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When Market Competition Benefits Firms

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【キーワード】competition, oligopoly, R&D, heterogeneity, entry
【要約】A conventional wisdom in economics posits that more intense market competition, measured in almost any way, reduces firm profit. In this paper, we challenge this conventional wisdom in a simple Cournot model with strategic R&D investments wherein an efficient firm (dominant firm) competes against less efficient firms (fringe firms). We find that an increase in the number of fringe firms can stimulate R&D by the dominant firm, while it always reduces R&D by each of the fringe firms. More importantly, this force can be strong enough to compensate for the loss that arises from more intense market competition: the dominant firm's profit may indeed increase with the number of fringe firms, quite contrary to the conventional wisdom. An implication of this result is far-reaching, as it gives dominant firms to help, rather than harm, fringe competitors. We relate this implication to a practice know as open knowledge disclosure, especially Ford's strategy of disclosing its know-how publicly and extensively at the beginning of the 20th century.

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Abstract

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JEL classification: L13, O32, M21

Key words: competition, oligopoly, R&D, heterogeneity, entry

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1 Introduction

A conventional wisdom in economics posits that more intense market competition, measured in almost any way, reduces firm profit. Consequently, it should be in the best interest of profit-maximizing firms to reduce the degree of market competition, especially if it can be done through some legal means. There are indeed several means to achieve this end. One way is to gain some market power through product differentiation. Another, though it is sometimes subject to some legal restrictions, is to reduce the number of competitors in a given market through means such as collusion, entry deterrence, predation and horizontal merger. In any event, firms are supposed to earn higher profits if they can place themselves in less competitive environments.

In this paper, we challenge this conventional wisdom, in particular asking the following question: do firms *always* dislike intense market competition? At a glance, this negative relationship between market competition and firm profit in fact seems to stand fairly robustly in standard oligopoly models. As a typical example, consider an n-firm Cournot model. In this setup, one can easily show that each individual firm's profit declines as n increases, thereby lending strong support to the conventional wisdom that firms dislike intense competition. This conclusion also seems to be robust when the baseline model is augmented with cost-reducing R&D investments. The intuition behind this is simple and goes as follows. Since an increase in R&D investments lowers the marginal cost at the expense of a rise in the fixed cost, the investing firm can naturally gain more when it produces more. Since an increase in n generally lowers each firm's output, that makes it harder for each firm to exploit the scale of economies. As it stands, therefore, an increase in n unambiguously lowers the investment level and, as a natural consequence of this, the equilibrium profit.

Despite this seemingly convincing intuition, however, we claim in this paper that this is not a result that generally holds true; we rather argue that this is a mere artifact of the specification that the firms are symmetric with respect to the initial efficiency (which may be defined broadly).[1] In this paper, we provide a simple model which defies the conven-

[1] In this paper, the initial productivity is defined in terms of the *ex ante* marginal cost (before any investment takes place), but a similar conclusion is expected to arise when it is defined in a variety of other

tional wisdom that firms always dislike intense market competition in an otherwise standard Cournot environment with strategic R&D investments. Our model does not rely on strategic market preemption or collusion; we instead take a more direct route, focusing on the (somewhat neglected) nature of cost-reducing R&D investments. The basic setup is deceptively simple and standard except for one twist: the firms are asymmetric with respective to the initial productive efficiency.[2] More precisely, we consider a market consisting of one dominant firm and many fringe firms that are equally less efficient. Within this setup, we explore how the intensity of market competition, measured by an increase in the number of (fringe) firms, affects each firm's incentive for R&D investments as well as its resulting equilibrium profit.

This simple and seemingly standard setting yields several counterintuitive results, two of which are particularly illuminating and worth emphasizing here. First, we show that an increase in the number of firms *increases* the dominant firm's incentive for R&D investments for a wide range of parameter values. Second, when this effect is strong enough, an increase in the number of firms also *increases* the dominant firm's equilibrium profit, quite contrary to the conventional wisdom. These results are in stark contrast to the standard setup where an increase in the number of firms typically reduces the amount of R&D investments, not to mention the equilibrium profits. Our model suggests that more intense competition, in the sense that there are more competitors in the market, is not always a bad news for firms with advanced technologies. This fact leads to implications that are rather far-reaching, because it gives those dominant firms a reason to help, rather than harm, fringe competitors in the market. We later relate our results to a practice known as open knowledge disclosure, especially focusing on a compelling case of Ford Motor Company back at the turn of the 20th century, when we discuss this implication of the model.

ways. We make some comments on this in the concluding section.

[2] The standard model of strategic R&D is formulated by Brander and Spencer (1983), and the literature dealing with strategic R&D competition is now fairly abundant. See Spence (1984), d'Aspremont and Jacquemin (1988), Suzumura (1992), Lahiri and Ono (1999), and Kitahara and Matsumura (2006). The last two papers also focus on initial cost difference among firms but dos not investigate the relationship between market competition and R&D.

The key to our argument is the *ex ante* productivity differential among competing firms and its impact on the strategic incentive for R&D investments. The investment to reduce its own marginal cost works as a commitment to expand its production, which crowds out the other firms' output by some margin. We show that this strategic gain is actually increasing in the number of competitors, as a unit decrease in the marginal cost can affect more firms when more of them are around. When this gain is sufficiently large, it can actually induce some firms to invest more and consequently makes them more efficient. As we will see, this could indeed happen for the dominant firm, but never for the fringe firms, implying that more intense market competition tends to widen the dispersion in the *ex post* marginal costs. As the market becomes more competitive, each fringe firm becomes less motivated and less efficient, consequently leaving more rents to be exploited by the dominant firm. This gives competitive edges to the dominant firm and, when this effect is strong enough to compensate for the loss that arises from more intense market competition, so does its profit.

Several recent studies have raised instances where an increase in the number of firms may actually increase firm profit. Coughlan and Soberman (2005), Chen and Riordan (2007), and Ishibashi and Matsushima (Forthcoming) belong to this strand, but the underlying mechanism of our model differs substantially from that in theirs.[3] In those previous studies, market entry works as a commitment device to soften market competition, so that the market actually becomes less competitive as firms enter into it. In contrast, the dominant firm in our model benefits more directly from intense market competition. This difference is summarized most succinctly by the following (possibly empirically testable) feature of our model: the equilibrium market price is decreasing in the number of firms in our model while it is increasing in theirs. To the best of our knowledge, this paper is the first to suggest a channel though which more intense market competition *per se* benefits individual firms.[4] Although we do not know *a priori* which scenario is more plausible, as it certainly depends

[3] There is also a line of works which argue that having weak competitors might help because it keeps stronger competitors out of the market. See, for instance, Ashiya (2000) on this point.

[4] In a bilateral oligopoly model, Naylor (2002) shows that an increase in the number of downstream firms could increase aggregate industry profits because more intense downstream competition shifts bargaining power in favor of the downstream firms.

on the minute details of the underlying structure, this fact may be used to discriminate between our mechanism and other existing ones when examining more specific cases.

