



Review of the Fiscal Theory of the Price Level

April 25, 2022

Hidekazu Niwa*

Assistant Professor, Osaka School of International Public Policy (OSIPP)

【JEL Classification】 E58, E63, E31

【Keywords】 Fiscal Theory of the Price Level; Fiscal-Monetary Interaction; Government Solvency

【Abstract】 This study reviews the fiscal theory of the price level (FTPL). Our goal is to briefly explain the following three points. First, how is an equilibrium determined in a simple model where prices are perfectly flexible and only one-period government bonds? Second, what is the intuition for equilibrium determination? Third, how does introducing long-term bonds or nominal price rigidities change results in the simplest case?

* Email: niwa@osipp.osaka-u.ac.jp

This paper is a revised version of the second chapter of my Ph.D. dissertation at Hitotsubashi University. I am deeply grateful to Etsuro Shioji for his special advice and encouragement. Any remaining errors are the sole responsibility of the author.

1 Introduction

This study reviews the fiscal theory of the price level (FTPL). The foundation of this literature is developed by Leeper (1991), Sims (1994), Woodford (1995, 1996, 2001), and Cochrane (2001). They stress the importance of a role that expectations about the future conduct of fiscal policy play in determining the equilibrium price level. In standard models for monetary policy analysis, it is implicitly assumed that the fiscal authority commits to adjusting the present discounted value (PDV) of primary surpluses to ensure solvency of the consolidated government. Therefore, the central bank achieves independent control of inflation by setting the short-term nominal interest rate appropriately. However, if the fiscal authority does not make such adjustments, the equilibrium price level must be determined so as to maintain the sustainability of public debt.

The objective of this study is to briefly explain the following three points. First, how is an equilibrium determined in a simple model in which prices are perfectly flexible and all government bonds are one-period? Second, what is the intuition for the equilibrium determination? Third, how does introducing long-term bonds or nominal price rigidities change results in the simplest case? We provide a textbook-style explanation by using a toy model. The discussion in this study is mainly based on Woodford (1995, 1996, 2001), Cochrane (2001, 2005, 202), Iwamura and Watanabe (2004), and Shioji (2018).

2 Two-period Model

We work with a deterministic two-period model with sticky prices. The two periods are $t = 0, 1$. The economy is populated by a representative household, a continuum of firms in the unit interval, the fiscal authority, and the central bank. In period 0, firms face adjustment costs in changing their prices so that prices are rigid. In period 1, they can change their prices at no costs so that prices are perfectly flexible. The fiscal authority issues government bonds, which are held by the households. At the beginning of period 0, the household holds initial wealth one-period and long-term government bonds. The important assumption in the FTPL is that the fiscal authority does not adjust primary surpluses to maintain government solvency. The central bank controls the short-term nominal interest rate. Finally, it should be noted that the model described below encompasses the simplest case in which prices are perfectly flexible, and all government bonds are one-period as a special case.

2.1 Households

The representative household has the following utility function

$$\log(C_0) - N_0 + \beta[\log(C_1) - N_1], \quad (1)$$

where C_t is an aggregate of consumption, N_t is labor supplied, and $\beta \in (0,1)$ is the discount factor. The aggregate consumption C_t is defined as

$$C_t \equiv \left[\int_0^1 c_t(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}, \quad (2)$$

where $c_t(j)$ denotes the quantity of good $j \in [0,1]$ consumed by the households. $\theta > 1$

parameterizes the elasticity of substitution across goods. The aggregate price index is

$$P_t \equiv \left[\int_0^1 p_t(j)^{1-\theta} dj \right]^{\frac{1}{1-\theta}}, \quad (3)$$

where $p_t(j)$ denotes the price of good j .

