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Strategic Intertemporal Budget Allocation of Local Governments in the Model with Spillovers and Mergers*

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【Abstract】Mergers of local governments, commonly referred to as municipal mergers, have been implemented widely to internalize spillover effects. Many empirical studies point out that municipalities strategically increase their debt issuance before mergers, creating the `fiscal common pool problem', because of pooled budgets after mergers. However, this phenomenon has not yet been analyzed theoretically. Therefore, this paper examines the mechanism of increased debt issuance before municipal mergers. Our results show that three different effects influence intertemporal budget allocations of municipalities at the time of a merger and the existence of externalities may reduce the severity of the fiscal common pool problem.

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1 Introduction

Mergers of local governments, sometimes referred to as municipal mergers, have occurred in many countries[*1]. According to the theory of fiscal federalism, such mergers are expected to internalize the externalities of local governments and improve welfare (Oates, 1972). However, municipalities have an incentive to issue debt excessively just before a merger because the debt of each municipality will be shared among the merged municipalities after the consolidation. Such a strategic intertemporal budget allocation that entails excessive debt issuance is called the 'fiscal common pool problem' because each entity can exploit a commonly pooled budget instead of using their own budget. Indeed, many empirical papers suggest that the fiscal common pool problem is caused by municipal mergers (Hinnerich, 2009; Jordahl and Liang, 2010; Hansen, 2014; Fritz and Feld, 2015; Saarimaa and Tukiainen, 2015; Nakazawa, 2016; Hirota and Yunoue, 2017); however, the mechanism of excessive debt issuance by municipalities has not yet been examined theoretically. In addition, whether a municipal merger enhances the welfare of the municipalities is unknown.

Therefore, this paper clarifies why and how municipalities determine their intertemporal budget allocation, and whether welfare is improved by municipal mergers even if excessive debt is issued. To see this, we construct a simple two-period model with N municipalities, where municipalities decide their debt at period 1 and distribute a local public good at each period to maximize their resident's utility. Using the model, we show that the strategic intertemporal budget allocation that entails excessive debt issuance occurs before the merger and is affected not only by (1) the fiscal common pool problem, but also by other mechanisms that arise after the merger, namely (2) internalization of spillovers and (3) equalization of the budget. We can confirm that the excess debt issuance can be relieved by the existence of spillover. Moreover, we clarify how social welfare is affected by these three effects.

Furthermore, because separations of governments occur in the real world[*2], we examine the case where a single municipality is separated into plural municipalities. In particular, we investigate whether there

[*1] For example, Australia, Denmark, Finland, Germany, Ireland, Israel, Japan, South Africa, Sweden, and the United States have implemented jurisdictional mergers since the mid-20th century.

[*2] For example, there have been the movement toward separations of governments in France, Former-Soviet Union, Japan, Spain, the United Kingdom and the United States since the mid-20th century.

is an incentive to increase debt before the separation and how welfare changes. As a result, we find that separation of municipalities leads to an increase in debt issuance and the size of spillovers affects both debt issuance and welfare.

Our results can be interpreted by considering the three effects mentioned above. The first effect is the fiscal common pool problem. After a merger, the budget is merged and the repayment of the debt issued before the merger is shared. Thus, the marginal cost of debt issuance before the merger is small, which increases the incentive to issue more debt. Therefore, the fiscal common pool problem involving an excessive issuance of debt arises. However, it is interesting to note that the fiscal common pool problem does not arise in the separation case because municipalities are merged at the time of issuance of debts and the incentive problem does not exist.

The second effect is internalization of spillovers. When spillovers of public goods exist among municipalities, mergers make internalization of the spillovers possible and allocations become efficient. When spillovers are assumed to be positive, internalization for positive spillovers requires more resources in the period following the merger. This is the opposite to the change in the budget allocation created by the fiscal common pool problem. In the separation case, municipalities are merged in period 1, and separated in period 2. Therefore, for internalizing positive spillovers in period 1, the resource moves from period 2 to period 1, which induces the excessive debt, despite the absence of the fiscal common pool problem in the separation case.

The third effect is equalization of budgets. When there exist differences of endowments among municipalities, equalization of marginal benefits through transfers across the budgets of municipalities improves social welfare. The social welfare in each case can be evaluated through these three effects.

We can determine whether the merger case or the separation case is superior to the non-merger case under special settings. Consider the following examples. One example is the case where the fiscal common pool problem by merger is fully offset by the internalization of the spillovers. Then equalization of budgets by merger creates a desirable effect additionally. Therefore, the merger case is superior to the non-merger case. Another example is the case where there is no disadvantage in period 2 following separation in an economy without spillovers. Then the internalization of spillovers and equalization of budgets in the merged period 1 create a desirable effect additionally. Therefore, the separation case is superior to the non-merger case.

We also extend the model by considering a private good. The results obtained in the extended model are similar to the results without a private good, confirming the existence of the fiscal common pool problem.

Although our paper is inspired by empirical studies on the fiscal common pool problem following municipal mergers, our work is related to several theoretical studies. Weingast et al. (1981) and Persson and Tabellini (1994) identify the mechanism of the fiscal common pool problem using a static political pork barrel model. Velasco (2000) and Krogstrup and Wyplosz (2010) construct a dynamic framework by considering the intertemporal budget allocation in a political economy. Velasco (2000) considers a dynamic political economy where special interest groups exploit the government budget as common property, and show that the government overissues debt as a result, which makes the economy unsustainable. Krogstrup and Wyplosz (2010) present a similar result such that governments issue excessive debt because of the special interest groups funded by governments. Although Velasco (2000) and Krogstrup and Wyplosz (2010) focus on the fiscal common pool problem in a dynamic setting, these papers do not consider municipal mergers as a source of the fiscal common pool problem. To our knowledge, in the context of local public finances, no theoretical paper focuses on municipal mergers, although these mergers are recognized as one of the reasons for the fiscal common pool problem by many empirical papers.

Given this background, this paper develops a basic two-period model and explains how the strategic intertemporal budget allocation by local governments causes the fiscal common pool problem and under what circumstances the merger or separation improves welfare in the model with spillovers[*3].

Thus, the result of this paper sheds light on the mechanism of strategic intertemporal budget allocation of local government using debt, and the effect that both the spillovers and the number of regions have on the debt level, which is useful for an understanding of local government behavior and the institutional design of local public finances.

This paper is organized as follows. In Section 2, we develop and analyze the model. The four cases are analyzed and first-order conditions with respect to the debt levels are derived. In Sections 3 and 4, we discuss these four cases and present the welfare implications, respectively. Section 5 extends the model by incorporating private consumption. Section 6 discusses related matters and Section 7 concludes.

2 The Model

2.1 The model setting

Consider an economy with two periods (denoted by $\tau = 1, 2$) and N municipalities (denoted by $i = 1, \cdots, N$). In each municipality i, there is a government and a representative consumer who enjoys a public good $g_{i\tau}$ at period τ[*4]. Public good $g_{i\tau}$ has a spillover and its extent at period τ is defined as $\theta_\tau \in [0, 1]$, which is assumed as common in each area, but different in each period[*5]. $g_{i\tau}$ is financed by an endowment $\omega_{i\tau}$ or a debt d_i. Governments set their public good levels in each period and decide their debt level d_i at period 1. A consumer in i at τ has the utility function $u_{i\tau} = u(g_{i\tau} + \theta_\tau \sum_{j\neq i} g_{j\tau})$, where $u' > 0 > u''$ is assumed[*6].

Consumers obtain endowment $\omega_{i\tau}$ in each period. The $\omega_{i\tau}$ can be varied in each area and each period. Thus, this model allows areas to be asymmetric. The net income of the consumer in i is

$$\begin{cases} g_{i1} = \omega_{i1} + b_i \\ g_{i2} = \omega_{i2} - (1+r)b_i \end{cases} \tag{1}$$

[*3] Nagami and Ogawa (2011) focus on policy coordination in a migration economy and analyze the effects of coordination in local debt issuance and repayment policies.

[*4] To focus on local government behavior and the mechanism of the fiscal common pool problem, we develop the model without private consumption here. The model with private consumption is examined in Section 5.

[*5] To capture the effect of the municipal mergers, it is valuable to allow differences in the degree of the spillover between ex ante and ex post stage.

[*6] For simplicity, we adopt the same utility function for all municipalities.

where b_i is a debt and r is the interest rate for savings[*7].

The total utility of consumer i is

$$W_i \equiv u(g_{i1} + \theta_1 \sum_{j \neq i} g_{j1}) + \delta u(g_{i2} + \theta_2 \sum_{j \neq i} g_{j2}). \tag{2}$$

Note that $\delta(\in [0,1])$ is the discount factor for period 2, which is common in each area. Benevolent governments provide public goods to maximize the utility of consumers. As the game has two stages, the game is solved backwardly.

2.2 First-best case

In this case, we consider the total utilities of municipalities merged throughout two periods. The objective function of the government at period 2 is

$$\max_{g_{12},\cdots,g_{N2}} \sum_i u(g_{i2} + \theta_2 \sum_{j \neq i} g_{j2}) \quad \text{s.t.} \quad \sum_i (\omega_{i2} - (1+r)b_i) = \sum_i g_{i2}. \tag{3}$$

We define the Lagrangian for period 2 as

$$\sum_i V_i = \sum_i u(g_{i2} + \theta_2 \sum_{j \neq i} g_{j2}) + \mu(\sum_i \omega_{i2} - (1+r)b_i - g_{i2}). \tag{4}$$

Solving this,

$$\frac{\partial \sum_i V_i}{\partial g_{i2}} = \sum_j \frac{\partial u_{j2}}{\partial g_{i2}} - \mu = 0 \tag{5}$$

can be obtained.