The paper proceeds as follows. The next section outlines the basic environment and illustrates fundamental forces behind our setup. Building on this intuition, section 3 provides a more detailed analysis of the model and derives main results. Section 4 discusses implications of the model, especially relating it to a practice known as open knowledge disclosure and what we call the Ford story. Finally, section 5 offers some concluding remarks.

2 Cournot with strategic R&D investments revisited

2.1 Setup

In order to make our point as emphatically as possible, we stick with a simple and standardized version of Cournot competition, augmented with cost-reducing R&D investments, as much as possible. Consider an industry consisting of n firms, each denoted by $i \in \{1, 2, ..., n\} \equiv N$. The model has two stages: each firm first chooses the investment level, which subsequently determines its marginal cost, and then engages in Cournot competition, given the realized marginal costs.

More precisely, in the first stage, each firm determines how much to invest in cost-reducing R&D activities. Let x_i denote the investment level chosen by firm i. A unit increase in the investment decreases the firm's marginal cost by the same margin. The total production cost incurred by the firm is thus given by $(z_i - x_i)q_i$, where q_i denotes the output level chosen by firm i. In this specification, z_i signifies the *ex ante* marginal cost (before the investment), while $c_i \equiv z_i - x_i$ the *ex post* marginal cost (after the investment). The cost of the investment is denoted by $I(x_i)$ and assumed to satisfy the usual properties: $I' > 0$, $I'' > 0$, $I'(0) = 0$ and $\lim_{x\to\infty} I'(x) = \infty$.

In the second stage, upon observing x_i for all $i \in N$, the firms engage in standard Cournot (quantity) competition. The inverse demand function is specified as

$$p = 1 - Q, \tag{1}$$

where $Q = \sum_{i\in N} q_i$ is the total output. Each firm simultaneously chooses q_i so as to maximize its own profit.

As can be seen, the basic setup is a text-book Cournot model with strategic R&D investments, only with one exception: we allow z_i to differ across firms in a particular way. More specifically, we consider a situation where there is one dominant firm and $n-1$ fringe firms by assuming that

$$0 < z_1 < z_f < 1, \text{ where } z_f \equiv z_2 = \ldots = z_n.$$

For expositional clarity, in what follows, we refer to firm 1 as dominant while all of the others as fringe (each of which is often denoted by subscript f in the subsequent analysis). Evidently, the key factor here is that the firms are *ex ante* asymmetric, not that there is one firm that stands out.[5]

2.2 Equilibrium

In this subsection, we go through the optimization problems faced by each firm. Since the model is standard enough, we only briefly sketch the outline of the analysis.

In the second stage, given the realized marginal costs for all $i \in N$, each firm chooses its output q_i. Each firm's problem in this stage can be written as

$$\max_{q_i} \quad \pi_i \equiv \left(1 - \sum_{i=1}^{n} q_i - c_i\right) q_i.$$

Assuming that the interior solutions exist, the equilibrium output is obtained as

$$q_i = \frac{1 + \sum_{j \neq i}(z_j - x_j) - n(z_i - x_i)}{n+1}. \tag{2}$$

It follows from this that the optimal output is a function of the investment levels, and we write $q_i^E(x_1, x_2, \ldots, x_n)$, where superscript E henceforth denotes the equilibrium value of any respective endogenous variable. This immediately leads us to obtain the equilibrium (gross) profit of each firm:

$$\pi_i^E(x_1, x_2, \ldots, x_n) = \left(1 - \sum_{i=1}^{n} q_i^E - c_i\right) q_i^E = \frac{(1 + \sum_{j \neq 1}(z_j - x_j) - n(z_i - x_i))^2}{(n+1)^2}, \tag{3}$$

which is also a function of the investment levels.

[5] Our main results hold as long as the firms are asymmetric with respect to the *ex ante* marginal cost, although this specification illustrates our point in perhaps the most striking way.

In the first stage, each firm simultaneously decides how much to invest in reducing the marginal cost. The first-period problem faced by each firm is defined as choosing x_i to maximize the net profit Π_i, taking the other firms' choices as given:

$$\max_{x_i} \quad \Pi_i(x_1, x_2, \ldots, x_n) \equiv \pi_i^E(x_1, x_2, \ldots, x_n) - I(x_i).$$

For expositional purposes, we dissect each firm's gains into two segments and term them (somewhat loosely) as strategic and non-strategic. The investment to reduce the marginal cost works as a strategic commitment to expand its production, which crowds out the other firms' output by some margin. This in turn raises the equilibrium price and consequently benefits the investing firm even with its output fixed. We refer to this gain as the strategic gain of the R&D investment. In contrast, we refer to any other benefits that accrue from the R&D investment broadly as the non-strategic gain, mostly for expositional purposes. Using these terms, the first-order condition can be decomposed as

$$\underbrace{q_i^E + (p^E - c_i - q_i^E)\frac{\partial q_i^E}{\partial x_i}}_{\text{non-strategic gain}} \underbrace{- \sum_{j \neq i}^{n} \frac{\partial q_j^E}{\partial x_i} q_i^E}_{\text{strategic gain}} = I'(x_i). \tag{4}$$

For the moment, we suppose that $I''(x_i^E)$ is large enough to satisfy the second-order condition.

2.3 Conventional wisdom?

It is pervasively believed that, in standard oligopoly models, an increase in the number of firms strictly decreases firm profit. It is also believed that this conclusion is robust to the addition of cost-reducing R&D investments because an increase in the number of firms lowers each firm's output, which only reduces the incentive to invest in reducing the marginal cost. Despite this seemingly robust intuition, however, we claim that this is not a result that generally holds true. Here, we illustrate how this slight alteration to the basic structure of the model potentially changes the market outcomes.

Our focus is on the marginal gain of the R&D investment, which is captured by the left-hand side of (4), and especially how it responds to a change in the number of firms n. Although the marginal gain is believed to be strictly decreasing in n, this does not necessarily hold when the firms are inherently asymmetric. We show that a situation may arise where an increase in n actually raises the marginal value of the R&D investment for the dominant firm and hence its investment level. More importantly, when this investment-stimulating effect is strong enough, the dominant firm can benefit from an increase in n, i.e., intense competition is not always a bad news for firms with advanced technologies.