At the beginning of period 0, the household holds one-period, risk-free nominal bonds B_{-1} and long-term bonds D_{-1} . Long-term government bonds outstanding at the beginning of period 0 pay one dollar in period 0 and ρ dollars in period 1. In a special case with $\rho = 0$, all government bonds outstanding in period 0 are one-period. The price of a one-period government bond is denoted by $1/(1 + i_t)$, where i_t is the short-term nominal interest rate, and Q_t denotes price of long-term bonds. The household earns labor income $N_t W_t$, where W_t is the nominal wage, receives profits $Z_t(j)$ from firm j , and pays lump-sum taxes T_t . Since all government bonds newly issued in period 0 are one-period, the budget constraint is given by

$$P_0 C_0 + \frac{B_0}{1 + i_0} \leq B_{-1} + (1 + Q_0) D_{-1} + W_0 N_0 + \int_0^1 Z_0(j) dj - P_0 T_0, \quad (4)$$

The budget constraint in period 1 can be written as

$$P_1 C_1 + \frac{B_1}{1 + i_1} \leq B_0 + (\rho + Q_1) D_{-1} + W_1 N_1 + \int_0^1 Z_1(j) dj - P_1 T_1. \quad (5)$$

The household is also subject to a constraint that prevents it from dying with debt at the end of period 1:

$$\frac{B_1}{1 + i_1} \geq 0. \quad (6)$$

2.2 Firms

There is a continuum of monopolistically competitive firms indexed by $j \in [0,1]$. Firm j produces goods using the production technology

$$y_t(j) = n_t(j). \quad (7)$$

where $n_t(j)$ is the labor hired. Firm j faces a downward-sloping demand curve given by

$$y_t(j) = \left(\frac{p_t(j)}{P_t} \right)^{-\theta} Y_t. \quad (8)$$

where $Y_t \equiv \left[\int_0^1 y_t(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}$ denotes aggregate output. In period 0, firms face adjustment costs in changing their prices. Following Rotemberg (1982), firm j faces price adjustment costs:

$$\frac{\gamma}{2} \left(\frac{p_0(j)}{p_{-1}(j)} - 1 \right)^2 Y_0. \quad (9)$$

γ is the parameter that controls the degree of nominal price rigidities. When $\gamma = 0$, firms can change their prices at no costs so that prices are perfectly flexible in both periods.

The profits of firm j in period 0 is then expressed as

$$Z_0(j) = \left[p_0(j)y_0(j) - W_0y_0(j) - P_0 \frac{\gamma}{2} \left(\frac{p_0(j)}{p_{-1}(j)} - 1 \right)^2 Y_0 \right]. \quad (10)$$

As in period 1, there are no price adjustment costs; profits in this period can be rewritten as

$$Z_1(j) = [p_1(j)y_1(j) - W_1y_1(j)]. \quad (11)$$

2.3 Government

The fiscal authority imposes lump-sum taxes on the households and issues bonds, one-period bonds B_t^F and long-term bonds D_t^F , respectively. The flow government budget constraints in each period are given by

$$(1 + Q_0)D_{-1}^F + B_{-1}^F = \frac{B_0^F}{1 + i_0} + P_0T_0, \quad (12)$$

$$(\rho + Q_1)D_{-1}^F + B_0^F = \frac{B_1^F}{1 + i_1} + P_1T_1. \quad (13)$$

They can be rewritten in real terms:

$$\frac{1}{P_0} [(1 + Q_0)D_{-1}^F + B_{-1}^F] = \frac{b_0^F}{1 + i_0} + T_0, \quad (14)$$

$$\frac{1}{P_1} [(\rho + Q_1)D_{-1}^F + B_0^F] = \frac{b_1^F}{1 + i_1} + T_1, \quad (15)$$

where $b_t^F \equiv B_t^F / P_t$ is the real value of one-period government bonds.

The central bank controls the short-term nominal interest rate i_t .