The objective function of the government at period 1 is

$$\max_{g_{11},\cdots,g_{N1},b_1,\cdots,b_N} \sum_i u(g_{i1} + \theta_1 \sum_{j \neq i} g_{j1}) + \delta \sum_i V_i \quad \text{s.t.} \quad \sum_i (\omega_{i1} + b_i) = \sum_i g_{i1}.$$

The Lagrangian for period 1 is

$$\sum_i W_i = \sum_i u(g_{i1} + \theta_1 \sum_{j \neq i} g_{j1}) + \lambda(\sum_i \omega_{i1} + b_i - g_{i1}) + \delta \sum_i V_i.$$

Solving this,

$$\begin{cases} \frac{\partial \sum_i W_i}{\partial g_{i1}} = \sum_j \frac{\partial u_{j1}}{\partial g_{i1}} - \lambda = 0 \\ \frac{\partial \sum_i W_i}{\partial b_i} = \lambda - \delta(1+r)\mu = 0 \end{cases}$$

are derived. Summarizing them and using (5),

$$\sum_j \frac{\partial u_{j1}}{\partial g_{i1}} = \delta(1+r) \sum_j \frac{\partial u_{j2}}{\partial g_{i2}} \tag{6}$$

[*7] To focus on the effect of the budget allocation of a municipal merger, it is assumed that the interest rate is exogenous in this model, and we assume that it is set in the international economy market.

can be obtained. Moreover, because $g_{i\tau} = g_{j\tau}$ is satisfied for any i, j in the equilibrium[*8], we obtain

$$\sum_j \frac{\partial u_{j\tau}}{\partial g_{i\tau}} = (1 + \theta_\tau(N-1))\frac{\partial u_{i\tau}}{\partial g_{i\tau}}.$$

Therefore, (6) is

$$(1 + \theta_1(N-1))\frac{\partial u_{i1}}{\partial g_{i1}} = (1 + \theta_2(N-1))\delta(1+r)\frac{\partial u_{i2}}{\partial g_{i2}}.$$

In addition, because $g_{i\tau} = g_{j\tau}$ is satisfied for any i, j[*9], we obtain

$$\frac{\partial u_{i\tau}}{\partial g_{i\tau}} = \frac{\partial u_{j\tau}}{\partial g_{j\tau}}, \quad \forall i, j. \tag{7}$$

Therefore, we can denote the marginal utility of public good provision, $\frac{\partial u_{i\tau}}{\partial g_{i\tau}} (= \frac{\partial u_{j\tau}}{\partial g_{j\tau}})$, as MU_τ. Using this, we can simplify this equation as

$$(1 + \theta_1(N-1))MU_1 = (1 + \theta_2(N-1))\delta(1+r)MU_2. \tag{8}$$

From this condition, we can see that the government issues debt considering the marginal (dis)utility of the debt issuance because the left-hand side of (8) shows the marginal utility of the debt issuance, while the right-hand side is the marginal disutility of the issuance. The marginal (dis)utility depends on the degree of spillover and the number of regions. This is because the government realizes that the spillovers exist after the municipalities have been merged.

2.3 Nonmerger case

In this case, municipalities are not merged or do not become merged, and thus exist independently[*10]. Each municipal government decides the amount of public goods provided and debt to maximize the utility of residents. Therefore, the objective function of a government at period 2 is

$$\max_{g_{i2}} u(g_{i2} + \theta_2 \sum_{j \neq i} g_{j2}) \quad \text{s.t.} \quad \omega_{i2} - (1+r)b_i = g_{i2} \tag{9}$$

and the Lagrangian for this objective function is

$$V_i = u(g_{i2} + \theta_2 \sum_{j \neq i} g_{j2}) + \mu_i(\omega_{i2} - (1+r)b_i - g_{i2}).$$

The first-order condition of this Lagrangian is

$$\frac{\partial V_i}{\partial g_{i2}} = \frac{\partial u_{i2}}{\partial g_{i2}} - \mu_i = 0. \tag{10}$$

[*8] See Appendix 1.
[*9] See Appendix 1.
[*10] We omit some similar calculations in the following cases.

For period 1, the objective function is

$$\max_{g_{i1},b_i} u(g_{i1} + \theta_1 \sum_{j\neq i} g_{j1}) + \delta V_i \quad \text{s.t.} \quad \omega_{i1} - b_i = g_{i1}$$

and the Lagrangian is

$$W_i = u(g_{i1} + \theta_1 \sum_{j\neq i} g_{j1}) + \lambda_i(\omega_{i1} + b_i - g_{i1}) + \delta V_i.$$

Solving this and using (10), we obtain

$$\frac{\partial u_{i1}}{\partial g_{i1}} = \delta(1+r)\frac{\partial u_{i2}}{\partial g_{i2}}. \tag{11}$$

Simplifying $\frac{\partial u_{i\tau}}{\partial g_{i\tau}}$ as $MU_{i\tau}$ gives

$$MU_{i1} = \delta(1+r)MU_{i2} \tag{12}$$

as the optimal condition for the non-merger case. Unlike the first-best case, governments do not consider the spillover in this case and it does not appear in (12). In addition, (7) is not satisfied here and instead

$$\frac{\partial u_{i\tau}}{\partial g_{i\tau}} \geq \frac{\partial u_{j\tau}}{\partial g_{j\tau}}, \quad \forall \omega_{i1} + \frac{\omega_{i2}}{1+r} \leq \omega_{j1} + \frac{\omega_{j2}}{1+r} \tag{13}$$

is obtained because the level of the marginal utility is different in each area and depends on the level of endowment.

2.4 Merger case

The merger case deals with the situation where municipalities exist independently at period 1 and become merged into a united municipality at period 2. At period 2, the government provides public goods to maximize the utility of all residents of all areas, while each government provides public goods to maximize the utility of each area at period 1.

The objective function of a government at period 2 is

$$\max_{g_{12},\cdots,g_{N2}} \sum_i u(g_{i2} + \theta_2 \sum_{j\neq i} g_{j2}) \quad \text{s.t.} \quad \sum_i (\omega_{i2} - (1+r)b_i) = \sum_i g_{i2}$$

and the Lagrangian is

$$\sum_i V_i = \sum_i u(g_{i2} + \theta_2 \sum_{j\neq i} g_{j2}) + \mu(\sum_i \omega_{i2} - (1+r)b_i - g_{i2}).$$

In fact, this objective function and the Lagrangian are exactly the same as (3) and (4), respectively. Therefore, we can use the same result as for period 2 of the first-best case, where the first-best condition is (5). Using (5), we can derive $g_{i2} = g_{j2}$ because the shape of the utility function is the same in each area*11. Therefore, we obtain $V_i = V_j$ here. This means that V_j is determined as the average level of the

*11 See Appendix 1.

total of utilities, $\frac{\sum_i V_i}{N}$ for any j.

The objective function of government i is

$$\max_{g_{i1}, b_i} W_i = u(g_{i1} + \theta_1 \sum_{j \neq i} g_{j1}) + \delta V_i \quad \text{s.t.} \quad \omega_{i1} + b_i = g_{i1}.$$

Note that $V_i = \frac{\sum_i V_i}{N}$ here because $V_i = V_j$. The Lagrangian of the objective function is

$$W_i = u(g_{i1} + \theta_1 \sum_{j \neq i} g_{j1}) + \lambda_i(\omega_{i1} + b_i - g_{i1}) + \delta V_i.$$

Solving this Lagrangian and using the result of (4),

$$\frac{\partial u_{i1}}{\partial g_{i1}} = \delta \frac{1+r}{N} \sum_j \frac{\partial u_{j2}}{\partial g_{i2}}$$

is obtained for any i. From this equation, we can confirm that $g_{i\tau} = g_{j\tau}$ is satisfied for any i, j in the equilibrium, and $\sum_j \frac{\partial u_{j2}}{\partial g_{i2}} = (1 + \theta_2(N-1))\frac{\partial u_{i2}}{\partial g_{i2}}$ and (7) hold. Thus,

$$\frac{\partial u_{i1}}{\partial g_{i1}} = \frac{1 + \theta_2(N-1)}{N} \delta(1+r) \frac{\partial u_{i2}}{\partial g_{i2}}$$

can be derived. Substituting MU_τ to $\frac{\partial u_{i\tau}}{\partial g_{i\tau}} (= \frac{\partial u_{j\tau}}{\partial g_{j\tau}})$, we have

$$MU_1 = \frac{1 + \theta_2(N-1)}{N} \delta(1+r) MU_2. \tag{14}$$

This implies that the marginal disutility of the debt issuance (the right-hand side of (14)) is discounted by the number of merged municipalities N. This is because the cost of debt repayment is shared among the merged municipalities after merger if a municipality issues debt before the merger. Therefore, the fiscal common pool problem may exist here. We also confirm that the spillover is considered here and is internalized at period 2.

2.5 Split case

In the separation case, each area is merged as a united municipality at period 1 and is divided into N municipalities at period 2. When municipalities are merged at period 1, a government provides public goods to maximize the welfare of residents of all areas. However, each government provides public goods to its own residents to maximize the utility of residents at period 2.

The objective function of each government at period 2 is

$$\max_{g_{i2}} u(g_{i2} + \theta_2 \sum_{j \neq i} g_{j2}) \quad \text{s.t.} \quad \omega_{i2} - (1+r)b_i = g_{i2}$$

and the Lagrangian for this objective function is

$$V_i = u(g_{i2} + \theta_2 \sum_{j \neq i} g_{j2}) + \mu_i(\omega_{i2} - (1+r)b_i - g_{i2}).$$

As these are the same functions as those at period 2 of the non-merger case, the first-order condition of the Lagrangian here is the same as (10).

The objective function of the government of the united municipality at period 1 is

$$\max_{g_{11},\cdots,g_{N1},b_{11},\cdots,b_{N1}} \sum_i W_i = \sum_i u(g_{i1} + \theta_1 \sum_{j\neq i} g_{j1}) + \delta \sum_i V_i \quad \text{s.t.} \quad \sum_i (\omega_{i1} + b_i) = \sum_i g_{i1},$$

and the Lagrangian is

$$\sum_i W_i = \sum_i u(g_{i1} + \theta_1 \sum_{j\neq i} g_{j1}) + \lambda \sum_i (\omega_{i1} + b_i - g_{i1}) + \delta \sum_i V_i.$$

Solving this Lagrangian and using (10), we obtain

$$\sum_j \frac{\partial u_{j1}}{\partial g_{i1}} = \delta(1+r)\frac{\partial u_{i2}}{\partial g_{i2}}.$$

As this condition holds for any i, $g_{i\tau} = g_{j\tau}$ can be derived from here*12. Therefore, we obtain

$$(1+\theta_1(N-1))\frac{\partial u_{i1}}{\partial g_{i1}} = \delta(1+r)\frac{\partial u_{i2}}{\partial g_{i2}}.$$

Simplifying $\frac{\partial u_{i\tau}}{\partial g_{i\tau}}$ as MU_τ gives

$$(1+\theta_1(N-1))MU_1 = \delta(1+r)MU_2. \tag{15}$$

This implies that separation of a municipality does not involve the cost-sharing effect among municipalities and the common pool effect disappears in the separation case. Moreover, we can see that the spillover effect is internalized at period 1 although it does not work at period 2.