We now examine more closely whether and when the marginal gain increases with n. To see this, we fix c_i $(i = 1, 2, \ldots, n)$ (so that they are not necessarily the equilibrium values) and see how an exogenous increase in n affects the marginal gain. Since the fringe firms are all symmetric, the marginal gain for the dominant firm can be written as

$$\frac{\partial \pi_1^E}{\partial x_1} = q_1^E + (p^E - c_1 - q_1^E)\frac{\partial q_1^E}{\partial x_1} - (n-1)\frac{\partial q_f^E}{\partial x_1} q_1^E, \tag{5}$$

where subscript f represents each fringe firm. Since $\partial q_i^E/\partial x_i = n/(n+1)$ and $\partial q_f^E/\partial x_i = -1/(n+1)$ from (2), the marginal gain is further reduced to

$$\frac{\partial \pi_1^E}{\partial x_1} = \underbrace{\frac{q_1^E + n(p^E - c_1)}{n+1}}_{\text{non-strategic gain}} + \underbrace{\frac{(n-1)q_1^E}{n+1}}_{\text{strategic gain}}. \tag{6}$$

A unit decrease in the marginal cost lowers the fringe firms' total output by $(n-1)/(n+1)$ and hence raises the dominant firm's profit by $q_1^E(n-1)/(n+1)$. The strategic gain is increasing in n since a unit decrease can affect more firms when n is large, which proves to be critical in giving rise to our main results.

Substituting $p^E = 1 - q_1^E - (n-1)q_f^E$ and $q_f^E = (1 + c_1 - 2c_f)/(n+1)$ into (6) yields

$$\frac{\partial \pi_1^E}{\partial x_1} = \frac{n(1 - (n-1)q_f^E - c_1)}{n+1} = \frac{2n(1 - c_f + n(c_f - c_1))}{(n+1)^2}, \tag{7}$$

from which we obtain

$$\frac{\partial^2 \pi_1^E}{\partial x_1 \partial n} = \frac{-2(n-1)(1-c_f) + 4n(c_f - c_1)}{(n+1)^3}. \tag{8}$$

This is positive if and only if

$$c_f - c_1 > \frac{(n-1)(1-c_f)}{2n}. \tag{9}$$

This condition illuminates when the presence of fringe firms stimulates the dominant firm's investment. To see this, suppose that the firms are *ex post* symmetric, i.e., $c_1 = c_f$. (9) then becomes $c_1 > 1$, which evidently never holds (as it is the necessary and sufficient condition for the dominant firm to produce strictly positive output). In this symmetric case, therefore, we end up with the result that we are all accustomed to: any increase in n always reduces the marginal gain and hence the investment level. Examining (9) more closely reveals, however, that this conclusion (that the marginal gain is decreasing in n) is not something that always holds true. As can easily be seen, (9) is more likely to hold for any given n when the dispersion in the *ex post* marginal costs, i.e., $c_f - c_1$, is sufficiently large.

The workings of the model perhaps become more transparent when we do the same exercise for the fringe firms. The marginal gain for each fringe firm can be written as

$$\frac{\partial \pi_f^E}{\partial x_f} = q_f^E + (p^E - c_f - q_f^E)\frac{\partial q_f^E}{\partial x_f} - \left(\frac{\partial q_1^E}{\partial x_f} + (n-2)\frac{\partial q_f^E}{\partial x_f}\right) q_f^E, \tag{10}$$

where, with a slight abuse of notation, $\partial \pi_f^E / \partial x_f$ denotes the marginal gain of the investment for each fringe firm. This is further simplified to

$$\frac{\partial \pi_f^E}{\partial x_f} = \underbrace{\frac{q_f^E + n(p^E - c_f)}{n+1}}_{\text{non-strategic gain}} + \underbrace{\frac{(n-1)q_f^E}{n+1}}_{\text{strategic gain}}. \tag{11}$$

Since each fringe firm is smaller than the dominant firm, i.e., $q_1^E > q_f^E$, the strategic gain is in general substantially weakened. Going through the same steps as above, we obtain

$$\frac{\partial \pi_f^E}{\partial x_f} = \frac{n(1 - q_1^E - (n-2)q_f^E - c_f)}{n+1} = \frac{2n(1 + c_1 - 2c_f)}{(n+1)^2}, \tag{12}$$

which is decreasing in n regardless of c_1 and c_f, as long as $1 + c_1 - 2c_f > 0$ (which is the necessary and sufficient condition for each fringe firm to produce strictly positive output). The conventional wisdom thus generally holds true for the fringe firms whose investment always decreases with n.

In sum, we can make the following two observations concerning the impact of the number of competitors on the incentive to invest:

Observation 1. The dominant firm's investment may increase with n.

Observation 2. Each fringe firm's investment always decreases with n.

An increase in n always reduces each fringe firm's investment while it may increase the dominant firm's. When this happens, intense market competition widens the dispersion in the *ex post* marginal costs. This is further reinforced by the fact that the investments are strategic substitutes: the dominant firm responds to this by increasing the investment even further to take advantage of the situation. When this effect works strongly enough, the dominant firm's profit may also increase with n. In the next section, we will explore this aspect of strategic investments in more depth, in an attempt to throw a new light at the relationship between competition and market outcomes.

3 Competition and market outcomes in asymmetric oligopoly

3.1 Competition stimulates investments by the rich

The driving force of our model is the endogenous nature of the marginal costs. We thus start with examining how an increase in n affects each firm's incentive to invest in reducing its marginal cost. Throughout this section, we work with a more tightly specified version of the setup described above: in particular, we assume that $I(x_i) = \gamma {x_i}^2$. This specification allows us to parameterize the importance of the R&D investment where a small γ means that endogenous cost reduction is an important part of the production process.

With this cost function, we can obtain a closed-form solution as the first-order condition (4) for the investment level is now modified as

$$x_1^E = \frac{n((n+1)\gamma - n + (n^2-1)\gamma z_f - n((n+1)\gamma - 1)z_1)}{((n+1)\gamma - n)((n+1)^2\gamma - n)}, \tag{13}$$

$$x_f^E = \frac{n((n+1)\gamma - n + (n+1)\gamma z_1 - (2(n+1)\gamma - n)z_f)}{((n+1)\gamma - n)((n+1)^2\gamma - n)}. \tag{14}$$

Notice that the second-order conditions are satisfied if and only if

$$\frac{n^2}{(n+1)^2} - \gamma < 0. \tag{15}$$

We assume that $\gamma \geq 1$ so that the second order condition is satisfied for any given n.

We also need to check whether the interior solutions indeed exist, which we have thus far taken for granted. To check this, we only need to look at the fringe firms since $q_1^E > 0$ and $x_1^E > 0$ if $q_f^E > 0$ and $x_f^E > 0$. For the fringe firms, $q_f^E > 0$ for any given n if and only if

$$z_1 > \frac{((2\gamma - 1)n + 2\gamma)z_f - ((\gamma - 1)n + \gamma)}{\gamma(n+1)} \equiv \underline{z}_1. \tag{16}$$

Notice that a firm would not invest in cost reduction if it is to produce no output; therefore, this is also the necessary and sufficient condition for $x_k > 0$. If (16) fails to hold, $x_f^E = \Pi_f^E = 0$, i.e., fringe firms virtually cease to exist. In this case, the dominant firm's behavior naturally becomes independent of n where

$$x_1^E = \begin{cases} 1 - 2z_f + z_1, & \text{if } \dfrac{1 - z_f - 2(1 - 2z_f)\gamma}{2\gamma} < z_1 \leq \underline{z}_1, \\ \dfrac{1 - z_1}{4\gamma - 1}, & \text{otherwise,} \end{cases} \tag{17}$$

$$\Pi_1^E = \begin{cases} (1 - z_f)^2 - (1 - 2z_f + z_1)^2\gamma, & \text{if } \dfrac{1 - z_f - 2(1 - 2z_f)\gamma}{2\gamma} < z_1 \leq \underline{z}_1, \\ \dfrac{\gamma(1 - z_1)^2}{4\gamma - 1}, & \text{otherwise,} \end{cases} \tag{18}$$

Since this case is apparently uninteresting, in the subsequent analysis, we restrict our attention to the case where $z_1 > \underline{z}_1$ so that fringe firms have some role to play.