2.4 Equilibrium conditions

This subsection describes the equations that characterize an equilibrium allocation. The household maximizes its lifetime utility (1) subject to the budget constraints (4) and (5) and the no-Ponzi game condition (6). The Euler equation in period 0 is given by

$$\frac{1}{1+i_0} = \beta \frac{C_0}{C_1} \Pi_1^{-1}, \quad (16)$$

where $\Pi_t \equiv P_t/P_{t-1}$ is the gross inflation. The optimality condition for labor supply is

$$\frac{W_t}{P_t} = C_t \text{ for } t = 0,1. \quad (17)$$

Given the no-arbitrage condition between one-period and long-term bonds, the price of long-term bonds must satisfy

$$Q_0 = \frac{1}{1+i_0} (\rho + Q_1), \quad (18)$$

$$Q_1 = 0 \quad (19)$$

Finally, since the household has no incentive to save in period 1, the household optimization requires that the following terminal condition holds:

$$\frac{B_1}{1+i_1} = 0 \quad (20)$$

Next, firms' profits maximization is considered. Firm j sets its price in each period $\{p_0(j), p_1(j)\}$ to maximize profits (10) and (11) subject to the downward-sloping demand curve (8). Deriving the first-order conditions in each period and focusing on a symmetric equilibrium where $p_t(j) = P_t$, the following conditions are obtained:

$$(1 - \theta) + \theta C_0 = \gamma \Pi_0 (\Pi_0 - 1), \quad (21)$$

$$C_1 = 1 - \theta^{-1}. \quad (22)$$

Equation (21) shows that when $\gamma = 0$, C_0 is also fixed at $1 - \theta^{-1}$. The Phillips curve in period 1 is vertical as prices are perfectly flexible.

Clearing in the goods market requires that

$$\left[1 - \frac{\gamma}{2}(\Pi_0 - 1)^2\right] Y_0 = C_0 \quad (23)$$

$$Y_1 = C_1. \quad (24)$$

Equilibrium in the bonds market requires that

$$B_t^F = B_t \quad \text{for } 0, 1 \quad (25)$$

$$D_{-1}^F = D_{-1}. \quad (26)$$

To complete the characterization of the equilibrium, we need to specify rules according to which fiscal and monetary policies are chosen. We assume that the fiscal authority pre-commits to a certain sequence of primary surpluses as follows:

$$T_t = \bar{T}_t \quad \text{for } 0, 1. \quad (27)$$

This means that the fiscal authority does not make fiscal adjustments needed to maintain government solvency, unlike standard models. As explained, this assumption plays a critical role in the FTPL. The central bank also exogenously sets the short-term nominal interest rate in period 0:

$$i_0 = \bar{i}_0. \quad (28)$$

Since the fiscal authority does not adjust primary surpluses, government solvency condition is relevant; endogenous variables must be determined so as to ensure government

solvency. Combining flow government budget constraints in each period, (14) and (15), and imposing the terminal condition (20), the government solvency condition in period 0 is obtained:

$$\begin{aligned} \frac{1}{P_0} [(1 + Q_0)D_{-1} + B_{-1}] &= \bar{T}_0 + \left(\frac{\Pi_1}{1 + i_0} \right) \bar{T}_1 \\ &= \bar{T}_0 + \beta \left(\frac{C_0}{C_1} \right) \bar{T}_1 \end{aligned} \tag{29}$$

When deriving equation (29), we have used the bonds market clearing condition (25) and (26) and the Euler equation (16) that describes the relationship between the real interest rate and the path of consumption. Imposing the terminal condition (20) on government budget constraint (15) gives the government solvency condition in period 1:

$$\frac{1}{P_1} [(\rho + Q_1)D_{-1} + B_0] = \bar{T}_1. \tag{30}$$

Equilibrium in this model is a collection of process $\{Y_t, C_t, \Pi_t, Q_t\}_{t=0}^1$ that satisfy (18), (19), (21)–(24), (29), and (30) given the predetermined variables $\{B_{-1}, D_{-1}, P_{-1}\}$ and policy variables $\{\bar{T}_0, \bar{T}_1, \bar{i}_0\}$. We need to choose a value for P_{-1} to uniquely determine the equilibrium path of the inflation rate.

