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Delegation of Taxation Authority and Multipolicy Commitment in a Decentralized Leadership Model*

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Nobuo Akai†

Professor, Osaka School of International Public Policy (OSIPP)

Takahiro Watanabe‡

PhD. Student, Graduate School of Economics, Osaka University

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【Abstract】This paper examines to what extent taxation authority should be delegated to local or lower-level government. Delegation of taxation authority can be regarded as a commitment to the local tax rate ex ante in a decentralized leadership model, in which local governments set policies ex ante and the central government decides transfer policies ex post. Previous papers point out that the ex post interregional transfers of the central government distort the ex ante regional policies of local governments. However, Silva (2014, 2015) clarify the case where efficient expenditure by local governments is achieved. This paper examines the delegation of taxation authority by extending Silva's model to include commitment to taxation and generally derives the conditions when efficient public expenditure by local governments can be achieved in relation to the delegation of taxation authority. The model adopted in this paper allows various levels of spillovers of local public goods and various types of multipolicy commitments of taxation and/or expenditure.

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† Corresponding author. E-mail: akai@osipp.osaka-u.ac.jp

‡ E-mail: sge028wt@student.econ.osaka-u.ac.jp

1 Introduction

Since the 1980s, government decentralization has occurred in many countries through the spreading of international liberalism and democracy, and efficient management of the government sector. As part of this process, taxation and expenditure authority has been delegated to local or lower-level governments.

There exist various rules for the setting of local tax rates across countries. For example, in Japan, which is a unitary state, the local tax system is uniformly designed by the central government based on the Local Tax Act and there is no discretion for local governments even for local consumption taxes. Alternatively, in the United States, which is a federal state, each state government can freely set the local tax rate. In the European Union, the value-added tax rate can be set differently across countries, but the standard tax rate is regulated to be at least 15%. However, it is natural in the EU that after delegating taxation authority to local or lower-level governments, the central or upper-level government still has an incentive to affect the regional resource allocation through ex post transfers among regions/countries. Such ex post interregional transfers may be designed to compensate for a lack of financial resources.

When the central government sets transfers ex post by considering differences in marginal utilities among regions from a social perspective, it might transfer resources from rich regions with lower marginal utilities to poor regions with higher marginal utilities. Given such ex post transfers, local governments will set tax rates that are too low (excessive consumption) or undertake excessive expenditure ex ante, if the authority to set tax rates or expenditure ex ante is delegated to them and the central government cannot change these effective levels ex post through transfers among regions. This is because they anticipate ex post central transfers to provide the goods that have not been committed to by local governments ex ante.

An important issue in this context is how central governments delegate authority to local governments in the decentralization process and what types of policy should be determined by the local governments. Silva (2014) analyzes the policy choices of local governments and considers national public goods with perfect spillovers as a type of local public good (such as reduction of greenhouse gas emissions) and shows that the first-best allocation can be achieved in the case where only national public goods are provided by local governments ex ante. [1] Silva (2015) analyzes a model with multiple public goods and shows that if the central government optimizes all goods ex post (no commitment by the local governments ex ante) including consumption (through income transfer) by setting transfers among regions ex post, then the first-best allocation can be achieved.

However, Silva (2014, 2015) does not discuss the delegation of taxation authority to local governments. In fact, commitment to the local tax rate is not examined in any previous papers. The question regarding how taxation authority should be delegated can be answered by analyzing a model that allows tax commitment, which is a generalization of Silva (2014, 2015). The present paper does not intend to analyze decentralized leadership. Rather, we aim to provide a general framework to synthesize the different views related to the various types of decentralized leadership models and propose the conditions for deriving efficient policies by local governments in relation to the delegation of taxation authority.

When the authority to set tax rates and/or expenditure levels are delegated to local governments, they can commit to these levels of expenditure and/or residents' consumption through the local income tax rate ex ante. Subsequently, the central government can carry out ex-post transfers to fund the

[1] Some previous studies have analyzed decentralized leadership. Akai and Sato (2008) analyze how the inefficiency of public goods, i.e., whether they are under- or oversupplied, changes depending on the type of commitment. Caplan and Silva (2000, 2011) provide a similar analysis, but focus on the effect of labor mobility.

goods that have not been committed to by local governments ex ante.

For example, suppose that two types of public goods are financed by local income taxes. If local governments are allowed to commit to the amounts supplied of the two public goods but do not commit to the local income tax rate ex ante, the central government will optimize consumption through a change in the local income tax rate by central income transfers among regions ex post. However, in the case where the local governments commit to the amounts supplied of one public good and the tax rate ex ante, the central government will optimize the supply of other public goods through ex-post transfers among regions.

The former case where the local income tax rate is not committed to ex ante is a centralized economy in terms of taxation setting because the local governments have no authority regarding the tax rate decision. However, the latter case where the local income tax rate is committed to ex ante is a decentralized economy in terms of taxation setting because the local tax level can be committed to ex ante by the local government. Therefore, analyzing these cases is useful for determining how to evaluate the decentralization of local taxation authority.

To address these issues, we analyze all plausible cases of policies that local governments can commit to ex ante and clarify the general conditions under which the first-best allocation is achieved, depending on the various levels of spillovers of local public goods and various types of multipolicy commitments.

The main results are as follows. When taxation authority is not delegated and local governments commit to the amounts of two public goods ex ante, as long as both are pure public goods, the first-best allocation is achieved. This result is an extension of the result of Silva (2014). In addition, even when the local governments commit to only the amount of one pure public good ex ante, the first-best allocation is also achieved, regardless of the degree of spillovers of the other uncommitted public good. However, we show that, even if local governments only commit to the amount of one pure public good ex ante, the first-best allocation is not achieved when taxation authority is delegated.

These curious results, which are obtained by analyzing multipolicy commitment to the levels of the tax rate and expenditure, might be useful for considering how taxation authority should be delegated.

This paper is structured as follows. The basic model is explained in Section 2. Several cases are analyzed in Section 3. The results are presented in Section 4. Section 5 concludes the paper.

2 The Model

Our model is based on Silva (2015). Consider an economy with $I \geq 2$ regions. There are I local governments and one central government. We denote the size of the population in region i by n_i and the fixed amount of per capita income by y_i. Define the total population and income in this economy as $\sum_i^I n_i \equiv N$ and $\sum_i^I n_i y_i \equiv Y$. We assume that n_i and y_i may differ between regions, but each region is occupied by representative households, which therefore removes intraregional preference heterogeneity.

Local government i provides two local public goods, g_{1i} and g_{2i}, which are measured in per capita terms and may generate interregional spillovers. Local government i levies a lump sum tax t_i on all residents who live in region i and this revenue is used for the provision of the local public goods g_{1i} and g_{2i}. In the presence of central transfers for private consumption, the consumer's budget constraint becomes:

$$c_i + t_i = y_i + s_{ci}, \ i = 1, \cdots, I \tag{1}$$

where s_{ci} is the per capita income transfer allocated to region i. When local government i commits to t_i, then $s_{ci} = 0$. Therefore, ci is also committed to when t_i is committed to.

Suppose that the fiscal transfers are "earmarked," which means they are used for a specific purpose.[2] Thus, the public-good-specific budget constraint for the supply of public good m is:

$$e_{mi} = g_{mi} - s_{mi}, \;\; i = 1, \cdots, I, \;\; m = 1, 2 \tag{2}$$

where $s_{mi}, \; m = 1, 2$ denotes the per capita subsidy (or tax) for the supply of public good m from the central government to region i and g_{mi} is the actual provision of public good m in region i. When local government i commits to g_{mi}, then $s_{mi} = 0$. Furthermore, e_{mi} is the ex ante determined expenditure that the local government i tentatively allocates to the supply of public good m. Therefore, the budget constraint of the local government i is:

$$t_i = (g_{1i} - s_{1i}) + (g_{2i} - s_{2i}) = e_{1i} + e_{2i}, \;\; i = 1, \cdots, I \tag{3}$$

The central budget constraint is:

$$\sum_i^I n_i s_{ci} = 0, \;\; \sum_i^I n_i s_{1i} = 0, \;\; \sum_i^I n_i s_{2i} = 0 \tag{4}$$

s_{ci}, s_{1i}, and s_{2i} can be either positive or negative. The central government can control them to pursue its own objective, but cannot commit to the transfer policies; therefore, the transfers are optimized ex post.