We have seen that an increase in n may induce the dominant firm to invest more when the dispersion in the *ex post* marginal costs are large. As can easily imagined, we can make a similar statement in terms of the dispersion in the exogenously given *ex ante* marginal costs.

Proposition 1 *For any $n > 1$ and $\gamma \geq 1$, (i) there exists some nonempty interval $Z^x \equiv (\underline{z}_1, z_1^x)$ such that x_1^E is increasing in n if and only if $z_1 \in Z^x$; (ii) x_f^E is decreasing in n.*

Proof See Appendix.

The proposition makes two claims. The first claim is the more illuminating part, which says that the dominant firm's R&D investment can be increasing in n, quite contrary to the conventional wisdom. The key factor turns out to be the relative location of z_f and z_1, i.e., the dispersion in the *ex ante* marginal costs between the dominant and the fringe firms. The dominant firm is induced to make more investment as n increases when the fringe firms are *ex ante* sufficiently (but not too) inefficient in a relative sense. Moreover, it says that Z^x is generally nonempty, meaning that we can always find some z_1 such that $\partial x_1^E / \partial n > 0$. These results are in stark contrast to the standard setup where an increase in n typically reduces the investment level because it implies smaller rents for innovating firms.

These results imply that more intense competition tends to make the dominant firm even more dominant. To see this, note that the fringe firms always invest less as n increases, as in the standard setup (the third claim). More intense competition thus always discourages the fringe firms but that is not necessarily the case for the dominant firm. As the market become more competitive, therefore, the dispersion in the *ex post* marginal costs may become even larger. As we will see next, because of this effect, the dominant firm may actually benefit from an increase in n.

More intense competition is more likely to induce the dominant firm to invest more when the upperbound z_1^x is larger. Not surprisingly, this is the case when γ is relatively small, i.e., when R&D investments are sufficiently important in the production process. The next proposition is a formal representation of this fact.

Proposition 2 *For any $n > 1$ and $\gamma \geq 1$, $\partial z_1^x / \partial \gamma < 0$.*

Proof Differentiating z_1^x with respect to γ, we have

$$\frac{\partial z_1^x}{\partial \gamma} = \frac{-(1-z_f)(n-1)(n+1)^2((\gamma-1)n+\gamma)((n+1)^2(3n-2)\gamma - n(n^2+3n-2))}{n(2n-(n+2)(n+1)^2\gamma+2(n+1)^3\gamma^2)^2}. \quad (19)$$

Note that $(n+1)^2(3n-2)\gamma - n(n^2+3n-2) > 0$ for any $\gamma > 1$ because, when $\gamma = 1$,

$$(n+1)^2(3n-2)\gamma - n(n^2+3n-2) = 2n^3 + n^2 + n - 2 > 0. \quad (20)$$

This means that the numerator of $\partial z_1^x / \partial \gamma$ is always negative. Q.E.D.

The range for which the dominant firm's investment increases with n can be best seen graphically. Figure 1 illustrates the range for two different values of γ. These examples indicate that the dominant firm's investment increases with n for a wide range of parameter values.

[Figure 1 here]

3.2 Competition makes the rich get richer

When $z_1 \in Z^x$, more intense competition makes the dominant firm even more dominant in the sense that the dispersion in the *ex post* marginal costs becomes wider. When this effect is strong enough to compensate the loss from more intense market competition, the dominant firm's profit may actually increase as more fringe firms enter the market. This can indeed happen, as the next proposition indicates.

Proposition 3 *For any $n > 1$ and $\gamma \geq 1$, (i) there exists some nonempty interval $Z^P \equiv (\underline{z}_1, z_1^P)$ such that Π_1^E is increasing in n if and only if $z_1 \in Z^P$; (ii) Π_f^E is decreasing in n.*

Proof See Appendix.

Proposition 3 is the main result of the paper, which basically runs parallel to Proposition 1, making two claims of similar nature. In particular, it again shows that Z^P is nonempty for any given $n > 1$ and $\gamma \geq 1$, and hence we can always find z_1 such that $\partial \Pi_1^E / \partial n > 0$. As the market becomes more competitive, the dominant firm invests more (if $z_1 \in Z^x$) and the fringe firms invest less. As a consequence, the dispersion in the marginal costs gets even larger at the *ex post* stage when the firms engage in Cournot competition. The dominant firm can benefit from this, even though *ex post* market competition becomes more severe as n increases.

Competition induces the dominant firm to invest more when γ is relatively small, and this same logic apparently carries over to the dominant firm's profit. The next proposition is a sequel to Proposition 2, showing that the dispersion in the *ex ante* marginal costs needs to be small in environments where R&D investments are sufficiently important.

Proposition 4 *For any $n > 1$ and $\gamma \geq 1$, $\partial z_1^P / \partial \gamma < 0$.*

Proof See Appendix.

Figure 2 illustrates the range of z_1 for which the dominant firm's profit is increasing in n, again for two different values of γ. Naturally, the range for which the dominant firm's profit increases is narrower than that for which its investment increases. In fact, as can be expected, the dominant firm's profit increases with n only if its investment increases with n, i.e., the former is a necessary condition of the latter.

Proposition 5 *For any $\gamma \geq 1$ and $n > 1$, $Z^P \subset Z^x$.*

Proof With some algebra we obtain

$$
\begin{aligned}
z_1^x - z_1^P &= \frac{(1-z_f)(n+1)((n+1)\gamma-n)^2((n+1)^2\gamma-n)^2((n+1)^2\gamma-(n^2+1))}{n[2n-(n+2)(n+1)^2\gamma+2(n+1)^3\gamma^2]H} > 0, \\
z_1^P - \underline{z}_1 &= \frac{(1-z_f)n(n-1)((n+1)\gamma-n)((n+1)^2\gamma-n)}{\gamma(n+1)H} > 0, \\
&\quad \text{where } H \equiv [n^2(1-n)+n^3(n+1)^2\gamma-(n+1)^3(2n^2+1)\gamma^2+(n+1)^5\gamma^3],
\end{aligned}
$$

for any $\gamma \geq 1$ and $n > 1$, which proves the proposition (and also the nonemptiness of Z^x and Z^P. Q.E.D.