3 The Simplest Case in Which $\gamma = \rho = 0$

Using the above model, we study how the equilibrium is determined. First of all, the simplest case in which $\gamma = \rho = 0$ is considered: we assume that prices are perfectly flexible and all government bonds are one-period. This is a simplified version of models

considered by Leeper (1991), Sims (1994), and Woodford (2001). They study flexible-price models of endowment economy.²

3.1 Equilibrium determination

In a case with $\gamma = 0$, the goods market clearing condition in period 0 (23) can be rewritten as

$$Y_0 = C_0 \tag{31}$$

Since $\gamma = 0$ implies that $C_0 = 1 - \theta^{-1}$, the government solvency condition in period 0 (30) can be expressed as

$$\frac{1}{P_0}(D_{-1} + B_{-1}) = \bar{T}_0 + \beta\bar{T}_1. \tag{32}$$

This is the key condition to uniquely determining the equilibrium price level. Since in the case of flexible-prices, the real interest rate is fixed at $\beta^{-1} - 1$, the PDV of primary surpluses $\bar{T}_0 + \beta\bar{T}_1$ is unchanged. The nominal value of outstanding government bonds $B_{-1} + D_{-1}$ is predetermined in period 0. We can thus uniquely determine the current price level P_0 to satisfy the government solvency condition (32). In other words, P_0 is determined to equate the real value of outstanding government bonds and the PDV of primary surpluses. The important point here is that as the fiscal authority does not adjust primary surpluses, the endogenous variables (the current price level in this simplest case) should adjust so as to maintain government solvency in the equilibrium.

² To be precise, they consider infinite-horizon models in which money is demanded.

3.2 Intuition I: Government bonds as net wealth

What is the intuition for the equilibrium determination? To examine this, we focus on the optimizing decision of the household. Combining the flow budget constraints in each period (4) and (5) gives the household's intertemporal budget constraint

$$C_0 + \beta \left(\frac{C_0}{C_1} \right) C_1 \leq B_{-1} + D_{-1} + W_0 N_0 + \int_0^1 Z_0(j) dj + \beta \left(\frac{C_0}{C_1} \right) \left(W_1 N_1 + \int_0^1 Z_1(j) dj \right) - \left[\bar{T}_0 + \beta \left(\frac{C_0}{C_1} \right) \bar{T}_1 \right]. \quad (33)$$

Moreover, imposing the optimality conditions of the households (17) and (20), the optimality conditions of firms (21) and (22), and goods market clearing conditions (24) and (31), we obtain the following condition:

$$\frac{1}{P_0} (D_{-1} + B_{-1}) + (1 + \beta)(\theta - 1) - (\bar{T}_0 + \beta \bar{T}_1) = C_0 + \beta C_1. \quad (34)$$

Note that equation (34) is one of the equilibrium conditions as it contains information about the optimality conditions of the private sector.

The condition (34) is informative about how an increase in outstanding government bonds $D_{-1} + B_{-1}$ affects the optimizing decision of the household. Recall that since the fiscal authority is assumed not to adjust primary surpluses regardless of the amount of bonds outstanding, the PDV of primary surpluses $\bar{T}_0 + \beta \bar{T}_1$ is unchanged. An increase in outstanding bonds then leads to an increase in the PDV of lifetime income of the household, which is given by the left-hand side of the condition (34), and then stimulates the household's demand for goods. In other words, an increase in outstanding bonds induces a

positive wealth effect on the household. This expands aggregate demand and thereby requiring a rise in the price level P_0 . Since the aggregate supply of goods is fixed, the equilibrium condition (34) is restored solely by a change in the price level P_0 .

The above discussion confirms that when the fiscal authority does not adjust primary surpluses, the Ricardian equivalence does not hold. Indeed, in the FTPL, an increase in outstanding bonds induces a change in the optimizing decision of the household and therefore affects the equilibrium price level.

3.3 Intuition II: Stock analogy discussed by Cochrane (2005)

In this section, we introduce another explanation for the economic mechanism behind the adjustment in the price level in the FTPL, which is highlighted in Section 3.1. Cochrane (2005) draws an analogy between the FTPL in which the equilibrium price level is endogenously determined to maintain government solvency—and the theory of stock price determination.³ More specifically, he argues that government bonds, including monetary base, share a similar property with stocks, which is the security that private corporations issue with a promise of future dividends. As well-known, the stock price is determined to equate its market value (stock price \times number of stocks) and the PDV of future dividends. This implies that stock prices reflect how market participants evaluate the ability of the corporation to make profits in the future. For example, a decrease in the PDV of future dividends leads to a decline in stock price.