3 Implications of the model

The results of the cases in the previous section are summarized in Table 1. The first-order conditions for determining the debt b_i is shown in Table 1.

Table 1: Summary of first-order conditions of b_i in each case

Case	b_i	
First-best	$(1+\theta_1(N-1))MU_1 = (1+\theta_2(N-1))\delta(1+r)MU_2$	(8)
Nonmerger	$MU_{i1} = \delta(1+r)MU_{i2}$	(12)
Merger	$MU_1 = \frac{1+\theta_2(N-1)}{N}\delta(1+r)MU_2$	(14)
Split	$(1+\theta_1(N-1))MU_1 = \delta(1+r)MU_2$	(15)

The results in Table 1 imply the following points. First, each condition shows the marginal utility and marginal disutility of debt issuance. The left-hand side of each condition in Table 1 is the marginal utility of public good provision at period 1. As increases in debt allow increases in public good provision at period 1 and the opposite at period 2, the left-hand side of each condition shows the marginal utility

*12 See Appendix 1.

of the debt issuance, while the right-hand side shows the marginal disutility. Moreover, because the marginal utility of public good provision at period 1 corresponds to the marginal utility of debt issuance, we can interpret the marginal disutility of debt issuance as the marginal utility of public good provision at period 2 evaluated at period 1. Second, we can see how the spillover effect is internalized in each case. The condition in the first-best case considers the spillover of both periods, while the conditions in other cases contain the spillover term in either period or neither period. The existence of a spillover effect influences whether municipalities become merged or not in this model. Third, the division of debt by N only exists in the merger case. This is because the debt repayment cost is shared between the N merged municipalities and this only occurs in the merger case. As we explain later, this is the fiscal common pool problem.

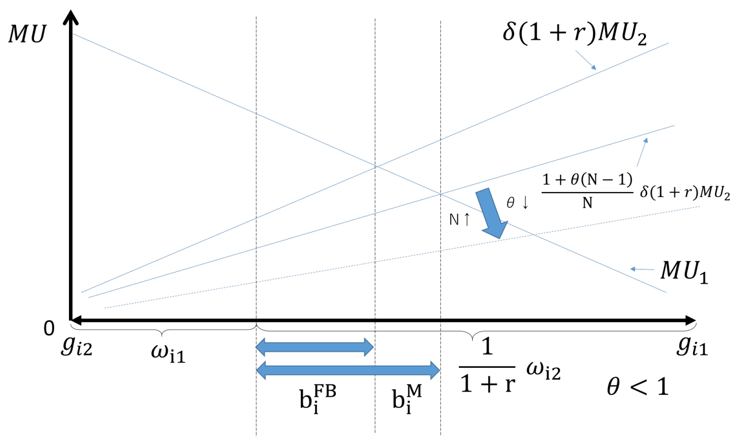
Referring to the conditions in Table 1, we obtain the following four propositions. First, we focus on the fiscal common pool problem.

Proposition 1 In the merger case, we find the existence of the fiscal common pool problem, namely, for any i, the debt issuance b_i is larger than the first-best case for $\theta_1 < 1$. For $\theta_1 = 1$, this problem vanishes in the sense that the debt issuance in the merger case corresponds to the first-best case.

(Proof) See Appendix 2.

This proposition shows that the amount of debt in the merger case is larger than in the first-best case except for the case of the perfect spillover. This phenomenon explains the empirical results, which show that municipalities issue excessive debt just before their mergers. As well as the debt, public goods at period 1 are provided excessively in the merger case compared with the first-best case because their cost is shared by N areas in period 2. However, the existence of a spillover in period 1 in which municipalities are not merged creates another adverse effect. The higher level of spillovers requires a higher level of public good provision because the marginal benefit is higher. When $\theta_1 = 1$ (the perfect spillover), the required higher level of public good provision in period 1 is matched by the excessive level of public good provision caused by the fiscal common pool problem, which achieves the first-best level.

Figure 1: Image of intertenporal budget allocation of the first-best case and the merger case.



This result is illustrated in Figure 1, which shows how b_i is determined in the first-best case or the merger case when $\theta_1 = \theta_2 = \theta$ is satisfied. The vertical axis in Figure 1 shows the marginal (dis)utility of the debt issuance and the horizontal axis shows the amount of public goods, endowment, and debt. The total endowment $\omega_{i1} + \frac{1}{1+r}\omega_{i2}$ equals the length of the horizontal axis, and the left part of it is ω_1, while the right part is $\frac{1}{1+r}\omega_2$. MU_1 is the left-hand side of (8) or (14) and is the marginal utility of public good provision at period 1, g_{i1}. MU_1 slopes down from left to right because the amount of g_{i1} is measured from the origin to the right and the marginal utility of g_{i1} decreases. If g_{i1} exceeds ω_{i1}, it is financed by debt b_i and repaid from $\frac{1}{1+r}\omega_{i2}$. The right-hand side of (8) and (14) correspond to $\delta(1+r)MU_2$ and $\frac{1+\theta(N-1)}{N}\delta(1+r)MU_2$, respectively. These are the marginal disutility of the debt issuance, or alternatively can be interpreted as the marginal utility of public good provision at period 2, g_{i2}, evaluated at period 1. These slope upward from left to right because a decrease in g_{i1} leads to an increase in g_{i2} and a decrease in b_i. From (8) and (14), we can see that amounts of g_{i1}, g_{i2}, and b_i are determined at the intersection of the marginal (dis)utility of debt. As the slope of $\frac{1+\theta(N-1)}{N}\delta(1+r)MU_2$ is flatter than $\delta(1+r)MU_2$, the intersection of the merger case is to the right of the intersection with the first-best case. As the intersection on the right requires more debt issuance, b^m becomes larger than b^{fb}. If N increases or θ decreases, then the slope of $\frac{1+\theta(N-1)}{N}\delta(1+r)MU_2$ becomes much flatter, which increases b^m.

This result shows that the fiscal common pool problem is less severe when the spillover of the public goods is large. However, even at higher levels of spillover, the fiscal common pool problem remains. Some empirical studies such as Hinnerich (2009) also show that smaller municipalities have a stronger incentive to free ride than larger municipalities. Related to this, we have the following proposition.

Proposition 2 In the merger case, a municipality with a smaller endowment issues more debt than one with a larger endowment.

(Proof) See Appendix 3.

In the merger case, the budget of each municipality is pooled at period 2, irrespective of its endowment, and the amount of public goods is equalized at period 2. As a result, a municipality with a smaller endowment issues more debt than one with a larger endowment. This result shows that poorer municipalities enjoy the benefits from the municipal merger more than richer municipalities.

Figure 2: Image of equalization effect

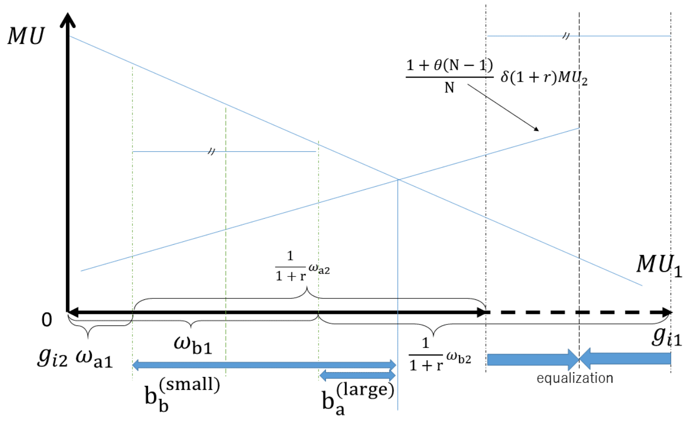


Figure 2 illustrates Proposition 2. Figure 2 is similar to Figure 1, but the two figures have different endowment sizes for a and b. We can see $\omega_{a1} > \omega_{b1}$ leads to $b_b > b_a$ from Figure 2[*13]. As explained in Figure 1, the amount of g_{i1} is determined at the intersection of MU_1 and $\frac{1+\theta(N-1)}{N}\delta(1+r)MU_2$. However, since the endowment ω_{i1} is not enough for providing g_{i1}, governments borrows b_i from period 2. All municipalities in this case face the same intersection of MU_1 and $\frac{1+\theta(N-1)}{N}\delta(1+r)MU_2$ because of equalization, while endowments differ between regions. Therefore, a municipality with a small endowment borrows a substantial amount, while one with a large endowment does not borrow as much.

Note that in Proposition 2, only the endowment at period 1 is important and the endowment at period 2 does not matter. The budget constraint at period 2 is merged into one budget constraint such that $\sum(\omega_{i2} - (1+r)b_i) = \sum g_{i2}$ and b_i will be

$$b_i = \frac{1}{1+r}[\sum(\omega_{i2} - g_{i2})] - \sum_{j \neq i} b_j.$$

Thus, the size of b_i does not depend on the size of each endowment ω_{i2}, instead depends on the size of the summed-up value of them.

For the separation case, we show two similar propositions to the merger case, although the mechanism there is different.

Proposition 3 In the separation case, the debt issuance b_i for any i is larger than the first-best case for $\theta_2 > 0$. For $\theta_2 = 0$, the debt issuance in the separation case corresponds to the first-best case.

(Proof) See Appendix 4.

As a result, this proposition is very similar to Proposition 1. However, interestingly, they are very different in the sense that the factors causing the increase in b_i differ. In the merger case, there exists the fiscal common pool effect, where the burden of debt issuance is shared and is $\frac{1}{N}$ of the original burden, and this effect causes overissuance of b_i. However, in the separation case, the spillover effect is internalized at period 1 because all municipalities are merged at that point. This means that the government evaluates the spillover effect and provides the public good at the correct level at period 1. However, governments in this case provide fewer public goods than the first-best case at period 2 because they do not consider the spillover effect. As a result, the resources used for public good provision are allocated more to period 1 and less to period 2, which makes the debt issuance in the separation case larger than the first-best case. However, if there is no spillover effect at period 2, the public good provision at period 2 will be the proper level and corresponds to the first-best case. Therefore, the debt level in the separation case is the same as in the first-best case at $\theta_2 = 0$.

Similar to the merger case, we present a proposition about the endowment in the separation case.