[Figure 2 here]

3.3 What happens at the aggregate level?

Several recent studies have raised instances where incumbents benefit from a new market entry. In those previous cases, however, the equilibrium price actually rises with a new entrant coming into the market: market competition becomes less severe despite the fact that there are now more firms in the market. What is common among those previous studies is therefore that they find a channel through which a market entry somehow makes the market less competitive. In Ishibashi and Matsushima (Forthcoming), for instance, a new entry into the low-end market works as a commitment device for the incumbents not to supply to that market (hence not to lower the price to accommodate low-end consumers). This is actually profit-enhancing for the incumbents as they can focus on the high-end market, allowing them to charge a higher price to high-end consumers.

In contrast, our model works in a totally different way. The difference is succinctly summarized by a distinguishing, and empirically testable, feature of our model: the equilibrium price is indeed decreasing in n so that the market becomes truly more competitive as n increases. In our model, there is nothing unusual about the impact that an increase in n has on the equilibrium market price. To see this, the equilibrium price is computed as

$$p^E = \frac{(n+1)(1+(n-1)z_f+z_1)\gamma-n}{(n+1)^2\gamma-n}, \tag{21}$$

which straightforwardly leads to the next result.

Proposition 6 *For any $n > 1$ and $\gamma \geq 1$, p^E is decreasing in n.*

Proof Differentiating p^E with respect to n, we have

$$\frac{\partial p^E}{\partial n} = -\frac{\gamma[(n+1)^2(1-2z_f+z_1)\gamma - (1-z_f)n^2 + (z_f - z_1)]}{((n+1)^2\gamma - n)^2}. \tag{22}$$

For $z_1 \geq \underline{z}_1$, the partial derivative is maximized at $z_1 = \underline{z}_1$. Substituting $z_1 = \underline{z}_1$ into the partial derivative, we have

$$\left.\frac{\partial p^E}{\partial n}\right|_{z_1 = \underline{z}_1} = -\frac{1-z_f}{(n+1)((n+1)^2\gamma - n)} < 0. \tag{23}$$

Therefore, p^E is decreasing in n. Q.E.D.

To the best of our knowledge, our model is the first to show that an incumbent can benefit from a market entry in an environment where the equilibrium price is decreasing in n. Of course, our intention is not to exclude those existing views because there can be many channels through which an incumbent benefits from a market entry. Which view is more plausible depends certainly on the underlying structure and is purely an empirical matter left for future research. We argue, however, that this fact provides a nice discriminating factor when one examines what forces are more likely to be at work in each specific case.

The equilibrium price is decreasing in n necessarily means that the total quantity supplied is increasing. This naturally raises a question about the composition of the total output (or the market share). Let Q^E_{-1} denote the total quantity supplied by the fringe firms, which is given by

$$Q^E_{-1} = \frac{\gamma(n^2-1)[(1-2z_f+z_1)(n+1)\gamma - (1-z_f)n]}{((n+1)^2\gamma - n)((n+1)\gamma - n)}. \tag{24}$$

Surprisingly, the total output produces by the fringe firms may decrease with n.

Proposition 7 *(i) For any $n > 1$ and $\gamma \geq 1$, (i) there exists some nonempty interval $Z^Q \equiv (\underline{z}_1, z_l^Q)$ such that Q^E_{-1} is decreasing in n if $z_1 \in Z^Q$; (ii) $Z^P \subset Z^Q \subset Z^x$.*

Proof See Appendix.

It directly follows from Proposition 6 that the total quantity supplied is increasing in n, as in standard Cournot models. This implies that if Q_{-1}^E decreases with n, then q_1^E must increase – yet another unusual property. This also implies that $z_1 \in Z^Q$ is a sufficient condition for the dominant firm's market share to increase with n.

Finally, we briefly discuss the impact of market competition on the industry-wide rate of innovation. The question we ask here is whether market competition spurs or inhibits innovations in our setup. To this end, suppose that we measure the industry-wide rate of innovation by the (effective) total investment M:

$$M \equiv \gamma x_1^2 + (n-1)\gamma x_f^2. \tag{25}$$

Figure 3 provides some numerical examples of the relationship between M and n. As the figure indicates, almost anything goes in our setup, and no clear prediction can hence be made from this. First, if the dispersion in the *ex ante* marginal costs is small and $z_1 > z_l^x$, the dominant firm's investment decreases with n, and the total investment is more likely to decrease (see two examples on the left-hand side of Figure 3). On the contrary, when the dispersion is large, the dominant firm invests more as n increases, even to the extent that it more than offset a decrease in the total investment of the fringe firms. As a consequence, the total investment is more likely to be increasing in n (see two examples on the right-hand side). If the dispersion is in some intermediate range, the total investment is non-monotone with respect to n (see examples at the middle). The last case where the dispersion falls into some intermediate range is perhaps most interesting, as recent evidence seems to suggest that the relationship between market competition and innovation is inverted U-shape, e.g., Aghion and Griffith (2005).

[Figure 3 here]

The literature examining the relationship between market competition and innovation is very large and diverse, as it is certainly an old issue which have attracted attention of many economists (at the very least from the days of Schumpeter), and many models have been proposed to account for observed patterns of the relationship between market competition and innovation, e.g., Aghion et al. (2005). We thus do not intend to make too much out of this, because we only analyze a very specific industry structure. If there is anything we can insist on this, though, at least this much is certain: the distribution of (initial) productivity matters, even in a simple Cournot framework like ours. This is an insight which, in our view, has not received enough attention, and it is of some interest to approach this issue from this perspective, both theoretically and empirically.

4 Open knowledge disclosure: the Ford story

In the previous sections, we have analyzed a simple Cournot model with strategic R&D investments when the firms are inherently asymmetric. The most important message here is that firms with more advanced technologies may benefit from having more (fringe) competitors. We argue that an implication that this leads to is rather far-reaching, because those dominant firms actually have an incentive to help, rather than harm, minor competitors so that they can remain just sufficiently competitive to stay in the market. In this section, we focus on this implication of the model, relating it to a practice known as open knowledge disclosure and especially the case of Ford back at the turn of the 20th century.

To discuss this issue, let us start with the following question: why are some firms more productive than others? There is one straightforward answer to this: if there is any sure way to gain market power, it is to make new innovations over the existing ways of production and commercialization. In fact, innovations, in a broad sense, are the typical and sometimes the only source of competitive edges that a firm can gain over its existing or potential competitors. For instance, a firm may drive competitors out of the market if it can attain a level of efficiency that no one can catch up to. New ideas are also indispensable to invent a unique, differentiated, product that cannot easily be imitated. The problem, as it has been clearly recognized, is that although innovative ideas are surely hard to come by, they are very easy to copy once they are created: in fact, most ideas can be copied at almost no cost. It is hence critical for innovating firms to establish and protect their innovative ideas, provided that they can benefit from being in less competitive environments.