³ His idea is also introduced by Iwamura and Watanabe (2004), Shioji (2018), and Cochrane (2019).

One of the main messages of Cochrane (2005) is that the same logic determines the price of government bonds (the inverse of the price level) in the FTPL. He writes that “The fiscal theory of the price level recognizes that nominal debt, including the monetary base, is a residual claim to government primary surpluses, just as Microsoft stock is a residual claim to Microsoft’s earnings” (p.502). As explained in the previous section, in the FTPL, the price of government bonds is endogenously determined so as to equate the real value of government bonds and the PDV of primary surpluses. In this sense, the price of government bonds reflects how the public evaluates the government’s ability (or willingness) to raise primary surpluses in order to return goods to bond holders in the future. Indeed, the government solvency condition in period 0 (32) shows that when the PDV of primary surpluses decreases, the real value of government bonds declines (the price level increases). In other words, in the FTPL, the real value of government bonds is endogenously determined by public confidence in the government.

4 Case with Long-Term Bonds

Next, the case in which $\gamma = 0$ and $\rho > 0$ is considered; we assume that long-term bonds are outstanding at the beginning of period 0. Cochrane (2001) and Woodford (2001) study the FTPL with long-term bonds. For simplicity, prices are assumed to be perfectly flexible, as in the previous case.

4.1 Equilibrium determination

When $\rho > 0$, the government solvency condition can be written as

$$\begin{aligned} \frac{1}{P_0} [(1 + Q_0)D_{-1} + B_{-1}] &= \frac{1}{P_0} \left[\left(1 + \frac{\rho}{1 + \bar{i}_0} \right) D_{-1} + B_{-1} \right] \\ &= \bar{T}_0 + \beta \bar{T}_1 \end{aligned} \tag{35}$$

As prices are perfectly flexible, the real interest rate is fixed, and so is the PDV of primary surpluses. The important point in the case with long-term bonds is that the nominal value of outstanding government bonds $(1 + Q_0)D_{-1} + B_{-1}$ is no longer predetermined in period 0. The reason is that the price of long-term bonds Q_0 depends on the current short-term nominal interest rate \bar{i}_0 . Then, the current price level P_0 is uniquely determined to satisfy the government solvency condition (35), given the policy variables $\{\bar{T}_0, \bar{T}_1, \bar{i}_0\}$ and the predetermined variables $\{B_{-1}, D_{-1}\}$. It is worth noting that when long-term bonds are outstanding, not only fiscal policy but also monetary policy plays a role in determining the price level since it affects the price of long-term bonds.

4.2 Numerical illustration

This section presents a numerical example to show how a change in the price of long-term bonds affects the dynamics of the equilibrium price level. We adopt $\beta = 0.5$ and $\theta = 10$. Outstanding bonds in period 0 are given by $B_{-1} = 1$ and $D_{-1} = 1$. The price level in the period - 1 is set to $P_{-1} = 1$. Primary surpluses in periods 0 and 1 are set to $\bar{T}_0 = \bar{T}_1 = 1$.

Figure 1 displays a numerical example of the price level in periods 0 and 1. We report the results for alternative values of \bar{i}_0 . When the nominal interest rate is increased, the price