Given the central and local budget constraints, we can describe the overall resource constraint as follows:

$$\sum_i n_i c_i + \sum_i n_i g_{1i} + \sum_i n_i g_{2i} = \sum_i n_i y_i = Y \tag{5}$$

The residents' preferences are assumed to be expressed by the following local utility function [3]:

$$\begin{aligned} U(c_i, g_{1i}, g_{2i}) = u(c_i) &+ \left\{ (1-\lambda) v(g_{1i}) + \lambda V \left(\sum_j^I n_j g_{1j} \right) \right\} \\ &+ \left\{ (1-\mu) h(g_{2i}) + \mu H \left(\sum_j^I n_j g_{2j} \right) \right\}, \;\; i, j = 1, \cdots, I \end{aligned} \tag{6}$$

where $c_i = y_i - t_i + s_{ci}$ is private consumption, and λ and μ are parameters that represent the degree of spillovers of g_{1i} and g_{2j}. $u(c_i)$ represents utility from consumption. $(1-\lambda)v(g_{1i})$ and $(1-\mu)h(g_{2i})$ are the direct effect, and $\lambda V \left(\sum_j^I n_j g_{1j} \right)$ and $\mu H \left(\sum_j^I n_j g_{2j} \right)$ are the spillover effects from g_{1i} and g_{2i} provided by all regions, respectively.

To abstract considerations of political economy and address the commitment problem, suppose that the central and local governments are benevolent. To be precise, the central government determines transfers to maximize a utilitarian objective, i.e., the sum of local utilities:

$$\begin{aligned} W = \sum_i^I n_i \Bigg[u(c_i) &+ \left\{ (1-\lambda) v(g_{1i}) + \lambda V \left(\sum_j^I n_j g_{1j} \right) \right\} \\ &+ \left\{ (1-\mu) h(g_{2i}) + \mu H \left(\sum_j^I n_j g_{2j} \right) \right\} \Bigg] \end{aligned} \tag{7}$$

[2] Boadway (2004) calls the transfer type $g_{1i} + g_{2i} = t_i + s_i$ a "net" grant.

[3] Akai and Sato (2008) also use the same functional form. This form of public goods can reflect all levels of externalities between national public goods and private goods by changing λ and μ.

However, the local government aims to maximize the welfare of its own region expressed by Eq.(6) with contributions of other regions in $V(\cdot)$ and $H(\cdot)$, i.e., $g_{mj}(j \neq i)$ being taken as given.

The timing of the game is defined as follows:

0. Nature determines the type of commitment policy variables for each local government.
1. Local government i chooses the levels of the policy variables determined in Stage 0, taking the choices made by the other local governments as given, $i = 1, \cdots, I$.
2. Having observed the local policies determined in Stage 1, the central government designs the transfer policies with the levels that have not been committed by the local governments. Finally, given all policies by governments, each individual consumes and gains utility.

Because the concept of a subgame perfect Nash equilibrium is applied, we solve the model backwardly.

3 First-best allocation

Before illustrating the subgame-perfect equilibrium, let us consider as a reference the first-best allocation that is determined by maximizing social welfare, W, subject to the resource constraint:

$$\max_{c_i, g_{1i}, g_{2i}} \quad W$$
$$s.t. \; \sum_i n_i c_i + \sum_i n_i g_{1i} + \sum_i n_i g_{2i} = Y$$

The first-best allocation is characterized by:

$$\begin{aligned} u'(c_i) &= (1-\lambda)v'(g_{1i}) + \lambda N V'\left(\sum_i n_i g_{1i}\right) \\ &= (1-\mu)h'(g_{2i}) + \mu N H'\left(\sum_i n_i g_{2i}\right) = \gamma, \; i = 1, \cdots, I \end{aligned} \tag{8}$$

γ is the Lagrangian multiplier associated with the constraint of the above optimization problem. From the first-order conditions, we have:

$$c_i = c^{**}, g_{1i} = g_1^{**}, g_{2i} = g_2^{**} \quad \forall i \tag{9}$$

In the case of $\lambda = \mu = 1$, g_1^{**} and g_2^{**} are coincident with the well-known Samuelson condition:

$$\frac{NV'(Ng_1^{**})}{u'(c^{**})} = \frac{NH'(Ng_2^{**})}{u'(c^{**})} = 1 \tag{10}$$

However, when $\lambda = \mu = 0$, the first-order conditions are rewritten as follows:

$$\frac{v'(g_1^{**})}{u'(c^{**})} = \frac{h'(g_2^{**})}{u'(c^{**})} = 1 \tag{11}$$

Likewise, when $\lambda = 1, \mu = 0$ or $\lambda = 0, \mu = 1$, the first-order conditions become $NV'(Ng_1^{**})/u'(c^{**}) = h'(g_2^{**})/u'(c^{**}) = 1$ or $v'(g_1^{**})/u'(c^{**}) = NH'(Ng_2^{**})/u'(c^{**}) = 1$ respectively.

In the following sections, we compare the various cases of ex ante commitments of local governments and analyze whether the above first-best conditions are achieved.

4 Analysis of some cases

4.1 Various types of policy commitments of local governments

We analyze the various cases of local governments' policy commitments. In Case A, local governments tentatively choose e_{1i} and e_{2i} in Stage 1, and all policy variables g_{1i}, g_{2i} and t_i are determined through the central transfers ex post. Silva (2015) refers to this model as "universal decentralized leadership," but does not consider the spillover of local public goods. In contrast, this paper considers the spillover of local public goods and demonstrates whether efficient allocation will be achieved.

In the following, we mainly analyze the following two types of plausible situations: the situation where the authority to set the tax rate is not delegated to the local government (commitment without tax rate) and the situation where the authority to set the tax rate is delegated to the local government (commitment with tax rate). By comparing these two cases, we demonstrate whether it is socially desirable for the tax rate to be selected by the local governments. Furthermore, the former situation (commitment without tax rate) is divided into two cases. In Case B, local governments commit to two local public goods g_{1i} and g_{2i} ex ante. In Case C, local governments commit to only one public good g_{1i} and choose the tentative level of public good e_{2i} ex ante. By considering both Cases B and C, we compare the case where the local governments only commit to one specific public good with the more general case of committing to multiple public goods, under no commitment for the tax rate.

In addition, the latter situation (commitment with tax rate) is also divided into two cases. In Case D, local governments commit only to tax rate t_{1i}. Note that the commitment to the tax rate affects private consumption because the (actual) tax rate $t_i = y_i - c_i$ is not changed ex post by a central income transfer. Local governments only choose the tentative amounts of local public goods (e_{1i} and e_{2i}) in Stage 1. In Case E, local governments commit to both g_{1i} and t_i ex ante. By considering Cases D and E, we compare the case where the local governments only commit to the tax rates with the case where they commit to both the tax rate and the amount of public goods. Table shows the timing of policy variable determination in each case. Silva (2014,2015) calls the case where authority is partly delegated to the local government and the timing of the choices of local public goods is separated as "selective decentralized leadership."

In the following subsection, we analyze Cases A–E and clarify how resource allocation becomes efficient, which is the first-best allocation.

4.2 Case A: Universal decentralized leadership

In Case A, each local government never commits to all policies, which are the actual tax rate and amount of public goods, and they choose only the tentative level of expenditure on public goods 1 and 2, i.e., e_{1i} and e_{2i} ex ante. Therefore, the central government can adjust private consumption (the actual tax rate) and the two public goods by setting the appropriate transfers ex post. Now, the optimization problem solved by the central government ex post is given as follows:

$$\max_{s_{ci}, s_{1i}, s_{2i}} \sum_i n_i \left[u(c_i) + \left\{ (1-\lambda)v(g_{1i}) + \lambda V\left(\sum_j n_j g_{1j} \right) \right\} + \left\{ (1-\mu)h(g_{2i}) + \mu H\left(\sum_j n_j g_{2j} \right) \right\} \right]$$

$$s.t. \ \sum_i s_{ci} = 0, \sum_i s_{1i} = 0, \ \sum_i s_{2i} = 0$$

			Ex ante	Ex post
Universal decentralized leadership	No commitment	Case A		g_{1i}, g_{2i}, t_i
Selective decentralized leadership	Commitment without tax rate	Case B	g_{1i}, g_{2i}	t_i
		Case C	g_{1i}	t_i, g_{2i}
	Commitment with tax rate	Case D	t_i	g_{1i}, g_{2i}
		Case E	g_{1i}, t_i	g_{2i}

Table: Various types of policy commitments of local governments

The first-order conditions become:

$$u'(c_i^A) = \gamma_c^A,\ i = 1, \cdots, I \tag{12}$$

$$(1-\lambda)v'(g_{1i}^A) + \lambda N V'\left(\sum_i n_i g_{1i}^A\right) = \gamma_1^A,\ i = 1, \cdots, I \tag{13}$$

$$(1-\mu)h'(g_{2i}^A) + \mu N H'\left(\sum_i n_i g_{2i}^A\right) = \gamma_2^A,\ i = 1, \cdots, I \tag{14}$$

where γ_c^A, γ_1^A and γ_2^A are the Lagrangian multipliers associated with the constraint of the above optimization problem $\sum_i s_{ci} = 0, \sum_i s_{1i} = 0$ and $\sum_i s_{2i} = 0$, respectively. Condition (12) states that the central transfers earmarked for consumption are chosen to equalize the marginal utilities of consumption across regions. Conditions (13) and (14) show that the central transfers earmarked for expenditure on public goods are chosen to equalize the marginal utilities of consumption of public goods across regions. Therefore, we obtain that $c_i^A = c^A,\ g_{1i}^A = g_1^A,\ g_{2i}^A = g_2^A$ for all i.