In reality, though, firms do not seem overly concerned about keeping "secrets" to themselves – at least not always. For instance, it is often suggested that a large fraction of patentable innovations are not patented (Mansfield, 1986).[6] Firms are not simply reckless with their secrets; they often go beyond just that as they intentionally and freely disclose what appears to be critical knowledge, even to their direct competitors. Informal know-how trading between competitors is very active, often though informal networks of process engineers (von Hippel, 1988). Employees frequently give technical information to colleagues in other firms, including direct competitors (Schrader, 1991). All of these seem to suggest that firms do not protect their innovations as carefully as the theory predicts.

Of course, some fraction of know-how trading occurs in a closed setting – the practice often referred to as closed knowledge disclosure.[7] In this case, some reciprocal agreements, either explicitly or implicitly, can in principle be made between the giver and the recipients, barring many difficulties associated with trading ideas.[8] In many instances, though, firms not just freely and but also publicly give away critical knowledge through open channels such as publications in scientific journals, presentation in conferences or more informal communication – a practice often referred to as open knowledge disclosure.[9] This practice is highly puzzling because, when a firm makes its critical knowledge publicly accessible, it inevitably loses some, if not all, control over the diffusion of the know-how. Once the know-how becomes publicly available, it is virtually impossible for the disclosing firm to take it back or to make any profit from it. No future exchange of favors can be expected either because it is

[6] A typical view on this is that patents are not always the best way to protect their ideas. This might be due to the limited power of Intellectual Property (IP) rights. Anton and Yao (2004) focus on this aspect.

[7] There is a scope for know-how trading especially if the industry profit increases with the number of firms. In this case, the concerned parties could reach an agreement if they can somehow manage to find a way to appropriately divide the surplus (perhaps through some explicit contracts).

[8] Trading an idea is difficult especially when its quality is uncertain: a recipient must see the idea to evaluate the quality but, once the idea is observed, there is no reason to buy it. Partial disclosure may be optimal in a situation like this. See Anton and Yao (2004) for this.

[9] A seminal work on open knowledge disclosure is Allen (1983) who argues that many new production technologies have been developed by a process called "collective invention." See Penin (2007) for further evidence on open knowledge disclosure.

nearly impossible to identify the beneficiaries of the know-how when it is disclosed via open channels.

One story stands out in this respect. At the beginning of the 20th century, Ford was the dominant automobile producer which had attained an unprecedented level of efficiency with radically new technologies. The pace of growth was astonishing, especially considering the fact that the entire industry had also been growing at a rapid pace. Ford's share of all automobile production grew from 9.4% in 1908 to 48% in 1914.[10] In it main filed of competition, i.e., the cheap car filed, Ford's share was 96%, practically making it a monopolist. The figures suggest Ford's incredible presence in the industry, given that the industry was not as concentrated as it is today.[11] There were certainly many factors that had contributed to Ford's success, but the main source of its competitive edges was undoubtedly its production efficiency made possible by several innovations and inventions such as the moving assembly line system and Henry Ford's scientific management (or so-called "Fordism"): in 1914, Ford, only with 13,000 employees, manufactured 260,720 while all of the other companies, with 66,350 employees combined, manufactured mere 286,770. There was virtually no competition; Ford was simply too good. Surprisingly, though, Ford had no intention of "hiding its secrets." Nevins (1954, p.508) notes:

> Engineers came from all over America and Europe to study this achievement in efficiently standardized and specialized production. Nothing was concealed. Indeed, Henry Ford and his associates this year cooperated with the editors of *Engineering* in laying before the world, in the technically detailed and richly illustrated pages of Arnold and Faurote's *Ford Methods and the Ford Shops*, ...

It is hard to believe that Ford expected something in return from those fringe competitors which seemed to have nothing worthwhile to offer at the time. Why did Ford give away its critical knowledge so generously? Our model provides a partial answer to this and, more broadly, a sensible reason for open knowledge disclosure: there is a channel through which

[10] All figures are taken from Nevins (1954).

[11] At the time, the number of manufacturers exceeded well above one hundred.

the innovating firm can benefit from disseminating its innovative ideas publicly. The Ford story fits our description particularly well for two reasons: (i) endogenous cost reduction – a driving force of our model – is evidently an important factor in the automobile industry; (ii) Ford possessed apparently superior technologies and cost advantages. In a situation like this, market competition tends to make a dominant firm even more dominant, and hence eliminating fringe competitors was not necessarily in its best interest. When there are more fringe competitors, competition among them becomes severer and that discourages them. Fringe competitors make less investment and, consequently, become less efficient, which actually works for the dominant firm. According to this logic, Ford had every reason to help, rather than harm, fringe competitors so that they can remain (just sufficiently) competitive in the market.

5 Conclusion

In this paper, we revisit a fundamental question of market competition: do firms always dislike intense market competition? We identify a situation under which the conventional wisdom that intense market competition decreases firm profit fails to ascertain itself. In a market consisting of inherently asymmetric firms, firms with advanced technologies can in fact benefit from having more (fringe) competitors, indicating that the conventional wisdom may not be as robust as generally believed. Our model also implies that in some cases, there is a reason for dominant firms to help, rather than harm, fringe competitors, just for their own sake (even without any spillover or network effects). This implication of the model provides a plausible explanation for open knowledge disclosure, especially Ford's strategy at the beginning of the 20th century.

While our analysis takes the number of (fringe) firms as exogenous, we can easily extend our model to incorporate free entry by fringe firms. Suppose that firms enter the market at their own discretion by incurring some fixed entry cost. In a setup like this, an equilibrium number of firms is determined by the entry cost, and a question in the spirit of the present paper is whether a decrease in the entry cost, which typically intensifies market competition, ever raises the dominant firm's profit. In the context of strategic entry deterrence, Etro (2004,

2006) show that in free entry markets, a leaders engaging in strategic commitment prior to the entry of followers make aggressive investments to deter their entry. Note, however, that in his setting, the leader's profit is never decreasing in the entry cost of followers: in this sense the models of entry deterrence do not yield our implication on market competition and firm profit.[12] In contrast, in our setup, one can show that a decrease in the entry cost can indeed raise the dominant firm's profit, even when firms first enter and then simultaneously determine the investment levels.

It is our view that the present analysis provides only a first step to better understand the nature of market competition in asymmetric oligopoly, and hence that there are several avenues to extend the current analysis. First, our results should hold even the firms are *ex ante* asymmetric in more broad senses. More specifically, although the initial efficiency is defined only in terms of the *ex ante* marginal cost of production, a similar conclusion holds when it is defined, for instance, in terms of the efficiency of cost reduction, (measured by γ in the model). It is hence an important task to see how far we can push the logic present in our model.

Second, we only examine a particular industry structure – one dominant firm and $n-1$ equally inefficient fringe firms – to make our points in a relatively clear manner, but the model's implications are certainly not restricted to this structure. Our main contention is rather that the distribution of initial productivity matters, for the incentive for R&D investments and the resulting equilibrium profits. We believe that this is an important insight especially when we examine the relationship between market competition and innovation, and it is of some interest to pursue this aspect, both theoretically and empirically, in future.