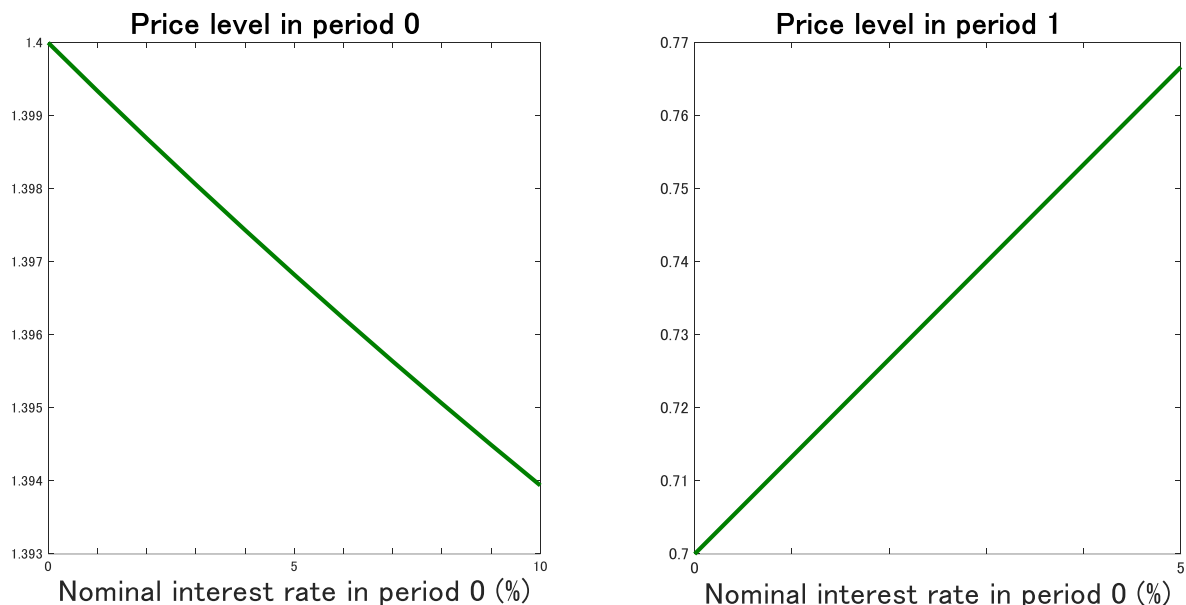


Figure 1. A numerical example of the price level in periods 0 and 1 at alternative values of \bar{t}_0 .

of long-term bonds declines. This leads to a decrease in the price level in period 0 and an increase in the price level in period 1. This result suggests that when long-term bonds are outstanding, the government can choose the timing of inflation needed to maintain its solvency by changing the price of bonds. For example, lowering the price of long-term bonds, which corresponds to a higher price level in the next period, reduces the reliance on current inflation.

An economic mechanism through which a decline in the price of bonds leads to a lower price level is clear in light of the fact that, in the FTPL, government bonds are net wealth for the households. When the fiscal authority does not adjust primary surpluses, a decline in the price of long-term bonds held by households induces negative wealth effects. This reduces aggregate demand and therefore lowers the price level.

5 Case with Nominal Rigidities

Finally, the case in which $\gamma > 0$ and $\rho = 0$ is considered; we assume that prices are rigid in period 0. We analyze this case following Woodford (1996), the first study that incorporate the FTPL framework into a New Keynesian framework. When prices are rigid, a fluctuation of aggregate demand is not absorbed entirely by a change in the price level. This also requires a variation in the level of real economic activities and then in the real interest rate. For simplicity, it is assumed that all government bonds are one period.

5.1 Equilibrium determination

When $\gamma > 0$ and $\rho = 0$, the government solvency condition in period 0 is given by

$$\begin{aligned}\frac{1}{P_0} [D_{-1} + B_{-1}] &= \bar{T}_0 + \left(\frac{\Pi_1}{1 + i_0} \right) \bar{T}_1 \\ &= \bar{T}_0 + \beta \left(\frac{C_0}{C_1} \right) \bar{T}_1\end{aligned}\tag{36}$$

In a case with $\rho = 0$, the nominal value of outstanding bonds $D_{-1} + B_{-1}$ is predetermined in period 0. The important point is that, in contrast to the previous two cases, due to nominal rigidities, the government solvency condition cannot be satisfied solely by a change in the price level P_0 . A change in the real interest rate is also needed. Given that the level of consumption in period 1 C_1 is fixed, not only the price level P_0 but also consumption C_0 should adjust to satisfy the government solvency condition (36).