Proposition 4 In the separation case, a municipality with a small endowment repays a smaller amount of debt b_i than one with a large endowment.

(Proof) See Appendix 5.

This proposition is very similar to Proposition 2 because the burden of debt repayment for a munici-

[*13] $\omega_{a2} = \omega_{b2}$ is assumed in Figure 2 for simplicity.

pality with a small endowment is lower than one with a large endowment. This is because the budget at period 1 is pooled and the amount of the public good for each municipality is equalized in the separation case. This proposition implies that only endowments in period 2 affect the levels of b_i and endowments at period 1 do not because the budget constraint at period 1 is merged and the endowment becomes equalized, while the budget constraint at period 2 is not merged and is not equalized in this case.

4 Welfare implications

In this section, we focus on social welfare. To obtain clear results, we calculate the welfare of each case by specifying the utility function as a natural logarithm function, $u(g) = \log g$. In addition, we denote $\bar{\omega}_\tau = \frac{1}{N}(\sum_i \omega_{i\tau})$. Now we obtain the following results:

$$\text{First-best}\begin{cases} g_1 = \frac{1+\theta_1(N-1)}{1+\theta_1(N-1)+\delta(1+\theta_2(N-1))}(\bar{\omega}_1 + \frac{1}{1+r}\bar{\omega}_2) \\ g_2 = \frac{\delta(1+\theta_2(N-1))}{1+\theta_1(N-1)+\delta(1+\theta_2(N-1))}((1+r)\bar{\omega}_1 + \bar{\omega}_2) \end{cases}$$
$$\text{Nonmerger}\begin{cases} g_{i1} = \frac{1}{1+\delta}(\omega_{i1} + \frac{1}{1+r}\omega_{i2}) \\ g_{i2} = \frac{\delta}{1+\delta}((1+r)\omega_{i1} + \omega_{i2}) \end{cases}$$
$$\text{Merger}\begin{cases} g_1 = \frac{N}{N+\delta(1+\theta_2(N-1))}(\bar{\omega}_1 + \frac{1}{1+r}\bar{\omega}_2) \\ g_2 = \frac{\delta(1+\theta_2(N-1))}{N+\delta(1+\theta_2(N-1))}((1+r)\bar{\omega}_1 + \bar{\omega}_2) \end{cases}$$
$$\text{Split}\begin{cases} g_1 = \frac{1+\theta_1(N-1)}{1+\theta_1(N-1)+\delta}(\bar{\omega}_1 + \frac{1}{1+r}\bar{\omega}_2) \\ g_2 = \frac{\delta}{1+\theta_1(N-1)+\delta}((1+r)\bar{\omega}_1 + \bar{\omega}_2) \end{cases}.$$

Furthermore, assume $\theta_1 = \theta_2 = \theta$, $\delta = 1$, and $\omega_i = \omega_{i1} = \omega_{i2}$, then we obtain the following utility level for area i:

$$\begin{aligned} W_i^{fb} &= 2\log\frac{(2+r)\bar{\omega}}{2} - \log(1+r) \\ W_i^{n} &= 2\log\frac{(2+r)\omega_i}{2} - \log(1+r) \\ W_i^{m} &= 2\log\frac{2+r}{N+1+\theta(N-1)}\bar{\omega} + \log\frac{N}{1+r} + \log(1+\theta(N-1)) \\ W_i^{s} &= 2\log\frac{2+r}{2+\theta(N-1)}\bar{\omega} + \log\frac{1+\theta(N-1)}{1+r}. \end{aligned}$$

Comparing them, we obtain

$$\sum_i (W_i^n - W_i^m) = \sum_i [2\log \frac{N + (1+\theta(N-1))}{2}\frac{\omega_i}{\bar{\omega}} - \log(N(1+\theta(N-1)))]$$
$$= \underbrace{2N\{\log \frac{N + (1+\theta(N-1))}{2} - \log(N(1+\theta(N-1)))^{1/2}\}}_{\geq 0} + \underbrace{2(\sum_i \log\omega_i - N\log\bar{\omega})}_{\leq 0} \tag{16}$$

$$\sum_i (W_i^n - W_i^s) = \sum_i [2\log \frac{1 + (1+\theta(N-1))}{2}\frac{\omega_i}{\bar{\omega}} - \log(1+\theta(N-1))]$$
$$= \underbrace{2N\{\log \frac{1 + (1+\theta(N-1))}{2} - \log(1+\theta(N-1))^{1/2}\}}_{\geq 0} + \underbrace{2(\sum_i \log\omega_i - N\log\bar{\omega})}_{\leq 0}. \tag{17}$$

These results have several implications. First, the second terms of (16) and (17) are nonpositive because the log function is concave. When there exists a difference between endowments of municipalities, it becomes negative. Second, the first terms of (16) and (17) are nonnegative because of the inequality of the arithmetic mean and geometric mean. In particular, when $\theta = 1$, the first term of (16) is zero. Therefore, when $\theta = 1$ and there exists a difference between endowments of municipalities, (16) becomes negative, meaning that a municipal merger improves social welfare. This is because the fiscal common pool problem is fully offset by the internalization effect of the spillover and the equalization by merger has a desirable effect. Third, when $\theta = 0$, the first term of (17) is zero. Therefore, when $\theta = 0$ and there exists a difference between the endowments of municipalities, (17) also becomes negative, meaning that the separation case improves social welfare. This is simply because the fiscal common pool problem does not exist in the separation case; therefore, there is no disadvantage from the separation in period 2 in the economy without spillovers and the equalization in the merged period 1 creates a desirable effect. Fourth, if $\omega_i = \omega_j$ holds for all i and j, these second terms will be zero. This implies that (16) and (16) are nonnegative. In particular, the first term of (16) is positive, when θ is between 0 and 1. Therefore, when θ is between 0 and 1 and there is no difference between the endowments of municipalities, the non-merger case becomes desirable. This is because the additional equalization benefit does not exist in either the merger or the separation cases, the fiscal common pool problem arises only in the merger case, and the internalization of spillovers only in period 1 induces biased intertemporal allocation of resources in the separation case. Finally, regarding the difference in welfare between the merger case and the separation case, we have

$$W_i^m - W_i^s = 2\{\log N^{1/2} - \log \frac{1+\theta(N-1)+N}{2+\theta(N-1)}\}.$$

From this, we can derive that $W_i^m > W_i^s$ holds if θ is larger than $\underline{\theta} \equiv \frac{\sqrt{N}-1}{N-1}$[*14].

[*14] Assuming $W_i^m \geq W_i^s$ and defining $A \equiv 1+\theta(N-1)$, we obtain

$$W_i^m - W_i^s = 2\{\log N^{1/2} - \log \frac{A+N}{A+1}\} \geq 0.$$

5 Extension: Model with private consumption

In this section, we consider a model that incorporates private consumption. The setting is basically the same as the model analyzed in the previous sections, although some of the implications will be modified. The existence of private consumption makes debt issuance in the merger case always excessive because private consumption has no spillover effect and the burden of the fiscal common pool problem will be excessive even if $\theta_1 = 1$. However, most of the results obtained in the previous sections still hold in this plausible setting.

5.1 Setting

The basic setting is similar to that in the previous sections. In each municipality $i = 1, \cdots, N$, there is a government and a representative consumer who enjoys private consumption $x_{i\tau}$ and public good $g_{i\tau}$ at period τ. The private consumption $x_{i\tau}$ is a numeraire, and can be consumed or used as an input for producing the public good $g_{i\tau}$. The resource for producing $g_{i\tau}$ is financed by lump-sum tax $T_{i\tau}$ or debt d_i. Governments set their tax rate in each period and decide their debt level d_i at period 1. A consumer in i at τ has the utility function $u_{i\tau} = u(x_{i\tau}, g_{i\tau} + \theta_\tau \sum_{j\neq i} g_{j\tau}) = u^x(x_{i\tau}) + u^g(g_{i\tau} + \theta_\tau \sum_{j\neq i} g_{j\tau})$, where $u'^x > 0 > u''^x$ and $u'^g > 0 > u''^g$ are assumed. In addition, the shape of the utility function is assumed to be the same among municipalities. In this model setting, denoting $G_{i\tau} \equiv g_{i\tau} + \theta_\tau \sum_{j\neq i} g_{j\tau}$, efficient public provision is satisfied when $(\frac{\partial u_{i\tau}}{\partial G_{i\tau}})/(\frac{\partial u_{i\tau}}{\partial x_{i\tau}}) + (\theta_\tau \sum_{j\neq i} \frac{\partial u_{j\tau}}{\partial G_{j\tau}})/(\frac{\partial u_{i\tau}}{\partial x_{i\tau}}) = 1$, or equivalently, $(\sum_j \frac{\partial u_{j\tau}}{\partial g_{i\tau}})/\frac{\partial u_{i\tau}}{\partial x_{i\tau}} = 1$ holds from Samuelson's condition.

Consumers buy private consumption within their budget constraint. They obtain endowment $\omega_{i\tau}$ in each period. The value of $\omega_{i\tau}$ can be varied in each area and each period. Thus, this model considers asymmetric areas. The net income of the consumer in i is

$$\begin{cases} x_{i1} + T_{i1} = \omega_{i1} - s_i \\ x_{i2} + T_{i2} = \omega_{i2} + (1+r)s_i \end{cases} \tag{18}$$

where s_i is savings and r is the interest rate on savings.

Governments can issue debt d_i at period 1, which must be repaid at period 2. Then, the governmental

This condition holds if and only if $\sqrt{N} \geq \frac{A+N}{A+1}$. Therefore, we can see that

$$\begin{aligned} &\sqrt{N} \geq \frac{A+N}{A+1} \\ &\sqrt{N}(A+1) \geq A+N \\ &A(\sqrt{N}-1) + \sqrt{N}(1-\sqrt{N}) \geq 0 \\ \text{and} \quad &(A-\sqrt{N})(\sqrt{N}-1) \geq 0 \end{aligned}$$

are the necessary and sufficient conditions for $W_i^m \geq W_i^s$. Note that $\sqrt{N} \geq 1$ holds for $N \geq 2$. Therefore, $W_i^m \geq W_i^s$ will hold if $A \geq \sqrt{N}$, i.e. $1 + \theta(N-1) \geq \sqrt{N}$. From this, we can obtain $\underline{\theta} = \frac{\sqrt{N}-1}{N-1}$.

budget is

$$\begin{cases} g_{i1} = T_{i1} + d_i \\ g_{i2} = T_{i2} - (1+r)d_i \end{cases} \tag{19}$$

where r is the common interest rate on debt and savings.