Let $\boldsymbol{e_m} = (e_{m1}, e_{m2}, \cdots, e_{mI})$ denote the vector of regional levels of expenditure on public good m, $m = 1, 2$.. Now, let $c^A(\boldsymbol{e_1}, \boldsymbol{e_2})$ and $s_{mi}(\boldsymbol{e_1}, \boldsymbol{e_2})$ denote the central government's best-response functions, $i = 1, \cdots, I$ and $m = 1, 2$. Substituting these functions into the central budget constraints, we obtain:

$$\sum_i n_i c^A(\boldsymbol{e_1}, \boldsymbol{e_2}) = Y - \sum_i n_i(e_{1i} + e_{2i}) \tag{15}$$

$$\sum_i n_i s_{mi}(\boldsymbol{e_1}, \boldsymbol{e_2}) = 0,\ m = 1, 2 \tag{16}$$

Differentiating Eqs.(15) and (16) with respect to $e_{mi},\ m = 1, 2$, we obtain:

$$\frac{\partial c^A(\cdot)}{\partial e_{mi}} = -\frac{n_i}{N} \tag{17}$$

$$\sum_i n_i \left(\frac{\partial s_{li}}{\partial e_{mi}}\right) = 0,\ l = 1, 2 \tag{18}$$

Now from Eqs.(13) and (14), we obtain:

$$g_{li}^{A}(\boldsymbol{e_1},\boldsymbol{e_2}) = g_{lj}^{A}(\boldsymbol{e_1},\boldsymbol{e_2}) = g_{l}^{A},\ i,j=1,\cdots,I,\ l=1,2 \tag{19}$$

where $g_{mi}^{A}(\boldsymbol{e_1},\boldsymbol{e_2}) = e_{mi} + s_{mi}(\boldsymbol{e_1},\boldsymbol{e_2})$. Differentiating Eq.(19) with respect to e_{mi} yields

$$\frac{\partial g_{mi}^{A}}{\partial e_{mi}} = 1 + \frac{\partial s_{mi}}{\partial e_{mi}} = \frac{\partial s_{mj}}{\partial e_{mi}} = \frac{\partial g_{mj}^{A}}{\partial e_{mi}},\ i,j=1,\cdots,I,\ i\neq j,\ m=1,2 \tag{20}$$

$$\frac{\partial g_{li}^{A}}{\partial e_{mi}} = \frac{\partial s_{li}}{\partial e_{mi}} = \frac{\partial s_{lj}}{\partial e_{mi}} = \frac{\partial g_{lj}^{A}}{\partial e_{2i}},,\ i,j=1,\cdots,I,\ l,m=1,2,\ l\neq m \tag{21}$$

Considering Eqs.(20) and (21) together with Eq.(18) yields

$$\frac{\partial s_{mi}}{\partial e_{mi}} = \frac{-(N-n_i)}{N},\ i=1,\cdots,I,\ m=1,2 \tag{22}$$

$$\frac{\partial s_{mj}}{\partial e_{mi}} = \frac{n_i}{N},\ i,j=1,\cdots,I,\ i\neq j,\ m=1,2 \tag{23}$$

$$\frac{\partial s_{lj}}{\partial e_{mi}} = 0,\ i,j=1,\ldots,I,\ l,m=1,2,\ l\neq m \tag{24}$$

In Stage 1, local government i chooses e_{1i} and e_{2i} to maximize the representative resident utility, taking the choices of all other local governments as given. The optimization problem considered by the local government i is given as follows:

$$\begin{aligned}\max_{e_{1i},e_{2i}}\ & u(c^{A}(\mathbf{e_1},\mathbf{e_2})) + (1-\lambda)v(g_1^{A}(\mathbf{e_1},\mathbf{e_2})) + \lambda V\left(Ng_1^{A}(\mathbf{e_1},\mathbf{e_2})\right) \\ & + (1-\mu)h(g_2^{A}(\mathbf{e_1},\mathbf{e_2})) + \mu H\left(Ng_2^{A}(\mathbf{e_1},\mathbf{e_2})\right)\end{aligned}$$

The first-order conditions become:

$$\begin{aligned} u'(\cdot)\frac{\partial c}{e_{1i}} &+ (1-\lambda)v'(\cdot)\frac{\partial g_1}{e_{1i}} + \lambda NV'(\cdot)\frac{\partial g_1}{e_{1i}} \\ &+ (1-\mu)h'(\cdot)\frac{\partial g_2}{e_{1i}} + \mu NH'(\cdot)\frac{\partial g_2}{e_{1i}} = 0,\ i=1,\cdots,I \end{aligned} \tag{25}$$

$$\begin{aligned} u'(\cdot)\frac{\partial c}{e_{2i}} &+ (1-\lambda)v'(\cdot)\frac{\partial g_1}{e_{2i}} + \lambda NV'(\cdot)\frac{\partial g_1}{e_{2i}} \\ &+ (1-\mu)h'(\cdot)\frac{\partial g_2}{e_{2i}} + \mu NH'(\cdot)\frac{\partial g_2}{e_{2i}} = 0,\ i=1,\cdots,I \end{aligned} \tag{26}$$

We can rewrite Eqs.(25) and (26) as follows:

$$u'(c^{A*}) = (1-\lambda)v'(g_1^{A*}) + \lambda NV'\left(Ng_1^{A*}\right) \tag{27}$$

$$u'(c^{A*}) = (1-\mu)h'(g_2^{A*}) + \mu NH'\left(Ng_2^{A*}\right) \tag{28}$$

A^* designates the solution to the problem in Case A. Eqs.(27) and (28) are consistent with the optimization condition Eq.(8). Therefore, the first-best allocation is achieved regardless of the degree of spillover, i.e., the values of λ and μ. Now, we obtain the following Lemma.

Lemma 1.
The first-best allocation is achieved regardless of the degree of spillover of public goods.

The intuition is as follows, First, in Case A, all goods including consumption are not committed ex ante and desirably equalized by central transfers ex post. Therefore, efficient allocation among goods is achieved. Second, note that the degree of spillover of public goods does not distort the resource allocation because the ex post transfers designed by the central government internalize any degree of spillover of public goods.

4.3 Selective decentralized leadership: Commitment without tax rate

In this section, we analyze commitment without tax rate, which corresponds to a centralized economy where the authority to set the local tax rate is not delegated to the local government.

4.3.1 Case B: g_{1i} and g_{2i} are chosen ex ante

Case B stipulates that g_{1i} and g_{2i} are chosen ex ante and t_i is adjusted after the ex post transfer to balance the local budget, i.e., taxation authority is centralized. In the following, we proceed backwardly, starting from Stage 2. The optimization problem solved by the central government in Stage 2 is given as follows:

$$\max_{s_{ci}} \sum_i n_i \left[u\left(y_i - t_i + s_{ci}\right) + \left\{ (1-\lambda)v(g_{1i}) + \lambda V\left(\sum_i n_i g_{1i}\right) \right\} + \left\{ (1-\mu)h(g_{2i}) + \mu H\left(\sum_i n_i g_{2i}\right) \right\} \right]$$

$$s.t. \ \sum_i n_i s_{ci} = 0$$

where s_{ci} is the per capita income transfer. As local governments commit to g_{1i} and g_{2i} ex ante, s_{1i} and s_{2i} are zero in Case B. The first-order conditions become:

$$u'(c_i^B) = \gamma^B, \ i = 1, \cdots, I \tag{29}$$

γ^B is the Lagrangian multiplier associated with the constraint of the above optimization problem. These conditions imply that the consumption level is perfectly equalized ex post, i.e., $c_i^B = c^B$ for all i. We obtain consumption from $c^B = y_i + s_{ci} - t_i$, $\sum_i n_i t_i = \sum_i n_i(g_{1i} + g_{2i})$:

$$c^B = \frac{1}{N}(Y - \sum_i n_i g_{1i} - \sum_i n_i g_{2i}) \equiv M \tag{30}$$

Therefore, c^B is given as increasing functions of M, $dc^B/dM = 1$.