Therefore in the case with nominal rigidities, the equilibrium price level is not uniquely determined solely by the government solvency condition, unlike in the two previous cases. We need another equilibrium condition to be combined with the government solvency condition to determine the price level and consumption. The Phillips curve (21) describes the relationship between inflation and consumption in period 0. The price level P_0 and consumption C_0 are jointly determined to satisfy both the Phillips curve (21) and the government solvency condition (36), given the policy variables $\{\bar{T}_0, \bar{T}_1, \bar{\tau}_0\}$ and the predetermined variables $\{B_{-1}, D_{-1}, P_{-1}\}$.

5.2 Numerical illustration

This section presents a numerical example to show how a change in the degree of nominal rigidities affects inflation and real interest rate in period 0. We use the same values for β , θ , B_{-1} , D_{-1} , \bar{T}_0 , \bar{T}_1 , and P_{-1} as in Section 4.2, and adopt $\rho = 0.1$.

Figure 2 displays the price level, the real interest rate, the net inflation rate, and consumption in period 0. We report results for ten values of γ , $\gamma \in \{0, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100\}$. The numerical result shows that when prices are stickier (i.e., as γ becomes larger), a larger decline in the real interest rate is needed to maintain government solvency. Given more sluggish adjustments in the price level, the government must put more reliance on a decline in the real interest rate to maintain its solvency.

It is also worth noting that consumption increases as prices are stickier. As explained in the FTPL, the outstanding government bonds put upward pressure on aggregate demand