The total utility of consumer i is

$$W_i \equiv u^x(x_{i1}) + u^g(g_{i1} + \theta_1 \sum_{j \neq i} g_{j1}) + \delta\{u^x(x_{i2}) + u^g(g_{i2} + \theta_2 \sum_{j \neq i} g_{j2})\}. \tag{20}$$

The consumer allocates his/her resources intertemporally by deciding savings, s_i, and the government does so intertemporally by setting the amount of bond, d_i. However, in this model, it is simply assumed that the benevolent government maximizes the utility of the representative consumer in the region. When combining the budget constraints of the consumer and the government in the region, we note that the utility in the region depends on only the net budget allocation between two periods, namely $d_i - s_i$, which is defined as b_i: the net debt. Therefore, we focus on the net debt level, b_i. Hereafter, we examine how the government (the representative consumer) adjusts b_i to maximize the utility in the region.

In addition to the model without private consumption, we consider four cases: 1) the first-best case, where municipalities are merged over two periods; 2) the non-merger case, where municipalities are independent over two periods; 3) the merger case, where municipalities become merged at period 2; and 4) the separation case, where merged municipalities are separated at period 2. Next, we present the objective function in each case and the results.

5.2 Solutions and results of cases analyzed

5.2.1 First-best case

The objective functions in this case are

$$\begin{aligned} &\max_{g_{12},\cdots,g_{N2},x_{12},\cdots,x_{N2}} \sum_i [u^x(x_{i2}) + u^g(g_{i2} + \theta_2 \sum_{j \neq i} g_{j2})] \\ &\text{s.t.} \quad \sum (x_{i2} + g_{i2}) = \sum (\omega_{i2} - (1+r)b_i) \end{aligned}$$

for period 2 and

$$\begin{aligned} &\max_{g_{11},\cdots,g_{N1},x_{11},\cdots,x_{N1},b_1,\cdots,b_N} \sum_i [u^x(x_{i1}) + u^g(g_{i1} + \theta_1 \sum_{j \neq i} g_{j1})] + \delta V \\ &\text{s.t.} \quad \sum (x_{i1} + g_{i1}) = \sum (\omega_{i1} + b_i) \end{aligned}$$

for period 1. Note that V is the optimized welfare at period 2, namely $V = \sum_i [u^x(x_{i2}^*) + u^g(g_{i2}^* + \theta_2 \sum_{j \neq i} g_{j2}^*)]$. Solving these, we obtain the following three first-order conditions for $(x_{i2}, g_{i2}), (x_{i1}, g_{i1})$, and b_i:

$$(\frac{\partial u_{i2}}{\partial G_{i2}})/(\frac{\partial u_{i2}}{\partial x_{i2}})+(\theta_2\sum_{j\neq i}\frac{\partial u_{j2}}{\partial G_{j2}})/(\frac{\partial u_{i2}}{\partial x_{i2}})=1 \tag{21}$$

$$(\frac{\partial u_{i1}}{\partial G_{i1}})/(\frac{\partial u_{i1}}{\partial x_{i1}})+(\theta_1\sum_{j\neq i}\frac{\partial u_{j1}}{\partial G_{j1}})/(\frac{\partial u_{i1}}{\partial x_{i1}})=1 \tag{22}$$

$$\frac{\partial u_{i1}}{\partial x_{i1}}/\frac{\partial u_{i2}}{\partial x_{i2}}=(1+r)\delta. \tag{23}$$

In addition, $x_{i\tau}=x_{j\tau}, g_{i\tau}=g_{j\tau}$ for any $i\neq j$ can be also derived in this case*[15].

5.2.2 Nonmerger case

The objective functions in this case are

$$\begin{aligned}&\max_{g_{i2},x_{i2}} u^x(x_{i2})+u^g(g_{i2}+\theta_2\sum_{j\neq i}g_{j2})\\ &\text{s.t.}\quad x_{i2}+g_{i2}=\omega_{i2}-(1+r)b_i\end{aligned}$$

for period 2 and

$$\begin{aligned}&\max_{g_{i1},x_{i1},b_i} u^x(x_{i1})+u^g(g_{i1}+\theta_1\sum_{j\neq i}g_{j1})+\delta V_i\\ &\text{s.t.}\quad x_{i1}+g_{i1}=\omega_{i1}+b_i\end{aligned}$$

for period 1. Note that V_i is the optimized i's welfare at period 2, namely $V_i=u^x(x^*_{i2})+u^g(g^*_{i2}+\theta_2\sum_{j\neq i}g^*_{j2})$. Solving these, we obtain the following three first-order conditions for $(x_{i2},g_{i2}),(x_{i1},g_{i1})$, and b_i:

$$\frac{\partial u_{i2}}{\partial g_{i2}}/\frac{\partial u_{i2}}{\partial x_{i2}}=1 \tag{24}$$

$$\frac{\partial u_{i1}}{\partial g_{i1}}/\frac{\partial u_{i1}}{\partial x_{i1}}=1 \tag{25}$$

$$\frac{\partial u_{i1}}{\partial x_{i1}}/\frac{\partial u_{i2}}{\partial x_{i2}}=(1+r)\delta. \tag{26}$$

5.2.3 Merger case

The objective functions in this case are

$$\begin{aligned}&\max_{g_{12},\cdots,g_{N2},x_{12},\cdots,x_{N2}} \sum_i[u^x(x_{i2})+u^g(g_{i2}+\theta_2\sum_{j\neq i}g_{j2})]\\ &\text{s.t.}\quad \sum(x_{i2}+g_{i2})=\sum(\omega_{i2}-(1+r)b_i)\end{aligned}$$

for period 2 and

$$\begin{aligned}&\max_{g_{i1},x_{i1},b_i} u^x(x_{i1})+u^g(g_{i1}+\theta_1\sum_{j\neq i}g_{j1})+\delta V_i\\ &\text{s.t.}\quad x_{i1}+g_{i1}=\omega_{i1}+b_i\end{aligned}$$

*[15] See Appendix 1.

for period 1. Note that V_i is the optimized i's welfare at period 2, namely $V_i = \frac{V}{N} = \frac{1}{N}\sum_i [u^x(x_{i2}) + u^g(g_{i2} + \theta_2 \sum_{j \neq i} g_{j2})]$. Solving these, we obtain three equations from the first-order conditions for $(x_{i2}, g_{i2}), (x_{i1}, g_{i1})$, and b_i:

$$(\frac{\partial u_{i2}}{\partial G_{i2}})/(\frac{\partial u_{i2}}{\partial x_{i2}}) + (\theta_2 \sum_{j \neq i} \frac{\partial u_{j2}}{\partial G_{j2}})/(\frac{\partial u_{i2}}{\partial x_{i2}}) = 1 \tag{27}$$

$$\frac{\partial u_{i1}}{\partial g_{i1}} / \frac{\partial u_{i1}}{\partial x_{i1}} = 1 \tag{28}$$

$$\frac{\partial u_{i1}}{\partial x_{i1}} / \frac{\partial u_{i2}}{\partial x_{i2}} = \frac{1+r}{N}\delta. \tag{29}$$

In addition, $x_{i\tau} = x_{j\tau}, g_{i\tau} = g_{j\tau}$ for any $i \neq j$ can also be derived in this case[*16].

5.2.4 Split case

The objective functions in this case are

$$\max_{g_{i2}, x_{i2}} u^x(x_{i2}) + u^g(g_{i2} + \theta_2 \sum_{j \neq i} g_{j2})$$
$$\text{s.t.} \quad x_{i2} + g_{i2} = \omega_{i2} - (1+r)b_i$$

for period 2 and

$$\max_{g_{11}, \cdots, g_{N1}, x_{11}, \cdots, x_{N1}, b_1, \cdots, b_N} \sum_i^n [u^x(x_{i1}) + u^g(g_{i1} + \theta_1 \sum_{j \neq i} g_{j1}) + \delta V_i]$$
$$\text{s.t.} \quad \sum(\omega_{i1} + b_i) = \sum(x_{i1} + g_{i1})$$

for period 1. Note that V_i is the optimized i's welfare at period 2, namely $V_i = u^x(x^*_{i2}) + u^g(g^*_{i2} + \theta_2 \sum_{j \neq i} g^*_{j2})$. Solving these, we obtain three equations from the first-order conditions for $(x_{i2}, g_{i2}), (x_{i1}, g_{i1})$, and b_i:

$$\frac{\partial u_{i2}}{\partial g_{i2}} / \frac{\partial u_{i2}}{\partial x_{i2}} = 1 \tag{30}$$

$$(\frac{\partial u_{i1}}{\partial G_{i1}})/(\frac{\partial u_{i1}}{\partial x_{i1}}) + (\theta_1 \sum_{j \neq i} \frac{\partial u_{j1}}{\partial G_{j1}})/(\frac{\partial u_{i1}}{\partial x_{i1}}) = 1 \tag{31}$$

$$\frac{\partial u_{i1}}{\partial x_{i1}} / \frac{\partial u_{i2}}{\partial x_{i2}} = (1+r)\delta. \tag{32}$$

In addition, $x_{i\tau} = x_{j\tau}, g_{i\tau} = g_{j\tau}$ for any $i \neq j$ can also be derived in this case[*17].

5.3 Implications of the model

We summarize the results from this model in Table 2.

Table 2: Summary of the first-order conditions in each case

[*16] See Appendix 1.
[*17] See Appendix 1.