In Stage 1, accounting for the ex post central policy, which is summarized by $c^B(M)$, the local governments independently choose g_{1i} and g_{2i} to maximize local utility in region i. Their optimization problem is expressed as:

$$\max_{g_{1i}, g_{2i}} \ u(c^B(M)) + (1-\lambda)v(g_{1i}) + \lambda V\left(\sum_i n_i g_{1i}\right) + (1-\mu)h(g_{2i}) + \mu H\left(\sum_i n_i g_{2i}\right)$$

The first-order conditions become:

$$(1-\lambda)v'(g_{1i}^{B*}) + \lambda n_i V'(\sum_i n_i g_{1i}^*) = \frac{n_i}{N}u'(c^{B*}),\ i = 1, \cdots, I \tag{31}$$

$$(1-\mu)h'(g_{2i}^{B*}) + \mu n_i H'(\sum_i n_i g_{2i}^*) = \frac{n_i}{N}u'(c^{B*}),\ i = 1, \cdots, I \tag{32}$$

B^* designates the solution to the problem in Case B. The right-hand side is the regionally perceived marginal benefit of the local public good, whereas the left-hand side represents the marginal cost from the regional perspective. $1 - n_i/N$ is the portion of the cost accruing to the other regions. The result of Case B is summarized as the following Lemma.

Lemma 2.

(a) Assume $\lambda = \mu = 1$. Then the first-best allocation will be achieved in the subgame-perfect equilibrium.

(b) Assume $\lambda, \mu \in [0, 1)$. Then:

(b-1) $\bar{g}_1^{B*} > g_1^{**}, \bar{g}_2^{B*} > g_2^{**}, c^{B*} < c^{**}$, where $\bar{g}_m^{B*} \equiv \sum_i n_i g_{mi}^{B*}/N,\ m = 1, 2$.

(b-2) g_{mi}^{B*} is overprovided in the sense that $\left.\frac{dW}{dg_{mi}}\right|_{g_{mi}=g_{mi}^{B*}} < 0$.

See Appendix 1 for the proof. With $\lambda = \mu = 1$, local public goods g_{1i} and g_{2i} have perfect spillover effects. Therefore, the amounts of public goods become too small as each local government accounts for only n_i/N of their benefits. However, ex post central transfers (s_{ci} in this case) give the incentive for local governments to supply the public goods excessively. The supply costs of public goods g_{1i} and g_{2i} become n_i/N. This means that these effects are perfectly offset, which ensures that the first-best allocation is achieved. By rewriting Eqs.(31) and (32), we can derive the first-best condition; $NV'(\cdot)/u'(\cdot) = NH'(\cdot)/u'(\cdot) = 1$. If $\lambda < 1 (\mu < 1)$, $g_{1i}(g_{2i})$ does not have a perfect spillover, then the oversupply incentives associated with the ex post transfers dominate the necessity level caused by the spillover effect. With $\lambda = 0 (\mu = 0)$, $g_{1i}(g_{2i})$ is similar to private consumption, and we have $v'(\cdot)/u'(\cdot) = (h'(\cdot)/u'(\cdot)) = n_i/N$ by rewriting Eq.(31) ((32)). Now, the per capita supply costs of the public goods become n_i/N; therefore, $g_{1i}(g_{2i})$ is overprovided from a social welfare viewpoint.

Finally, we can easily confirm that n_i/N is the inefficiency parameter that induces the biased incentive. As n_i/N approaches one, social welfare improves. This is because the free-rider incentive that leads to commitment to the smaller levels ex ante and the expectation of the additional transfer ex post is reduced as n_i/N approaches one.

4.3.2 Case C: g_{1i} is chosen ex ante

Case C stipulates that, while g_{1i} is chosen and committed to ex ante, g_{2i} and t_i can be adjusted after the ex post transfers to balance the local budget. Now the local governments choose the tentative amount of expenditure for g_{2i}, i.e., e_{2i}. Thus, the optimization problem solved by the central government in

Stage 2 is given as follows:

$$\max_{s_{ci},s_{2i}} \sum_i n_i \left[u\left(y_i - t_i + s_{ci}\right) + \left\{ (1-\lambda)v(g_{1i}) + \lambda V\left(\sum_i n_i g_{1i}\right) \right\} \right.$$
$$\left. + \left\{ (1-\mu)h(e_{2i} + s_{2i}) + \mu H\left(\sum_i n_i (e_{2i} + s_{2i})\right) \right\} \right]$$

$$s.t. \quad \sum_i n_i s_{ci} = 0, \; \sum_i n_i s_{2i} = 0$$

The first-order conditions become:

$$u'(c_i^C) = \gamma_c^C, \; i = 1, \cdots, I \tag{33}$$
$$(1-\mu)h'(g_{2i}^C) + \mu N H'(\sum_i n_i g_{2i}^C) = \gamma_2^C, \; i = 1, \cdots, I \tag{34}$$

where γ_c^C and γ_2^C are the Lagrangian multipliers corresponding to the constraint $\sum_i n_i s_{ci} = 0$ and $\sum_i n_i s_{2i} = 0$ respectively. Eq.(33) states that the central transfers earmarked for consumption are chosen to equalize marginal utilities of consumption across regions. Eq.(34) shows that the central transfers earmarked for expenditure on public good 2 are chosen to equalize the marginal utilities of consumption for public good 2 across regions. Therefore, we obtain that $c_i^C = c^C, \; g_{2i}^C = g_2^C$ for all i.

Let $\boldsymbol{g_1} = (g_{11}, g_{12}, \cdots, g_{1I})$ denote the vector of regional amounts of provision of public good 1. As with Case A, we can define c^C, g_2^C and s_{mi} as functions of $\boldsymbol{g_1}$ and $\boldsymbol{e_2}$, i.e., $c^C(\boldsymbol{g_1}, \boldsymbol{e_2}), g_2^C(\boldsymbol{g_1}, \boldsymbol{e_2})$ and $s_{2i}(\boldsymbol{g_1}, \boldsymbol{e_2}), \; i = 1, \cdots I$.

In Stage 1, accounting for the ex post central policy, local governments choose g_{1i} and e_{2i} independently to maximize the local utility in region i. Their optimization problem is expressed as:

$$\max_{g_{1i},e_{2i}} u(c^C(\boldsymbol{g_1}, \boldsymbol{e_2})) + (1-\lambda)v(g_{1i}) + \lambda V\left(\sum_i n_i g_{1i}\right)$$
$$+ (1-\mu)h(g_2^C(\boldsymbol{g_1}, \boldsymbol{e_2}) + \mu H\left(N g_2^C(\boldsymbol{g_1}, \boldsymbol{e_2})\right)$$

The first-order conditions become:

$$u'(\cdot)\frac{\partial c^C}{\partial g_{1i}} + (1-\lambda)v'(\cdot) + \lambda n_i V'(\cdot) = 0 \tag{35}$$
$$u'(\cdot)\frac{\partial c^C}{\partial e_{2i}} + (1-\mu)h'(\cdot)\frac{\partial g_2^C}{\partial e_{2i}} + \mu N H'(\cdot)\frac{\partial g_2^C}{\partial e_{2i}} = 0 \tag{36}$$

These conditions can be rewritten as follows: [4]:

$$(1-\lambda)v'(g_{1i}^{C*}) + \lambda n_i V'\left(\sum_i n_i g_{1i}^{C*}\right) = \frac{n_i}{N}u'(c^{C*}),\ i = 1, \cdots, I \tag{37}$$

$$(1-\mu)h'(g_2^{C*}) + \mu N H'(N g_2^{C*}) = u'(c^{C*}),\ i = 1, \cdots, I \tag{38}$$

C^* designates the solution to the problem in Case C. The right-hand side of Eq.(37) is the regionally perceived marginal benefit of the local public good g_{1i}, whereas the left-hand side of Eq.(37) represents the marginal cost from the regional perspective. $1 - n_i/N$ is the portion of the cost accruing to the other regions. Eq.(38) is also a condition of the supply of public good g_{2i}, which is not biased. From these conditions, we obtain the following Lemma.