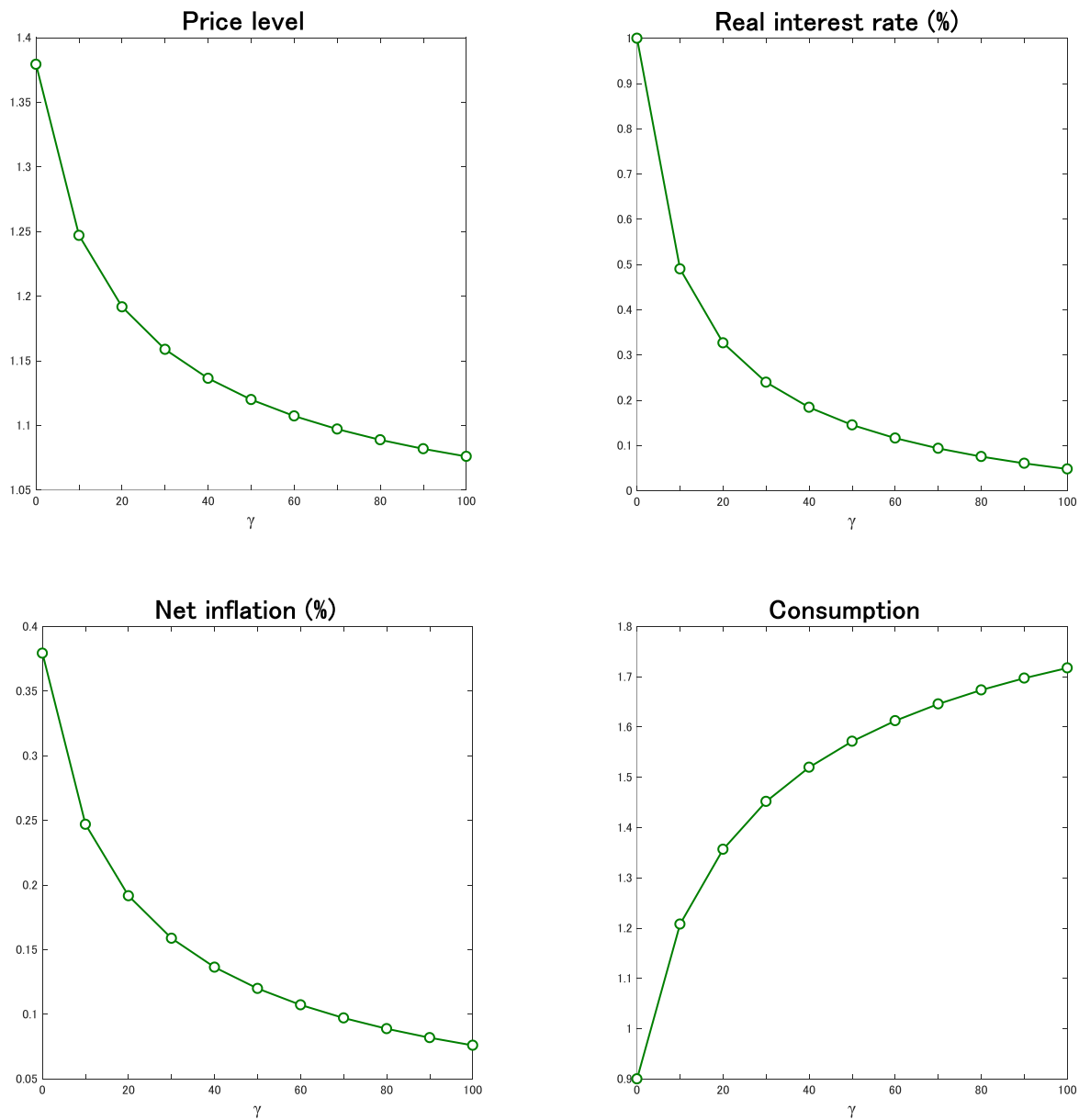


Figure 2. A numerical example of the price level, the real interest rate, the net inflation rate, and consumption in period 0 at alternative values of γ .

through the positive wealth effect. Given more sluggish adjustments in prices, an increase in consumption is needed to ensure goods market clearing.

6 Concluding Remarks

This study briefly reviewed the FTPL using a simple model. We have studied how the equilibrium is determined in three cases: (i), the simplest case in which prices are perfectly flexible, and all government bonds are one-period, (ii), the case with long-term bonds, and (iii), the case with nominal rigidities. The important point common to the three cases is that in the FTPL the Ricardian equivalence does not hold so that an increase in outstanding bonds induces a positive wealth effect on households and therefore affects the equilibrium allocations. Again, the assumption that the fiscal authority does not make fiscal adjustments needed to maintain government solvency plays a critical role.

References

- Barro, R. J. (1974). “Are Government Bonds Net Wealth?.” *Journal of Political Economy*, 82(6), 1095-1117.
- Cochrane, J. H. (2001). “Long-Term Debt and Optimal Policy in the Fiscal Theory of the Price Level.” *Econometrica*, 69(1), 69-116.
- Cochrane, J. H. (2005). “Money as Stock.” *Journal of Monetary Economics*, 52(3), 501-528.
- Cochrane, J. H. (2022). “The Fiscal Theory of the Price Level,” Book Draft.
- Iwamura, M., and Watanabe, T. (2004). *Atarashii bukka riron*. Iwanami Shoten, Tokyo (in Japanese).

- Leeper, E. M. (1991). "Equilibria under "Active" and "Passive" Monetary and Fiscal Policies." *Journal of Monetary Economics*, 27(1), 129-147.
- Rotemberg, J. J. (1982). "Sticky Prices in the United States." *Journal of Political Economy*," 90(6), 1187-1211.
- Sargent, T. J., and Wallace, N. (1981). Some Unpleasant Monetarist Arithmetic. Federal Reserve Bank of Minneapolis Quarterly Review, 5(3), 1-17.
- Shioji, E. (2018). "Fiscal Theory of the Price Level and Unconventional Fiscal and Monetary Policies: Overview." PRI Discussion Paper Series. No.18A-07 (in Japanese).
- Sims, C. A. (1994). "A Simple Model for Study of the Determination of the Price Level and the Interaction of Monetary and Fiscal Policy." *Economic Theory*, 4(3), 381-399.
- Sims, C. A. (2013). "Paper Money." *American Economic Review*, 103(2), 563-84.
- Walsh, C. (2017), *Monetary Theory and Policy*, MIT Press.
- Woodford, M. (1995). "Price-Level Determinacy without Control of a Monetary Aggregate," in *Carnegie-Rochester Conference Series on Public Policy*, 43, 1-46.
- Woodford, M. (1996). "Control of the Public Debt: A Requirement for Price Stability?." NBER Working Paper No. 5684.
- Woodford, M. (2001). "Fiscal Requirements for Price Stability." *Journal of Money, Credit, and Banking*, 33(3), 669.