Case	Period 1	Period 2	b_i
First-best	$(\frac{\partial u_{i1}}{\partial G_{i1}})/(\frac{\partial u_{i1}}{\partial x_{i1}}) + (\theta_1 \sum_{j \neq i} \frac{\partial u_{j1}}{\partial G_{j1}})/(\frac{\partial u_{i1}}{\partial x_{i1}}) = 1$	$(\frac{\partial u_{i2}}{\partial G_{i2}})/(\frac{\partial u_{i2}}{\partial x_{i2}}) + (\theta_2 \sum_{j \neq i} \frac{\partial u_{j2}}{\partial G_{j2}})/(\frac{\partial u_{i2}}{\partial x_{i2}}) = 1$	$\frac{\partial u_{i1}}{\partial x_{i1}} / \frac{\partial u_{i2}}{\partial x_{i2}} = (1+r)\delta$
Nonmerger	$\frac{\partial u_{i1}}{\partial g_{i1}} / \frac{\partial u_{i1}}{\partial x_{i1}} = 1$	$\frac{\partial u_{i2}}{\partial g_{i2}} / \frac{\partial u_{i2}}{\partial x_{i2}} = 1$	$\frac{\partial u_{i1}}{\partial x_{i1}} / \frac{\partial u_{i2}}{\partial x_{i2}} = (1+r)\delta$
Merger	$\frac{\partial u_{i1}}{\partial g_{i1}} / \frac{\partial u_{i1}}{\partial x_{i1}} = 1$	$(\frac{\partial u_{i2}}{\partial G_{i2}})/(\frac{\partial u_{i2}}{\partial x_{i2}}) + (\theta_2 \sum_{j \neq i} \frac{\partial u_{j2}}{\partial G_{j2}})/(\frac{\partial u_{i2}}{\partial x_{i2}}) = 1$	$\frac{\partial u_{i1}}{\partial x_{i1}} / \frac{\partial u_{i2}}{\partial x_{i2}} = \frac{1+r}{N}\delta$
Split	$(\frac{\partial u_{i1}}{\partial G_{i1}})/(\frac{\partial u_{i1}}{\partial x_{i1}}) + (\theta_1 \sum_{j \neq i} \frac{\partial u_{j1}}{\partial G_{j1}})/(\frac{\partial u_{i1}}{\partial x_{i1}}) = 1$	$\frac{\partial u_{i2}}{\partial g_{i2}} / \frac{\partial u_{i2}}{\partial x_{i2}} = 1$	$\frac{\partial u_{i1}}{\partial x_{i1}} / \frac{\partial u_{i2}}{\partial x_{i2}} = (1+r)\delta$

With private consumption, we can see which case satisfies Samuelson's condition, $(\frac{\partial u_{i\tau}}{\partial G_{i\tau}})/(\frac{\partial u_{i\tau}}{\partial x_{i\tau}}) + (\theta_\tau \sum_{j \neq i} \frac{\partial u_{j\tau}}{\partial G_{j\tau}})/(\frac{\partial u_{i\tau}}{\partial x_{i\tau}}) = 1$. It is clear that this condition only holds when municipalities become merged, and thus spillovers are internalized. In addition, the first-order condition about b_i is different only in the merger case compared with the other cases. This is because municipal mergers cause the fiscal common pool problem and the burden of issued debt is shared in the merged municipalities as a whole. However, in the other cases, such cost sharing does not occur.

The implications drawn from this model are similar to the model without private consumption. For example, Propositions 2-4 hold even in this model and their proofs are in the Appendix. However, Proposition 1 is modified in this model as follows.

Proposition 5 In the merger case in the model with private consumption, the fiscal common pool problem always occurs and, for any i, the debt issuance b_i is always larger than in the first-best case. This is satisfied under any levels of spillover.

(Proof) See Appendix 6.
Note that $b_i^{fb} < b_i^m$ always holds in this case regardless of the value of θ_1. This differs from the result of Proposition 1, but why does this result hold here? The reason is that private consumption does not have a spillover effect and the excessive resource allocation at period 1, because the cost shared among the merged municipalities at period 2 is still excessive even under a perfect spillover for the public good at period 1. In reality, we have both private consumption and the public good. Therefore, this result is compatible with the fact that municipal mergers in the real world involve the fiscal common pool problem. This may be the reason why most empirical studies observe excessive debt before municipal mergers.

6 Discussion

The results obtained in this paper imply that municipal mergers are usually accompanied by the fiscal common pool problem, namely municipalities issue excessive debt before their mergers within the framework of our two-period model. Welfare comparisons between cases show that municipal mergers may reduce welfare. In particular, this negative scenario occurs in the situation where the degree of spillover is lower and the number of municipalities is larger. However, welfare in the merger case may improve and the negative scenario may disappear when considering the more plausible general framework of the model, as described below.

The first plausible framework is a model with a longer time period $M(>2)$ than the two-period model considered above, keeping the number of municipalities fixed. The merger occurs in period $1 < h < M$. How does the merger affect welfare in this longer period situation? To understand the structure of the model with M periods, we consider the three effects separately: the fiscal common pool problem, internalization of externalities, and equalization of budgets. Consider the simple case where the municipality is symmetric and the degree of spillover is the same throughout the period to eliminate the equalization effect. In such a situation, the intertemporal budget allocation is determined repeatedly as in the non-merger case before the merger, while it is decided repeatedly as in the first-best case after the merger. This means that the intertemporal budget allocation between t and $t+1$ is

$$MU_t = \delta(1+r)MU_{t+1} \quad \text{for} \quad t \in [1, h-1] \tag{33}$$

$$(1+\theta_t(N-1))MU_t = \delta(1+r)(1+\theta_{t+1}(N-1))MU_{t+1} \quad \text{for} \quad t \in [h+1, M]. \tag{34}$$

Note that $\theta_t = \theta_{t+1}$ is assumed here and (34) can be reduced to (33). The fiscal common pool problem and the internalization of externalities occur only when the merger is executed, namely at period h[*18], and the intertemporal budget allocation will then be

$$MU_h = \delta(1+r)\frac{1+\theta_{h+1}(N-1)}{N}MU_{h+1}. \tag{35}$$

As the intertemporal budget allocations before and after merger are determined repeatedly by (33) and (34), respectively, we can focus on the timing when the merger occurs, namely period h, to examine the effects of strategic intertemporal budget allocation. This means that the relationship between the fiscal common pool problem and the internalization of externalities is unchanged even if the period lengthens. Furthermore, this implies that our two-period model successfully describes the essence of the strategic intertemporal budget allocation associated with the municipal merger.

In addition, in the situation where the municipality is not symmetric, namely there exist differences between municipalities, the equalization effect also occurs after the merger. This additional effect operates without increasing the fiscal common pool problem and the effect accumulates as the period lengthens, which increases social welfare. Therefore, mergers among asymmetric municipalities increase social welfare beyond the non-merger case and the effects of the merger improve as the time period lengthens.

The second plausible framework involves the existence of a corrective policy to solve the fiscal common pool problem in period 0. One possible corrective policy is to prohibit the pooling of debts (or savings) after a merger and force municipalities to repay their own debts (or enjoy their own savings). If the debts created in the period before a merger are not pooled in the period after the merger, then there is no incentive to issue debt excessively and the fiscal common pool problem disappears. This is a very simple policy. It may be difficult to implement once the government has been merged. However, we can observe such situations in the real world[*19]. Another corrective policy to solve the fiscal common pool problem is to regulate the issuance of debt, namely setting a debt ceiling.[*20]. If the ceiling is set at the correct level, the fiscal common pool problem will be solved because municipalities cannot issue debt excessively. However, if it cannot be set at the correct level because of imperfect information, the result will be a biased intertemporal budget allocation and reduction in welfare.

[*18] Please note that the equalization can be ignored in the symmetric case.

[*19] For example, Japanese municipalities can designate a part of their property as belonging to a special property ward (an organization given legal identity as a special local government). The properties that belong to the special property ward cannot be pooled for the merged municipalities and can only be used by the former municipalities.

[*20] In South Africa, municipalities to be merged are prohibited from entering new contracts to borrow money.

7 Conclusion

Using a simple model, this paper is the first attempt to examine the mechanisms of the fiscal common pool problem caused by municipal mergers, despite a substantial number of empirical studies about the fiscal common pool problem and mergers. The results of this paper show that municipal mergers cause the fiscal common pool problem, although the gravity of the problem is affected by the degree of the spillover effect of the public good. In addition, we also find that the separation of municipalities may increase debt issuance before separation, while the mechanism of debt issuance can be explained by the internalization of externalities, not by the fiscal common pool problem. These results can be applied to designing policies aimed at diminishing the fiscal common pool problem and increasing social welfare.

Appendix

Appendix 1: Derivation of $g_{i\tau} = g_{j\tau}, x_{i\tau} = x_{j\tau}$ for any $i \neq j$.

We show $g_{i\tau} = g_{j\tau}, x_{i\tau} = x_{j\tau}$ for any $i \neq j$. These conditions are derived if conditions $\frac{\partial u_{i\tau}}{\partial g_{i\tau}} = \frac{\partial u_{j\tau}}{\partial g_{j\tau}}$ and $\frac{\partial u_{i\tau}}{\partial x_{i\tau}} = \frac{\partial u_{j\tau}}{\partial x_{j\tau}}$ hold. These can be derived in the first-best case, the merger case, and the separation case. We provide the derivations here.

$\frac{\partial u_{i\tau}}{\partial g_{i\tau}} = \frac{\partial u_{l\tau}}{\partial g_{l\tau}}$ holds for any $i, l(i \neq l)$ at both periods 1 and 2 in the first-best case, at period 2 in the merger case, and at period 1 in the separation case.

(Proof)

From the first-order condition for each Lagrangian, $\sum_j \frac{\partial u_{j2}}{\partial g_{i2}} = \mu$ or $\sum_j \frac{\partial u_{j1}}{\partial g_{i1}} = \lambda$ holds. Thus, $\sum_j \frac{\partial u_{j\tau}}{\partial g_{i\tau}} = \sum_h \frac{\partial u_{h\tau}}{\partial g_{l\tau}}$ is shown for any $i, l(i \neq l)$ in each case.

