone convergence without oscillation reflects the smooth Bayesian learning process. As the incumbent accumulates uncontested victories, each additional signal provides diminishing information about the competitive environment, leading to progressively smaller

salary adjustments. The convexity of the rent function in beliefs ensures this adjustment path is smooth rather than erratic.