Lemma 3.

(a) Assume $\lambda = 1$. Then the first-best allocation will be achieved in the subgame-perfect equilibrium.

(b) Assume $\lambda < 1$. Then:

(b-1) $\bar{g}_1^{C*} > g_1^{**}, g_2^{C*} < g_2^{**}, c^{C*} < c^{**}$, where $\bar{g}_1^{C*} \equiv \sum_i n_i g_{1i}^{C*}/N$.

(b-2) g_{1i}^{C*} is overprovided in the sense that $\left.\frac{dW}{dg_{1i}}\right|_{g_{1i}=g_{1i}^{C*}} < 0$.

See Appendix 2 for the proof. Lemma 3 is similar to Lemma 2 in Case B. With $\lambda = 1$, the local public good g_{1i} has a perfect spillover effect; therefore, the amount of g_{1i} is too small. However, ex post central transfers create an incentive for local governments to supply more of public good 1. Therefore, these two effects are perfectly offset and the amount of g_{1i} supplied becomes efficient. However, the local public good g_{2i} with any level of spillover is internalized by the central ex post transfer. Thus the supply of g_{2i} also becomes efficient. Therefore, the first-best allocation is achieved regardless of the value of μ.

4.4 Selective decentralized leadership: Commitment with tax rate

In this section, we analyze commitment using the tax rate, which corresponds to a decentralized economy where the authority for setting the local tax rates is delegated to the local governments.

Lemmas 2 and 3 imply that the first-best allocation is achieved when taxation authority is not delegated. We examine how this desirable result is affected by a decentralized economy with a tax rate commitment.

[4] As $\sum n_i c^C = Y - \sum n_i g_{1i} - \sum n_i e_{2i}$, we obtain the following equation:

$$\frac{\partial c^C}{\partial e_{2i}} = \frac{\partial c^C}{\partial g_{1i}} = -\frac{n_i}{N},\ i = 1, \cdots I$$

And we also obtain

$$\frac{\partial g_{2i}^C}{\partial e_{2i}} = 1 + \frac{\partial s_{2i}}{\partial e_{2i}} = \frac{n_i}{N},\ i = 1, \cdots I$$

$$\frac{\partial g_{1i}}{\partial e_{2i}} = \frac{\partial g_{2i}^C}{\partial g_{1i}} = 0,\ i = 1, \cdots I$$

4.4.1 Case D: t_i is chosen ex ante

In Case D, local governments choose t_i ex ante and can commit to its level, i.e., taxation authority is delegated. Recall that $t_i = e_{1i} + e_{2i}$, therefore local governments determine e_{1i} and e_{2i} before the central government chooses the transfers. Now, because local governments commit to t_i in Stage 1, the central government chooses the transfers for the provision of public goods 1 and 2, i.e., s_{1i} and s_{2i}. Then, the optimization problem considered by the central government in Stage 2 is given as follows:

$$\max_{s_{1i},s_{2i}} \sum_i n_i \left[u(c_i) + \left\{ (1-\lambda)v(e_{1i}+s_{1i}) + \lambda V\left(\sum_i n_i(e_{1i}+s_{1i})\right)\right\} \right.$$
$$\left. + \left\{ (1-\mu)h(e_{2i}+s_{2i}) + \mu H\left(\sum_i n_i(e_{2i}+s_{2i})\right)\right\}\right]$$

$$s.t. \ \sum_i n_i s_{1i} = 0, \ \sum_i n_i s_{2i} = 0$$

The first-order conditions become as follows:

$$(1-\lambda)v'(g_{1i}^D) + \lambda N V'\left(\sum_i n_i g_{1i}^D\right) = \gamma_1^D, \ i = 1, \cdots, I \tag{39}$$

$$(1-\mu)h'(g_{2i}^D) + \mu N H'\left(\sum_i n_i g_{2i}^D\right) = \gamma_2^D, \ i = 1, \cdots, I \tag{40}$$

where γ_1^D and γ_2^D are the Lagrangian multipliers corresponding to the constraints $\sum_i n_i s_{1i} = 0$ and $\sum_i n_i s_{2i} = 0$ respectively. From these conditions, we have that $g_{1i}^D = g_1^D, g_{2i}^D = g_2^D$ for all i. As $s_{1i} = g_{1i} - e_{1i}$ and $s_{2i} = g_{2i} - e_{2i}$, we obtain:

$$g_1^D + g_2^D = \frac{1}{N}\sum_i n_i t_i \equiv R \tag{41}$$

where g_1^D and g_2^D are increasing functions of R, i.e., $dg_1^D(R)/dR + dg_2^D(R)/dR = 1$.

In Stage 1, local governments choose feasible $(e_{11}, e_{12}, \cdots, e_{1I})$ and $(e_{21}, e_{22}, \cdots, e_{2I})$. Let $\boldsymbol{e_m} = (e_{m1}, e_{m2}, \cdots, e_{mI}), \ m = 1, 2.$. Now we can denote g_m^D and s_{mi} as functions of $\boldsymbol{e_m}$, i.e., $g_m^D(\boldsymbol{e_1}, \boldsymbol{e_2})$ and $s_{mi}(\boldsymbol{e_1}, \boldsymbol{e_2}), \ m = 1, 2$.

In Stage 1, the local government i chooses e_{1i} and e_{2i} considering their influences on transfers and taking the choices of all other local governments as given. Then, the optimization problem considered by local government i is given as follows:

$$\max_{e_{1i},e_{2i}} u(y_i - t_i(\boldsymbol{e_1}, \boldsymbol{e_2})) + (1-\lambda)v(g_1^D((\boldsymbol{e_1}, \boldsymbol{e_2}))) + \lambda V(N g_1^D(\boldsymbol{e_1}, \boldsymbol{e_2}))$$
$$+ (1-\mu)h(g_2^D(\boldsymbol{e_1}, \boldsymbol{e_2})) + \mu H(N g_2^D(\boldsymbol{e_1}, \boldsymbol{e_2}))$$

The first-order conditions become:

$$-u'(\cdot)\frac{\partial t_i}{\partial e_{1i}} + (1-\lambda)v'(\cdot)\frac{\partial g_1^D}{\partial e_{1i}} + \lambda N V'(\cdot)\frac{\partial g_1^D}{\partial e_{1i}} + (1-\mu)h'(\cdot)\frac{\partial g_2^D}{\partial e_{1i}} + \mu N H'(\cdot)\frac{\partial g_2^D}{\partial e_{1i}} = 0, \ i = 1, \cdots, I \tag{42}$$

$$-u'(\cdot)\frac{\partial t_i}{\partial e_{2i}} + (1-\lambda)v'(\cdot)\frac{\partial g_1^D}{\partial e_{2i}} + \lambda N V'(\cdot)\frac{\partial g_1^D}{\partial e_{2i}} + (1-\mu)h'(\cdot)\frac{\partial g_2^D}{\partial e_{2i}} + \mu N H'(\cdot)\frac{\partial g_2^D}{\partial e_{2i}} = 0, \ i = 1, \cdots, I \tag{43}$$

Because $\partial g_1^D/\partial e_{1i} = 1 + \partial s_{1i}/\partial e_{1i} = n_i/N, \partial g_1^D/\partial e_{1i} = 1 + \partial s_{1i}/\partial e_{1i} = n_i/N, \partial g_2^D/\partial e_{2i} = 1 + \partial s_{2i}/\partial e_{2i} = n_i/N, \partial g_1^B/\partial e_{2i} = \partial s_{1i}/\partial e_{2i} = 0$, and $\partial g_2^D/\partial e_{1i} = \partial s_{2i}/\partial e_{1i} = 0$, we can rewrite these conditions as follows[5]:

$$u'(c_i^{D*}) = \frac{n_i}{N}\left[(1-\lambda)v'(g_1^{D*}) + \lambda N V'(N g_1^{D*})\right], \; i = 1, \cdots, I \tag{44}$$

$$u'(c_i^{D*}) = \frac{n_i}{N}\left[(1-\mu)h'(g_2^{D*}) + \mu N H'(N g_2^{D*})\right], \; i = 1, \cdots, I \tag{45}$$

D^* designates the solution to the problem in Case D. Interpretation of these equations is straightforward. The right-hand side is the regionally perceived marginal benefit of private consumption, whereas the left-hand side is the marginal cost from the regional perspective. $1 - n_i/N$ is the portion of the cost accruing to the other regions. The result of Case D is summarized as the following Lemma.