Denote $G_{i\tau}$ as $G_{i\tau} \equiv g_{i\tau} + \sum_{j\neq i} \theta g_{j\tau}$. As $u_{i\tau} = u(g_{i\tau} + \sum_{j\neq i} \theta g_{j\tau})$ (or $u_{i\tau} = u^x(x_{i\tau}) + u^g(g_{i\tau} + \sum_{j\neq i} \theta g_{j\tau})$), $\frac{\partial G_{i\tau}}{\partial g_{l\tau}} = \theta$, and $\frac{\partial G_{i\tau}}{\partial g_{i\tau}} = 1$ for $i, l(i \neq l)$ can be shown, $\frac{\partial u_{i\tau}}{\partial g_{l\tau}} = \frac{\partial u_{i\tau}}{\partial G_{i\tau}}\theta$, $\frac{\partial u_{i\tau}}{\partial g_{i\tau}} = \frac{\partial u_{i\tau}}{\partial G_{i\tau}}$ are satisfied for $i, l(i \neq l)$. Considering this, for any $i, l(i \neq l)$

$$
\begin{aligned}
&\frac{\sum_j^n \partial u_{j\tau}}{\partial g_{i\tau}} = \frac{\sum_h^n \partial u_{h\tau}}{\partial g_{l\tau}} \\
&\frac{\partial u_{i\tau}}{\partial g_{i\tau}} + \theta \sum_{j\neq i} \frac{\partial u_{j\tau}}{\partial G_{i\tau}} = \frac{\partial u_{l\tau}}{\partial g_{l\tau}} + \theta \sum_{h\neq l} \frac{\partial u_{h\tau}}{\partial G_{l\tau}} \\
&\frac{\partial u_{i\tau}}{\partial g_{i\tau}} - \frac{\partial u_{l\tau}}{\partial g_{l\tau}} = \theta(\sum_{h\neq l} \frac{\partial u_{h\tau}}{\partial G_{l\tau}} - \sum_{j\neq i} \frac{\partial u_{j\tau}}{\partial G_{i\tau}}) \\
&\frac{\partial u_{i\tau}}{\partial g_{i\tau}} - \frac{\partial u_{l\tau}}{\partial g_{l\tau}} = \theta(\frac{\partial u_{i\tau}}{\partial G_{i\tau}} - \frac{\partial u_{l\tau}}{\partial G_{l\tau}}) \\
&(1-\theta)(\frac{\partial u_{i\tau}}{\partial g_{i\tau}} - \frac{\partial u_{l\tau}}{\partial g_{l\tau}}) = 0
\end{aligned}
$$

holds. When $\theta \neq 1$, $\frac{\partial u_{i\tau}}{\partial g_{i\tau}} = \frac{\partial u_{l\tau}}{\partial g_{l\tau}}$ is derived for any $i, l(i \neq l)$. When $\theta = 1$, for $i, l(i \neq l)$, $G_{i\tau} = G_{l\tau} \equiv G$ can be shown. Thus, $\frac{\partial u_{i\tau}}{\partial G} = \frac{\partial u_{i\tau}}{\partial g_{i\tau}}$ is satisfied. As the shape of the utility function in each area is the same, $\frac{\partial u_{i\tau}}{\partial G} = \frac{\partial u_{j\tau}}{\partial G}$ holds and $\frac{\partial u_{i\tau}}{\partial g_{i\tau}} = \frac{\partial u_{l\tau}}{\partial g_{l\tau}}$ is derived for any $i, l(i \neq l)$.

(Q.E.D.)

$\frac{\partial u_{i\tau}}{\partial x_{i\tau}} = \frac{\partial u_{j\tau}}{\partial x_{j\tau}}$ holds for any $i, j (i \neq j)$ at both periods 1 and 2 in the first-best case, at period 2 in the merger case, and at period 1 in the separation case.

(Proof)

From the first-order condition for each Lagrangian, $\frac{\partial u_{i2}}{\partial x_{i2}} = \mu$ or $\frac{\partial u_{i1}}{\partial x_{i1}} = \lambda$ for any i can be derived in either case.

(Q.E.D.)

Finally, we can see that $\frac{\partial u_{i\tau}}{\partial x_{i\tau}} = \frac{\partial u_{j\tau}}{\partial x_{j\tau}}$ and $\frac{\partial u_{i\tau}}{\partial g_{i\tau}} = \frac{\partial u_{l\tau}}{\partial g_{l\tau}}$ hold at period 1 in the merger case and at period 2 in the separation case from (29) and (32). Therefore, we can derive $x_{i\tau} = x_{j\tau}, g_{i\tau} = g_{j\tau}$ for any $i \neq j$ in the first-best case, the merger case, and the separation case

Appendix 2: Proof of Proposition 1

(Proof)

When $\theta_1 = 1$, substituting $\theta_1 = 1$ into (14), (14) becomes $N \times MU_1 = (1 + \theta_1(N-1))\delta(1+r)MU_2$. This equation corresponds to (8) for $\theta_1 = 1$.

From now on, variables with subscripts m and fb relate to the merger case and the first-best case, respectively. For $\theta_1 < 1$, assume $b^{fb} \leq b^m$. As $g_\tau^\xi \equiv g_{i\tau}^\xi = g_{j\tau}^\xi$ is satisfied for $\xi \in \{fb, m\}$, we can derive

$$g_2^m = \omega_{i2} - (1+r)b^m \geq \omega_{i2} - (1+r)b^{fb} = g_2^{fb}.$$

This means that the marginal utility evaluated at g_2^m is smaller than the one at g_2^{fb}. Therefore,

$$\delta(1+r)(1+\theta_2(N-1))\frac{\partial u}{\partial g_2^m} \leq \delta(1+r)(1+\theta_2(N-1))\frac{\partial u}{\partial g_2^{fb}}$$

holds. Using the intertemporal budget allocation conditions in the first-best case and the merger case, this equation becomes

$$N\frac{\partial u}{\partial g_1^m} \leq (1+\theta_1(N-1))\frac{\partial u}{\partial g_1^{fb}}.$$

As $1 + \theta_1(N-1) < N$ holds, we obtain

$$(1+\theta_1(N-1))\frac{\partial u}{\partial g_1^m} < (1+\theta_1(N-1))\frac{\partial u}{\partial g_1^{fb}}. \tag{A1}$$

As the utility function is concave, the public good must be $g_1^m > g_1^{fb}$ for all i. Thus, using the budget constraint, we derive $\omega_{i1} + b^m > \omega_{i1} + b^{fb}$ for all i and, as a result, $b^m > b^{fb}$.

However, this contradicts the assumption, $b^{fb} \leq b^m$. Therefore, we can derive $b^m > b^{fb}$ when $\theta_1 < 1$.(Q.E.D.)

Appendix 3: Proof of Proposition 2

(Proof)

(7) holds in this case. Consider two areas denoted as a and b, where $\omega_{a1} > \omega_{b1}$ is satisfied. Under this setting, the budget constraints will be

$$\omega_{a1} + b_a = g_{a1} = g_{b1} = \omega_{b1} + b_b.$$

From this we can easily obtain

$$\omega_{a1} - \omega_{b1} = b_b - b_a > 0. \tag{36}$$

Therefore, $b_b > b_a$ can be derived for $\omega_{a1} > \omega_{b1}$. (Q.E.D.)

Appendix 4: Proof of Proposition 3

(Proof)

When $\theta_2 = 0$, by substituting $\theta_2 = 0$ into (15), (15) becomes $(1 + \theta_1(N-1))MU_1 = \delta(1+r)MU_2$. This equation corresponds to (8) for $\theta_2 = 0$.

From now on, we use the subscripts s and fb to denote variables in the separation case and in the first-best case, respectively. For $\theta_2 > 0$, we assume $b^{fb} \leq b^s$. As $g_\tau^\xi \equiv g_{i\tau}^\xi = g_{j\tau}^\xi$ is satisfied for $\xi \in \{fb, s\}$, we derive the following:

$$g_2^s = \omega_{i2} - (1+r)b^s \geq \omega_{i2} - (1+r)b^{fb} = g_2^{fb}.$$

This means that the marginal utility evaluated at g_2^s is lower than that at g_2^{fb}. Therefore,

$$\delta(1+r)\frac{\partial u}{\partial g_2^s} \leq \delta(1+r)\frac{\partial u}{\partial g_2^{fb}}$$

is derived. As $\delta(1+r) < \delta(1+r)(1+\theta_2(N-1))$ holds, we have

$$\delta(1+r)\frac{\partial u}{\partial g_2^s} < \delta(1+r)(1+\theta_2(N-1))\frac{\partial u}{\partial g_2^{fb}}.$$

From the intertemporal budget allocation condition, we can also show

$$(1+\theta_1(N-1))\frac{\partial u}{\partial g_1^s} < (1+\theta_1(N-1))\frac{\partial u}{\partial g_1^{fb}}. \tag{37}$$

As the utility function is concave, $g_1^s > g_1^{fb}$ for all i. Substituting the budget constraint to this, we derive $\omega_{i1} + b^s > \omega_{i1} + b^{fb}$ for all i and $b^s > b^{fb}$. However, this contradicts the assumption, $b^{fb} \leq b^s$. Therefore, $b^s > b^{fb}$ when $\theta_2 > 0$.(Q.E.D.)

Appendix 5: Proof of Proposition 4

(Proof)

(7) holds in this case. Consider the two areas denoted a and b, where $\omega_{a2} > \omega_{b2}$ is satisfied. Under this setting, the budget constraints are

$$\omega_{a2} - (1+r)b_a = g_{a2} = g_{b2} = \omega_{b2} - (1+r)b_b.$$

We then obtain

$$\omega_{a2} - \omega_{b2} = (1+r)(b_a - b_b) > 0. \tag{38}$$

Therefore, $b_a > b_b$ can be derived for $\omega_{a2} > \omega_{b2}$. (Q.E.D.)

Appendix 6: Proof of Proposition 5

(Proof)

The subscripts m and fb denote variables in the merger case and in the first-best case, respectively. We derive $x_\tau^\xi \equiv x_{i\tau}^\xi = x_{j\tau}^\xi$ and $g_\tau^\xi \equiv g_{i\tau}^\xi = g_{j\tau}^\xi$ for each $\xi \in \{m, fb\}$ In addition, we denote $\bar{\omega}$ as the mean value of ω_i. From Table 2, we can derive the following equations for the first-best case and the merger case, respectively,

$$(1+\theta_1(N-1))\frac{\partial u_{i1}}{\partial g_{i1}^{fb}} = \frac{\partial u_{i1}}{\partial x_{i1}^{fb}} = \delta(1+r)\frac{\partial u_{i2}}{\partial x_{i2}^{fb}} = \delta(1+r)(1+\theta_2(N-1))\frac{\partial u_{i2}}{\partial g_{i2}^{fb}} \tag{39}$$

$$N\frac{\partial u_{i1}}{\partial g_{i1}^{m}} = N\frac{\partial u_{i1}}{\partial x_{i1}^{m}} = \delta(1+r)\frac{\partial u_{i2}}{\partial x_{i2}^{m}} = \delta(1+r)(1+\theta_2(N-1))\frac{\partial u_{i2}}{\partial g_{i2}^{m}} \tag{40}$$

where we use $\frac{\partial u_{i\tau}}{\partial g_{i\tau}} = \frac{\partial u_{i\tau}}{\partial G_{i\tau}} = \frac{\partial u_{j\tau}}{\partial G_{j\tau}}$ because the shape of the utility function is the same across all municipalities. Assuming $b_i^m \leq b_i^{fb}$, we obtain

$$\bar{\omega}_2 - (1+r)b_i^{fb} \leq \bar{\omega}_2 - (1+r)b_i^m.$$

Substituting the budget constraint into this, we obtain

$$x_2^{fb} + g_2^{fb} \leq x_2^m + g_2^m.$$

From (39) and (40), it is clear that the marginal utilities of private consumption and the public good at period 2 move by the same proportion in the first-best case and merger case. In addition, the utility function is concave. Therefore, we have

$$\frac{\partial u_{i2}}{\partial x_{i2}}|_{x_2^{fb}} + \frac{\partial u_{i2}}{\partial g_{i2}}|_{g_2^{fb}} \geq \frac{\partial u_{i2}}{\partial x_{i2}}|_{x_2^m} + \frac{\partial u_{i2}}{\partial g_{i2}}|_{g_2^m}. \tag{41}$$