[12] For the discussion of R&D competition in Stackelberg settings, see also Ino and Kawamori (Forthcoming).

Appendix: proofs

Proof of Proposition 1 (i) From (13) we have

$$\frac{\partial x_1^E}{\partial n} = \frac{\gamma[n^2(n^2+1) - 3n^2(n+1)^2\gamma + (n+1)^3(3n-1)\gamma^2]z_f}{((n+1)\gamma - n)^2((n+1)^2\gamma - n)^2},$$
$$- \frac{\gamma[(n^2-1)((n+1)\gamma - n)^2 + (2n^2 - n(n+1)^2(n+2)\gamma + 2n(n+1)^3\gamma^2)z_1]}{((n+1)\gamma - n)^2((n+1)^2\gamma - n)^2}, \quad (26)$$

which is positive if and only if

$$z_1 < \frac{n^2(1-n^2) + 2(n-1)n(n+1)^2\gamma - (n-1)(n+1)^3\gamma^2}{n(2n - (n+2)(n+1)^2\gamma + 2(n+1)^3\gamma^2)}$$
$$+ \frac{[n^2(1+n^2) - 3n^2(n+1)^2\gamma + (3n-1)(n+1)^3\gamma^2]z_f}{n(2n - (n+2)(n+1)^2\gamma + 2(n+1)^3\gamma^2)} \equiv z_1^x; \quad (27)$$

For the nonemptiness, see the Proof of Proposition 5.

(ii) From (14) we have

$$\frac{\partial x_f^E}{\partial n} = -\gamma\left[\frac{2[(n-1)(n+1)^3\gamma^2 - n^2(n+1)^2\gamma + n^2]z_1}{2((n+1)\gamma - n)^2((n+1)^2\gamma - n)^2}\right.$$
$$+ \frac{n[n(n^2-2) - (n-2)(n+1)^2\gamma]}{2((n+1)\gamma - n)^2((n+1)^2\gamma - n)^2}$$
$$\left. - \frac{[2(n-1)(n+1)^3\gamma^2 - n(n+1)^2(3n-2)\gamma + n^4]z_f}{2((n+1)\gamma - n)^2((n+1)^2\gamma - n)^2}\right], \quad (28)$$

which is maximized at $z_1 = \underline{z}_1$ (because $\partial x_f^E/\partial n$ is strictly decreasing in z_1). Evaluating the partial derivative at this value, we have

$$\left.\frac{\partial x_f^E}{\partial n}\right|_{z_1 = \underline{z}_1} = -\frac{n(1-z_f)}{2(n+1)((n+1)\gamma - n)((n+1)^2\gamma - n)} < 0. \quad (29)$$

Therefore, under the maintained assumptions, x_f^E is strictly decreasing in n as long as $z_1 > \underline{z}_1$ and $x_f^E > 0$. Q.E.D.

Proof of Proposition 3 (i) Given the equilibrium investment levels, we can obtain the net profits as follows:

$$\Pi_1^E = \frac{\gamma((n+1)^2\gamma - n^2)((n+1)\gamma - n + (n^2-1)\gamma z_f - n((n+1)\gamma - 1)z_1)^2}{((n+1)\gamma - n)^2((n+1)^2\gamma - n)^2}, \quad (30)$$
$$\Pi_f^E = \frac{\gamma((n+1)^2\gamma - n^2)((n+1)\gamma - n + (n+1)\gamma z_1 - (2(n+1)\gamma - n)z_f)^2}{((n+1)\gamma - n)^2((n+1)^2\gamma - n)^2}. \quad (31)$$

From (30), we have

$$\frac{\partial \Pi_1^E}{\partial n} = 2\gamma^2((n+1)\gamma - n + (n^2-1)\gamma z_f - n((n+1)\gamma - 1)z_1)\times$$
$$\left\{\frac{n^2(n^3+1) - n(n+1)^2(3n^2-n+2)\gamma + (n+1)^3(3n^2+n+1)\gamma^2 - (n+1)^5\gamma^3}{((n+1)\gamma-n)^3((n+1)^2\gamma-n)^3}\right.$$
$$-\frac{[n^2(1-n) + n^3(n+1)^2\gamma - (n+1)^3(2n^2+1)\gamma^2 + (n+1)^5\gamma^3]z_1}{((n+1)\gamma-n)^3((n+1)^2\gamma-n)^3}$$
$$\left.-\frac{[n^3(1+n^2) - n(n+1)^2(4n^2-n+2)\gamma + (n+1)^3(5n^2+n+2)\gamma^2 - 2(n+1)^5\gamma^3]z_f}{((n+1)\gamma-n)^3((n+1)^2\gamma-n)^3}\right\}, \tag{32}$$

which is positive if and only if

$$z_1 < -\frac{(n+1)((n+1)\gamma-n)^2((n+1)^2\gamma-(n^2-n+1))}{[n^2(1-n)+n^3(n+1)^2\gamma-(n+1)^3(2n^2+1)\gamma^2+(n+1)^5\gamma^3]}$$
$$+\left[\frac{[-n^3(n^2+1)+n(n+1)^2(4n^2-n+2)\gamma]z_f}{[n^2(1-n)+n^3(n+1)^2\gamma-(n+1)^3(2n^2+1)\gamma^2+(n+1)^5\gamma^3]}\right.$$
$$\left.+\frac{[-(n+1)^3(5n^2+n+2)\gamma^2+2(n+1)^5\gamma^3]z_f}{[n^2(1-n)+n^3(n+1)^2\gamma-(n+1)^3(2n^2+1)\gamma^2+(n+1)^5\gamma^3]}\right] \equiv z_1^P \tag{33}$$

For the nonemptiness, see the Proof of Proposition5.

(ii) From (31) we have

$$\frac{\partial \Pi_f^E}{\partial n} = -2\gamma^2(\gamma(n+1)z_1 + ((\gamma-1)n+\gamma) - ((2\gamma-1)n+2\gamma)z_f)K(n,\gamma), \tag{34}$$

where

$$K(n,\gamma) = \frac{[(n+1)^5\gamma^3 - (n+1)^3(2n^2+n+2)\gamma^2 + n(n+1)^2(n^2+2)\gamma - n^3]z_1}{((n+1)\gamma-n)^3((n+1)^2\gamma-n)^3}$$
$$-\frac{[2(n+1)^5\gamma^3 - (n+1)^3(5n^2+2n+3)\gamma^2 + n(n+1)^2(4n^2-n+4)\gamma - n^2(n^3+n+1)]z_f}{((n+1)\gamma-n)^3((n+1)^2\gamma-n)^3}$$
$$+\frac{(n+1)((n+1)\gamma-n)^2((n+1)^2\gamma-(n^2-n+1))}{((n+1)\gamma-n)^3((n+1)^2\gamma-n)^3}. \tag{35}$$

For any $z_1 \geq \underline{z}_1$, $(\gamma(n+1)z_1 + ((\gamma-1)n+\gamma) - ((2\gamma-1)n+2\gamma)z_f)$ is nonnegative. We can thus prove the proposition if $K(n,\gamma) > 0$. Note that since $K(n,\gamma)$ is increasing in z_1, it suffices to show this at $z_1 = \underline{z}_1$:

$$\frac{(1-z_f)((\gamma-1)n^2+2\gamma n+\gamma)}{(n+1)\gamma((n+1)\gamma-n)^2((n+1)^2\gamma-n)^2} > 0.$$

This shows that for any n, Π_f^E is decreasing in n. Q.E.D.