Lemma 4.

(a) The first-best allocation is never achieved in the subgame-perfect equilibrium, irrespective of the level of spillovers, λ and μ.

(b) For any λ and μ:

(b-1) $g_1^{D*} < g_1^{**}, g_2^{D*} < g_2^{**}$, and $c^{**} < \bar{c}^{D*}$, where $\bar{c}^{D*} \equiv \sum_i n_i c_i^{B*}/N$.

(b-2) c_i^{D*} is overprovided in the sense that $\left.\frac{dW}{dc_i}\right|_{c_i = c_i^{D*}} < 0$.

See Appendix 3 for the proof. Ex post equalization of the marginal utilities of g_{1i} and g_{2i} gives rise to an ex ante free-riding motive among regions that set and commit to the lower tax rate ex ante, implying a higher level of consumption. In contrast to Case B, overconsumption is not solved even in the case of $\lambda = 1 (\mu = 1)$. Eq.(44) ((45)) becomes $u'(c_i^{D*}) = n_i V(N g_1^{D*}) (= n_i H'(N g_2^{B*}))$, which is different from the Samuelson condition (10). In this case, there is no commitment to supplying public goods; therefore, there is no incentive to supply greater amounts of public goods, unlike in case C. However, as shown in Case A, the ex post transfers successfully internalize the spillovers in the case of noncommitment to public goods. As a result, the spillover effect of public goods is internalized in this case. However, the result is not efficient because the central government tries to internalize the spillover effect of public goods by ex post transfers among regions, but the lower tax rate (overconsumption) is committed to ex ante and cannot be adjusted ex post, i.e., the supply of public goods is biased as a result. This is because the tax rate is committed to ex ante; therefore, ex post income transfers are not allowed. Setting a lower tax rate to increase consumption is desirable for local governments.

[5] As $\sum_i n_i s_{1i} = 0$, $\sum_i n_i s_{2i} = 0$ and $g_{mi}^D(\boldsymbol{e_1}, \boldsymbol{e_2}) = e_{mi} + s_{mi}(\boldsymbol{e_1}, \boldsymbol{e_2}),\; m = 1, 2\, i = 1, \ldots, I$, we obtain:

$$\frac{\partial s_{mi}}{\partial e_{mi}} = \frac{-(N - n_i)}{N}, \; m = 1, 2, \; i = 1, \ldots, I$$
$$\frac{\partial s_{mj}}{\partial e_{mi}} = 1 + \frac{\partial s_{mi}}{\partial e_{mi}} = 1 + \frac{-(N - n_i)}{N} = \frac{n_i}{N}, \; m = 1, 2, \; i, j = 1, \cdots I, \; i \neq j$$
$$\frac{\partial s_{mj}}{\partial e_{li}} = 0, \; l, m = 1, 2, \; l \neq m, \; i, j = 1, \cdots I$$

4.4.2 Case E: g_{1i} and t_i are chosen ex ante

In Case D, we show that only tax rate commitment is always inefficient. However, we have already confirmed that the expenditure commitments in Cases B and C may be desirable. Can we restore efficiency (relative to Case D) when there is commitment with respect to one public good, as inspired by the efficient result in Case C? To answer this question, we analyze Case E.

Case E is basically the same as Case C, except that local governments commit to t_i in addition to g_{1i}. As g_{1i} and t_i are committed to ex ante, s_{1i} and s_{ci} are equal to zero, and the central government can adjust public good g_{2i} by setting the ex post transfer. Therefore, the optimization problem solved by the central government in Stage 2 is given as follows:

$$\max_{s_{2i}} \sum_i n_i \left[u\left(c_i\right) + \left\{ (1-\lambda) v(g_{1i}) + \lambda V\left(\sum_i n_i g_{1i}\right)\right\} \right.$$
$$\left. + \left\{ (1-\mu) h(e_{2i}+s_{2i}) + \mu H\left(\sum_i n_i (e_{2i}+s_{2i})\right)\right\}\right]$$

$$s.t. \sum_i n_i s_{2i} = 0$$

The first-order conditions become:

$$(1-\mu) h'(g_{2i}^E) + \mu N H'(\sum_i n_i g_{2i}^E) = \gamma^E, \ i = 1, \cdots, I \tag{46}$$

where γ^E is the Lagrangian multiplier associated with the constraint of the above optimization problem. The central government transfer for the supply of g_{2i} will perfectly equalize the supply of the public goods, $g_{2i}^E = g_2^E$ for all i. We obtain the following equation from $g_{1i} = e_{1i}$ and $g_2^E = e_{2i} + s_{2i}$,

$$g_2^E = \frac{1}{N} \sum_i n_i (t_i - g_{1i}) \equiv R' \tag{47}$$

where g_2^E is an increasing function of R', that is $dg_2^E/dR' = 1$.

In Stage 1, local governments maximize local utility with respect to g_{1i} and t_i. Their optimization problem is given as follows:

$$\max_{g_{1i}, t_i} u(y_i - t_i) + (1-\lambda) v(g_{1i}) + \lambda V\left(\sum_i n_i g_{1i}\right)$$
$$+ (1-\mu) h(g_2^E(R')) + \mu H\left(N g_2^E(R')\right)$$

The first-order conditions become:

$$u'(c_i^{E*}) = \frac{n_i}{N}\left[(1-\mu) h'(g_2^{E*}) + \mu N H'(N g_2^{E*})\right], \ i = 1, \cdots, I \tag{48}$$

$$(1-\lambda) v'(g_{1i}^{E*}) + \lambda n_i V'(\sum_i n_i g_{1i}^{E*}) = \frac{n_i}{N}\left[(1-\mu) h'(g_2^{E*}) + \mu N H'(N g_2^{E*})\right], \ i = 1, \cdots, I \tag{49}$$

where E^* designates the solution to the problem in Case E. The right-hand side is the regionally perceived marginal benefit of the public good g_{1i} and private consumption, whereas the left-hand side represents the marginal cost from the regional perspective. $1 - n_i/N$ is the portion of the cost accruing to the other regions. Comparing this to Lemma 3, the result of Case E is summarized as the following Lemma.

Lemma 5.

(a) The first-best allocation is never achieved in the subgame-perfect equilibrium, even if $\lambda = 1$.

(b) For any λ *and* μ:

(b-1) $\bar{g}_1^{E*} > g_1^{**}, g_2^{E*} < g_2^{**}, \bar{c}^{E*} > c^{**}$, *where* $\bar{g}_1^{E*} \equiv \sum_i n_i g_{1i}^{E*}/N,\ \bar{c}^{E*} \equiv \sum_i n_i c_i^{E*}/N$.

(b-2) g_{1i}^{E*} *and* c_i^{E*} *are overprovided in the sense that* $\left.\frac{dW}{dg_{1i}}\right|_{g_{1i}=g_{1i}^{E*}} < 0$, $\left.\frac{dW}{dc_i}\right|_{c_i=c_i^{E*}} < 0$.

See Appendix 4 for the proof. In Case E, the first-best allocation cannot be achieved even when the public goods have a perfect spillover, i.e., $\lambda = 1$, as opposed to Case C. This result is similar to Case D where the tax rate is also committed to ex ante, except for the existence of the commitment to g_{1i}. First, committing to the tax rate means commitment to residents' consumption. Local governments have an incentive to keep residents' consumption high by setting a lower tax rate. Secondly, commitment to g_{1i} creates an incentive to provide more public goods because of the expectation of ex post transfers. Both effects compete with each other. While, the second incentive is offset when the public goods have perfect spillover, the first incentive cannot be avoided. As a result, the first-best allocation cannot be achieved even when $\lambda = 1$. This negative answer to the question posed at the beginning of this subsection demonstrates that the inefficiency of the tax commitment is strong enough to offset the efficiency of the commitment to the perfect spillover public good.

5 Implications

We analyze Cases A–E and obtain the results summarized in Lemmas 1–5. From Lemmas 1–5, we obtain the following proposition.