Using the intertemporal budget allocation condition, this can be rewritten as

$$\frac{\partial u_{i1}}{\partial x_{i1}}|_{x_1^{fb}} + \frac{\partial u_{i1}}{\partial g_{i1}}|_{g_1^{fb}} \geq N\frac{\partial u_{i1}}{\partial x_{i1}}|_{x_1^m} + \frac{N}{1+\theta_2(N-1)}\frac{\partial u_{i1}}{\partial g_{i1}}|_{g_1^m}.$$

As $N > 1$, we can derive

$$\frac{\partial u_{i1}}{\partial x_{i1}}|_{x_1^{fb}} + \frac{\partial u_{i1}}{\partial g_{i1}}|_{g_1^{fb}} > \frac{\partial u_{i1}}{\partial x_{i1}}|_{x_1^m} + \frac{\partial u_{i1}}{\partial g_{i1}}|_{g_1^m}.$$

As $u''^x < 0$ and $u''^g < 0$, we have $x_1^{fb} + g_1^{fb} < x_1^m + g_1^m$. Substituting the budget constraint into this, we have $\omega_{i1} + b_i^{fb} < \omega_{i1} + b_i^m$, which gives $b_i^{fb} < b_i^m$. This obviously contradicts the assumption $b_i^m \leq b_i^{fb}$. Therefore, $b_i^{fb} < b_i^m$ holds. (Q.E.D.)

Appendix 7: Proof of Proposition 2 using the model with private consumption

(Proof) As (27), (28), (29), and $\frac{\partial u_{i2}}{\partial x_{i2}} = \frac{\partial u_{j2}}{\partial x_{j2}}$ hold in this case, we can derive

$$\frac{\partial u_{i2}}{\partial x_{i2}} = \frac{\partial u_{j2}}{\partial x_{j2}}$$
$$\frac{1+r}{N}\delta\frac{\partial u_{i2}}{\partial x_{i2}} = \frac{1+r}{N}\delta\frac{\partial u_{j2}}{\partial x_{j2}}$$
$$\frac{\partial u_{i1}}{\partial x_{i1}} = \frac{\partial u_{j1}}{\partial x_{j1}}$$
$$\frac{\partial u_{i1}}{\partial g_{i1}} = \frac{\partial u_{j1}}{\partial g_{j1}}.$$

Because the shape of the utility functions is the same for each area, $x_{i1} = x_{j1}$ and $g_{i1} = g_{j1}$ are derived. Consider the two areas denoted a and b, where $\omega_{a1} > \omega_{b1}$ is satisfied. Under this setting, the budget constraints are

$$\omega_{a1} + b_a = x_{a1} + g_{a1} = x_{b1} + g_{b1} = \omega_{b1} + b_b.$$

We then obtain

$$\omega_{a1} - \omega_{b1} = b_b - b_a > 0. \tag{42}$$

Therefore, $b_b > b_a$ can be derived for $\omega_{a1} > \omega_{b1}$. (Q.E.D.)

Appendix 8: Proof of Proposition 3 using the model with private consumption

(Proof) The subscripts s and fb denote the variables in the separation case and in the first-best case, respectively. We can derive $x_\tau^\xi \equiv x_{i\tau}^\xi = x_{j\tau}^\xi$ and $g_\tau^\xi \equiv g_{i\tau}^\xi = g_{j\tau}^\xi$ for each $\xi \in \{s, fb\}$. We can also show that $x_1^{fb} \geq x_1^s$ holds if and only if $\frac{\partial u_{i1}}{\partial x_{i1}}|_{x_1^{fb}} \leq \frac{\partial u_{i1}}{\partial x_{i1}}|_{x_1^s}$ because the utility function is concave. From (23) and (32), because $\frac{\partial u_{i1}}{\partial x_{i1}}|_{x_1^\xi} = (1+r)\delta\frac{\partial u_{i2}}{\partial x_{i2}}|_{x_2^\xi}$ holds in both the first-best case and the separation case, $\frac{\partial u_{i1}}{\partial x_{i1}}|_{x_1^{fb}} \leq \frac{\partial u_{i1}}{\partial x_{i1}}|_{x_1^s}$ means $\frac{\partial u_{i2}}{\partial x_{i2}}|_{x_2^{fb}} \leq \frac{\partial u_{i2}}{\partial x_{i2}}|_{x_2^s}$. This can be reduced to $x_2^{fb} \geq x_2^s$ because the utility function is concave. Thus, we can see that $x_1^{fb} \geq x_1^s$ holds if and only if $x_2^{fb} \geq x_2^s$.

In a similar manner, $g_1^{fb} \leq g_1^s$ holds if and only if $\frac{\partial u_{i1}}{\partial g_{i1}}|_{g_1^{fb}} \leq \frac{\partial u_{i1}}{\partial g_{i1}}|_{g_1^s}$. From (23), we can also derive $\frac{\partial u_{i1}}{\partial g_{i1}}|_{g_1^{fb}} = (1+r)\delta\frac{\partial u_{i2}}{\partial g_{i2}}|_{g_2^{fb}}$ in the first-best case. However, $\frac{\partial u_{i1}}{\partial g_{i1}}|_{g_1^s} = \frac{(1+r)\delta}{1+\theta_1(n-1)}\frac{\partial u_{i2}}{\partial g_{i2}}|_{g_2^s}$ is derived in the separation case from (32). Therefore, $\frac{\partial u_{i1}}{\partial g_{i1}}|_{g_1^{fb}} \leq \frac{\partial u_{i1}}{\partial g_{i1}}|_{g_1^s}$ can be rewritten as $\frac{\partial u_{i2}}{\partial g_{i2}}|_{g_2^{fb}} \leq \frac{1}{1+\theta(n-1)}\frac{\partial u_{i2}}{\partial g_{i2}}|_{g_2^s}$. As $\frac{1}{1+\theta_1(n-1)} < 1$ for $\theta_1 > 0$, $\frac{\partial u_{i2}}{\partial g_{i2}}|_{g_2^{fb}} < \frac{\partial u_{i2}}{\partial g_{i2}}|_{g_2^s}$ and $g_2^{fb} > g_2^s$ always hold when $\frac{\partial u_{i2}}{\partial g_{i2}}|_{g_2^{fb}} \leq \frac{1}{1+\theta(n-1)}\frac{\partial u_{i2}}{\partial g_{i2}}|_{g_2^s}$. To summarize, $g_2^{fb} > g_2^s$ holds if $g_1^{fb} \leq g_1^s$ holds.

Assume $b_i^{fb} \geq b_i^m$ and $\theta_1 > 0$. As (22) and (31) are the same, from the budget constraint we obtain

$$\bar{\omega}_1 + b_i^{fb} \geq \bar{\omega}_1 + b_i^s$$
$$x_1^{fb} + g_1^{fb} \geq x_1^s + g_1^s$$

where we use the budget constraint. When this holds, we can obtain

$$x_2^{fb} + g_2^{fb} > x_2^s + g_2^s.$$

Substitution of budget constraints lead to

$$\omega_{i2} - (1+r)b_i^{fb} > \omega_{i2} - (1+r)b_i^s$$
$$b_i^s > b_i^{fb}.$$

This contradicts the assumption, $b_i^{fb} \geq b_i^m$. Therefore, $b_i^{fb} < b_i^s$ holds if $\theta_1 > 0$. However, $x_1^{fb} = x_1^s, x_2^{fb} = x_2^s, g_1^{fb} = g_1^s, g_2^{fb} = g_2^s$ holds if $\theta_1 = 0$ because $\sum_j \frac{\partial u_{j\tau}}{\partial g_{i\tau}} = \frac{\partial u_{i\tau}}{\partial g_{i\tau}}$. From this, $b_i^{fb} = x_1^{fb} + g_1^{fb} - \bar{\omega}_1 = x_1^s + g_1^s - \bar{\omega}_1 = b_i^s$ holds if $\theta_! = 0$.(Q.E.D.)

Appendix 9: Proof of Proposition 4 using the model with private consumption

(Proof) As (30), (31), (32), and $\frac{\partial u_{i1}}{\partial x_{i1}} = \frac{\partial u_{j1}}{\partial x_{j1}}$ hold in this case, we have

$$\frac{\partial u_{i1}}{\partial x_{i1}} = \frac{\partial u_{j1}}{\partial x_{j1}}$$
$$(1+r)\delta\frac{\partial u_{i2}}{\partial x_{i2}} = (1+r)\delta\frac{\partial u_{j2}}{\partial x_{j2}}$$
$$\frac{\partial u_{i2}}{\partial x_{i2}} = \frac{\partial u_{j2}}{\partial x_{j2}}$$
$$\frac{\partial u_{i2}}{\partial g_{i2}} = \frac{\partial u_{j2}}{\partial g_{j2}}.$$

As the shape of the utility functions is identical for each area, we have $x_{i2} = x_{j2}$ and $g_{i2} = g_{j2}$. Consider the two areas denoted a and b, where $\omega_{a2} > \omega_{b2}$ is satisfied. Under this setting the budget constraints are

$$\omega_{a2} - (1+r)b_a = x_{a2} + g_{a2} = x_{b2} + g_{b2} = \omega_{b1} - (1+r)b_b.$$

Thus, we have

$$\omega_{a2} - \omega_{b2} = (1+r)(b_a - b_b) > 0. \tag{43}$$

Therefore, $b_a > b_b$ can be derived for $\omega_{a2} > \omega_{b2}$. (Q.E.D.)

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