Proposition.
The first-best allocation is never achieved as long as local governments can commit to the tax rate t_i *ex ante, i.e., taxation authority is delegated to the local government regardless of the degree of spillover.*

Given this Proposition, the first-best allocation is never achieved in the case with commitment to the tax rate. This means that the first-best allocation can be achieved only in the case without commitment to the tax rate. Figure shows the areas where first-best allocation is achieved in the three cases, A–C, where tax rate authority is not delegated to the local governments and is not committed to, dependent on the degrees of spillovers of the two public goods. As summarized in Lemma 1–3, (A) if nothing is committed to ex ante or (B) if g_{1i} and g_{2i} with perfect spillovers are committed to ex ante or (C) g_{1i} with perfect spillovers is committed to ex ante, then the allocation becomes efficient.

Case C $'$ is the case where local governments commit to g_{2i} instead of g_{1i} as in Case C. When local governments commit to policies, it is desirable that the policies they commit to are pure public goods with perfect spillovers. This result corresponds to that of Silva (2014) for selective decentralized leadership, where an efficient allocation is only achieved when the local governments commit to the supply of national public goods.

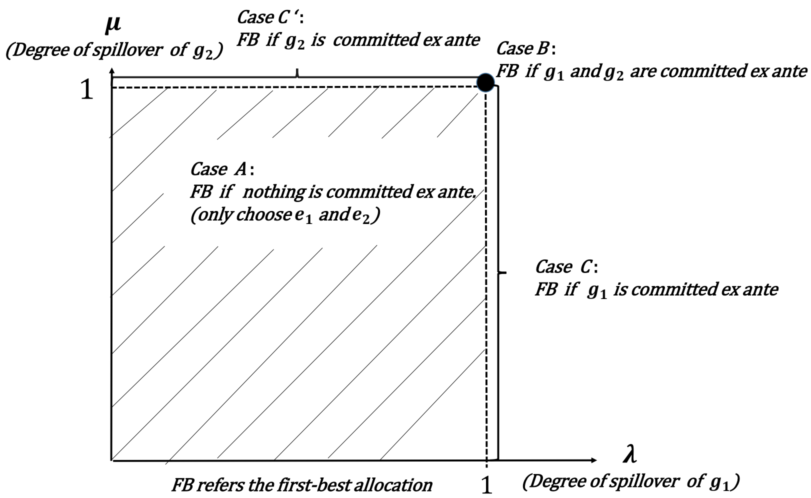

Figure: Areas where the first-best allocation is achieved when taxation authority is not delegated

6 Conclusion

This paper analyzes the decentralized leadership model with spillovers by focusing on which policy to commit to ex ante. In particular, we focus on the delegation of the authority to set tax rate (commitment to tax rate). When the tax rate is committed to ex ante, efficiency is never achieved, as shown in Case D. This inefficiency result remains even when the public good is additionally committed to as shown in Case E, as opposed to Case C where efficiency is achieved when the perfect spillover good is committed to ex ante. These results imply that the delegation of the authority to set the tax rate should be considered carefully.

Additionally, we show that when taxation authority is not delegated, the existence of spillover effects may work well when the good committed to ex ante has perfect spillover effects, as shown in Cases B and C.

The results of this paper expand the scope of our understanding of decentralized leadership and are useful for evaluating future decentralized institutional design for federalism.

Appendix

Appendix 1: Proof of Lemma 2

Suppose that $g_1^{**} > \bar{g}_1^{B*}, g_2^{**} > \bar{g}_2^{B*}$. Then we obtain:

$$\bar{M} \equiv \frac{Y}{N} - \bar{g}_1^{B*} - \bar{g}_2^{B*} > \frac{Y}{N} - g_1^{**} - g_2^{**} \equiv M^{**} \tag{50}$$

As $c^{B*}(M)$ is an increasing function of M, we have $c^{B*}(M) > c^{**}$. Now comparing the subgame-perfect equilibrium solution with the first-best allocation solution, we have:

$$(1-\lambda)\frac{N}{n_i}v'(g_{1i}^{B*}) + \lambda NV'(N\bar{g}_1^{B*}) = u'(c^{B*}(M)) \tag{51}$$

$$(1-\lambda)v'(g_1^{**}) + \lambda NV'(Ng_1^{**}) = u'(c^{**}) \tag{52}$$

Because $c^{B*}(M) > c^{**}$, we have $u'(c^{B*}(M)) < u'(c^{**})$. Then, we obtain the following inequality.

$$(1-\lambda)\left[\frac{N}{n_i}v'(g_{1i}^{B*}) - v'(g_1^{**})\right] + \lambda N\left[V'(N\bar{g}_1^{B*}) - V'(Ng_1^{**})\right] < 0 \tag{53}$$

As $g_1^{**} > \bar{g}_1^{B*}$ and $N/n_i > 1$, the above inequality holds only if $g_{1i}^{B*} > g_1^{**}$ for all i. However, this contradicts $g_1^{**} > \bar{g}_1^{B*}$. Therefore, $\bar{g}_1^{B*} > g_1^{**}$ holds. Using a similar procedure, we obtain $\bar{g}_2^{B*} > g_2^{**}$.

Social welfare in the equilibrium is given by:

$$\begin{aligned} W = Nu(c^{B*}(M^{B*})) &+ (1-\lambda)\sum_i n_i v(g_{1i}^{B*}) + \lambda NV(\sum_i n_i g_{1i}^{*A}) \\ &+(1-\mu)\sum_i n_i h(g_{2i}^{B*}) + \mu NH(\sum_i n_i g_{2i}^{B*}) \\ \textit{where } M^* &= \frac{1}{N}\left[Y - \sum_i n_i g_{1i}^{B*} - \sum_i n_i g_{2i}^{B*}\right] \end{aligned}$$

Differentiating the above equation with respect to g_{1i} and evaluating the equilibrium, we have:

$$\left.\frac{dW}{dg_{1i}}\right|_{g_{1i}=g_{1i}^{B*}} = -n_i u'(\cdot)\frac{dc^{B*}(M^*)}{dM^*} + (1-\lambda)n_i v'(g_{1i}^{B*}) + \lambda N n_i V'(\cdot) \tag{54}$$

Rewriting the above equation, we obtain:

$$\frac{1}{n_i}\left.\frac{dW}{dg_{1i}}\right|_{g_{1i}=g_{1i}^{B*}} = (1-\lambda)\left(1-\frac{N}{n_i}\right)v'(g_{1i}^{B*}) < 0 \tag{55}$$

Using a similar procedure, we can show that $\frac{1}{n_i}\frac{dW}{dg_{2i}}|_{g_{2i}=g_{2i}^{B*}} = (1-\mu)\left(1-\frac{N}{n_i}\right)h'(g_{2i}^{B*}) < 0$. □

Appendix 2: Proof of Lemma 3

Suppose that $g_1^{**} > \bar{g}_1^{C*}$. As $c^{C*} + g_2^{C*} = \frac{1}{N}(Y - \sum_i g_{1i}^{C*}) \equiv M'$, we obtain:

$$\begin{aligned} \bar{M}' &\equiv \frac{Y}{N} - \bar{g}_1^{C*} \\ &\geq \frac{Y}{N} - g_1^{**} \equiv M'^{**} \end{aligned} \tag{56}$$

As $c^{C*}(M')$ and $g_2^{C*}(M')$ are increasing functions of M', we have $c^{C*} > c^{**}$ and $g_2^{C*} > g_2^{**}$. Now comparing the subgame-perfect equilibrium solution with the first-best allocation solution, we have:

$$(1-\lambda)\frac{N}{n_i}v'(g_{1i}^{C*}) + \lambda NV'(N\bar{g}_1^{C*}) = u'(c^{C*}(M')) \tag{57}$$

$$(1-\lambda)v'(g_1^{**}) + \lambda NV'(Ng_1^{**}) = u'(c^{**}) \tag{58}$$

Because $c^{C*}(M') > c^{**}$, we obtain the following inequality:

$$(1-\lambda)\left[\frac{N}{n_i}v'(g_{1i}^{C*}) - v'(g_1^{**})\right] + \lambda N\left[V'(N\bar{g}_1^{C*}) - V'(Ng_1^{**})\right] < 0 \tag{59}$$

As $g_1^{**} > \bar{g}_1^{C*}$ and $N/n_i > 1$, the above inequality holds only if $g_{1i}^{C*} > g_1^{**}$ for all i. However, this contradicts $g_1^{**} > \bar{g}_1^{C*}$.

Social welfare in the equilibrium is given by:

$$W = Nu(c^{C*}(M^{C*})) + (1-\lambda)\sum_i n_i v(g_{1i}^{C*}) + \lambda NV(\sum_i n_i g_{1i}^{C*})$$
$$+(1-\mu)\sum_i Nh(g_2^{C*}) + \mu NH(Ng_2^{C*})$$
$$where\ M^{'*} = \frac{1}{N}\left[Y - \sum_i n_i g_{1i}^{C*}\right]$$

Differentiating the above equation with respect to c_i and evaluating the equilibrium, we have:

$$\left.\frac{dW}{dg_{1i}}\right|_{g_{1i}=g_{1i}^{C*}} = -\, n_i u'(\cdot)\frac{dc^{C*}(M^{'*})}{dM^{'*}} + (1-\lambda)n_i v'(g_{1i}^{C*}) + \lambda N n_i V'(\cdot)$$
$$-\, n_i[(1-\mu)h'(\cdot) + \mu NH'(\cdot)]\frac{dg^{C*(M^{'*})}}{dM^{'*}} \tag{60}$$

Rewriting the above equation, we obtain:

$$\frac{1}{n_i}\left.\frac{dW}{dg_{1i}}\right|_{g_{1i}=g_{1i}^{C*}} = (1-\lambda)\left(1-\frac{N}{n_i}\right)v'(g_{1i}^{C*}) < 0 \quad \square \tag{61}$$

Appendix 3: Proof of Lemma 4

Because $n_i c_i^{D*} = n_i y_i - n_i t_i$ and $\sum_i n_i t_i = Ng_1^{D*} + Ng_2^{D*}$, we have $(\sum_i n_i t_i)/N = g_1^{D*} + g_2^{D*} = Y/N - \bar{c}^{D*} \equiv \bar{R}$. Now, suppose that $c^{**} > \bar{c}^{D*}$. Then we obtain:

$$\bar{R} \equiv \frac{Y}{N} - \bar{c}^{D*} > \frac{Y}{N} - c^{**} \equiv R^{**} \tag{62}$$

As $g_1^{D*}(R)$ and $g_2^{D*}(R)$ are increasing functions of R, we have $g_1^{D*}(R) > g_1^{**}$ and $g_2^{D*}(R) > g_2^{**}$. Now comparing the subgame-perfect equilibrium solution with the first-best allocation solution, we have:

$$\frac{N}{n_i}u'(c_i^{D*}) = (1-\lambda)v'(g_1^{D*}) + \lambda NV'(Ng_1^{D*}) \tag{63}$$
$$u'(c^{**}) = (1-\lambda)v'(g_1^{**}) + \lambda NV'(Ng_1^{**}) \tag{64}$$

As $g_1^{D*} > g_1^{**}$, we have $(1-\lambda)v'(g_1^{D*}) + \lambda NV'(Ng_1^{D*}) < (1-\lambda)v'(g_1^{**}) + \lambda NV'(Ng_1^{**})$. Therefore, we obtain following inequality.

$$\frac{N}{n_i}u'(c_i^{D*}) - u'(c^{**}) < 0 \tag{65}$$

Because $N/n_i > 1$, the above inequality holds only if $c_i^{D*} > c^{**}$ for all i. However, this contradict $c^{**} > \bar{c}^{D*}$.

The social welfare in the equilibrium is given by:

$$W = \sum_i n_i u(c_i^{D*}) + (1-\lambda)Nv(g_1^{D*}(R^*)) + \lambda N\psi(Ng_1^{D*}(R^*))$$
$$+(1-\mu)Nh(g_2^{D*}(R^*)) + \mu NH(Ng_2^{D*}(R^*))$$
$$where\ R^* = \frac{1}{N}\left[Y - \sum_i n_i c_i^{D*}\right]$$

Differentiating the above equation with respect to c_i and evaluating the equilibrium, we have:

$$\left.\frac{dW}{dc_i}\right|_{c_i=c_i^{D*}} = n_i u'(\cdot) - \frac{n_i}{N}\left[(1-\lambda)Nv'(\cdot) + \lambda N^2 V'(\cdot)\right]\frac{dg_1^{D*}}{dR^*} - \frac{n_i}{N}\left[(1-\mu)Nh'(\cdot) + \mu N^2 H'(\cdot)\right]\frac{dg_2^{D*}}{dR^*} \tag{66}$$

Rewriting the above equation, we obtain as follows:

$$\frac{1}{n_i}\left.\frac{dW}{dc_i}\right|_{c_i=c_i^{D*}} = \left(1-\frac{N}{n_i}\right)u'(c_i^{D*}) < 0 \quad \square \tag{67}$$

Appendix 4: Proof of Lemma 5

Suppose that $g_1^{**} > \bar{g}_1^{E*},\ c^{**} > \bar{c}_i^{E*}$. Then we obtain:

$$\begin{aligned}\bar{R}' &\equiv \frac{Y}{N} - \frac{\sum_i n_i c_i^{E*}}{N} - \frac{\sum_i n_i g_{1i}^{E*}}{N} \\ &= \frac{Y}{N} - \bar{c}_i^{E*} - \bar{g}_1^{E*} \\ &\geq \frac{Y}{N} - c^{**} - g_1^{**} \equiv R'^{**}\end{aligned} \tag{68}$$

Since $g_2^{E*}(R')$ is increasing function of R', we have $g_2^{E*} > g_2^{**}$. Now comparing the subgame-perfect equilibrium solution with the first-best allocation solution, we have

$$\frac{N}{n_i}u'(c_i^{E*}) = (1-\mu)h'(g_2^{E*}) + \mu N H'(N g_2^{E*}) \tag{69}$$

$$u'(c^{**}) = (1-\mu)h'(g_2^{**}) + \mu N H'(N g_2^{**}) \tag{70}$$

From the assumption $g_2^{E*} > g_2^{**}$, we have $(1-\mu)h'(g_2^{E*})+\mu N H'(N g_2^{E*}) < (1-\mu)h'(g_2^{**})+\mu N H'(N g_2^{**})$. Therefore, we obtain following inequality.

$$\frac{N}{n_i}u'(c_i^{E*}) - u(c^{**}) < 0 \tag{71}$$

Because $N/n_i > 1$, the above inequality holds only if $c_i^{E*} > c^{**}$ for all i. However, this contradicts $c^{**} > \bar{c}^{E*}$. However, comparing the subgame-perfect equilibrium solution with the first-best allocation solution of g_1, we have:

$$(1-\lambda)\left[\frac{N}{n_i}v'(g_{1i}^{E*}) - v'(g_1^{**})\right] + \lambda N[V'(N\bar{g}_1^{E*}) - V'(N g_1^{**})] < 0 \tag{72}$$

As $g_1^{**} > \bar{g}_1^{E*}$ and $N/n_i > 1$, the above inequality holds only if $g_{1i}^{E*} > g_1^{**}$ for all i. However, this contradicts $g_1^{**} > \bar{g}_1^{E*}$.

The social welfare in the equilibrium is given by:

$$\begin{aligned}W = &\sum_i n_i u(c_i^{E*}) + (1-\lambda)\sum_i n_i v(g_{1i}^{E*}) + \lambda N V(\sum_i n_i g_{1i}^{E*}) \\ &+ (1-\mu)N h(g_2^{E*}(R')) + \mu N H(N g_2^{E*}(R')) \\ \textit{where } R'^* = &\frac{1}{N}[Y - \sum_i n_i c_i^{E*} - \sum_i n_i g_{1i}^{E*}]\end{aligned}$$

Differentiating the above equation with respect to c_i and evaluating the equilibrium, we have:

$$\left.\frac{dW}{dc_i}\right|_{c_i=c_i^{E*}} = n_i u'(\cdot) - N(1-\mu)h'(\cdot)\frac{dg_2^{E*}(R'^*)}{dR'^*}\frac{n_i}{N} - \mu N H'(\cdot) N \frac{dg_2^{E*}(R'^*)}{dR'}\frac{n_i}{N} \tag{73}$$

Rewriting the above equation, we obtain:

$$\frac{1}{n_i}\left.\frac{dW}{dc_i}\right|_{c_i=c_i^{E*}} = u'(\cdot) - \frac{N}{n_i}u'(\cdot) = (1-\frac{N}{n_i})u'(\cdot) < 0 \tag{74}$$

In a similar way, differentiating the above equation with respect to c_i and evaluating the equilibrium, we have:

$$\frac{1}{n_i}\left.\frac{dW}{dg_{1i}}\right|_{g_{1i}=g_{1i}^{E*}} = (1-\lambda)(1-\frac{N}{n_i})v'(\cdot) < 0 \quad \square \tag{75}$$

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