Proof of Proposition 4 Differentiating z_1^P with respect to γ, we have

$$\frac{\partial z_1^P}{\partial \gamma} = J(n,\gamma)\frac{(1-z_f)n(n+1)^2((\gamma-1)n+\gamma)}{[n^2(1-n)+n^3(n+1)^2\gamma-(n+1)^3(2n^2+1)\gamma^2+(n+1)^5\gamma^3]^2}, \tag{36}$$

where

$$\begin{aligned} J(n,\gamma) \;\equiv\;& n(n^5-3n^3+5n^2-3n+2)-(n+1)^2(3n^4-3n^3+2n^2+n+2)\gamma \\ &+(n+1)^4(3n^2-3n+4)\gamma^2-(n+1)^6\gamma^3. \end{aligned} \tag{37}$$

Note that $\partial z_1^P/\partial\gamma < 0$ if and only if $J(n,\gamma) < 0$. To show this, we first obtain

$$\begin{aligned} \frac{\partial J(n,\gamma)}{\partial\gamma} &= -(n+1)^2(3n^4-3n^3+2n^2+n+2) \\ &\quad +2(n+1)^4(3n^2-3n+4)\gamma-3(n+1)^6\gamma^2, \\ \frac{\partial^2 J(n,\gamma)}{\partial\gamma^2} &= 2(n+1)^4(3n^2-3n+4)-6(n+1)^6\gamma, \\ \frac{\partial^3 J(n,\gamma)}{\partial\gamma^3} &= -6(n+1)^6 < 0. \end{aligned}$$

Substituting $\gamma = 1$ into $J(n,\gamma)$, $\partial J(n,\gamma)/\partial\gamma$, and $\partial^2 J(n,\gamma)/\partial\gamma^2$ yield

$$\begin{aligned} J(n,1) &= -(7n^4+7n^3+9n^2-4n-1) < 0, \\ \left.\frac{\partial J(n,\gamma)}{\partial\gamma}\right|_{\gamma=1} &= -3(n+1)^2(n^3+6n^2+n-1) < 0, \\ \left.\frac{\partial^2 J(n,\gamma)}{\partial\gamma^2}\right|_{\gamma=1} &= -2(n+1)^4(9n-1) < 0, \end{aligned}$$

which assures that $J(n,\gamma)$ is always negative. Q.E.D.

Proof of Proposition 7 (i) From (24), we have

$$\begin{aligned} \frac{\partial Q_{-1}^E}{\partial n} = \gamma\times\Bigg\{ & \frac{((n+1)\gamma-n)(2(n+1)^2\gamma-(n^2+1))}{((n+1)\gamma-n)^2((n+1)^2\gamma-n)^2} \\ &+\frac{[n^2(n^2+1)-n(n+1)(5n^2+n+4)\gamma+(n+1)^2(7n^2+6n+3)\gamma^2-4(n+1)^4\gamma^3]z_f}{((n+1)\gamma-n)^2((n+1)^2\gamma-n)^2} \\ &+\frac{(n+1)\gamma[n(n^2-n+2)-2(n+1)(n^2+n+1)\gamma+2(n+1)^3\gamma^2]z_1}{((n+1)\gamma-n)^2((n+1)^2\gamma-n)^2}\Bigg\}. \end{aligned} \tag{38}$$

This is positive if and only if

$$z_1 < \frac{((n+1)\gamma - n)^2(2(n+1)^2\gamma - (n^2+1))}{(n+1)\gamma[n(n^2-n+2) - 2(n+1)(n^2+n+1)\gamma + 2(n+1)^3\gamma^2]}$$
$$+ \left[\frac{[-n^2(n^2+1) + n(n+1)(5n^2+n+4)\gamma]z_f}{(n+1)\gamma[n(n^2-n+2) - 2(n+1)(n^2+n+1)\gamma + 2(n+1)^3\gamma^2]}\right.$$
$$\left. + \frac{[-(n+1)^2(7n^2+6n+3)\gamma^2 + 4(n+1)^4\gamma^3]z_f}{(n+1)\gamma[n(n^2-n+2) - 2(n+1)(n^2+n+1)\gamma + 2(n+1)^3\gamma^2]}\right] \equiv z_1^Q. \quad (39)$$

For the nonemptiness, see the next part (ii) of this proof.

(ii) With some algebra we obtain

$$\begin{aligned}
z_1^x - z_1^Q &= \frac{2(1-z_f)((n+1)\gamma - n)^2((n+1)^2\gamma - n)^2((n+1)^2\gamma - (n^2+1))}{n(n+1)\gamma[2n - (n+2)(n+1)^2\gamma + 2(n+1)^3\gamma^2]H'} > 0, \\
z_1^Q - z_1^P &= \frac{(1-z_f)(n-1)((n+1)\gamma - n)^2((n+1)^2\gamma - n)^2((n+1)^2\gamma - (n^2+1))}{\gamma(n+1)HH'} > 0, \\
&\quad \text{where } H = [n^2(1-n) + n^3(n+1)^2\gamma - (n+1)^3(2n^2+1)\gamma^2 + (n+1)^5\gamma^3], \\
&\quad H' \equiv [n(n^2-n+2) - 2(n+1)(n^2+n+1)\gamma + 2(n+1)^3\gamma^2],
\end{aligned}$$

which proves the proposition. Q.E.D.

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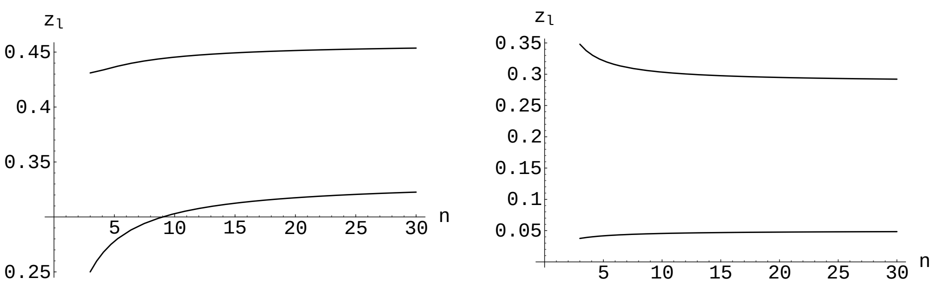
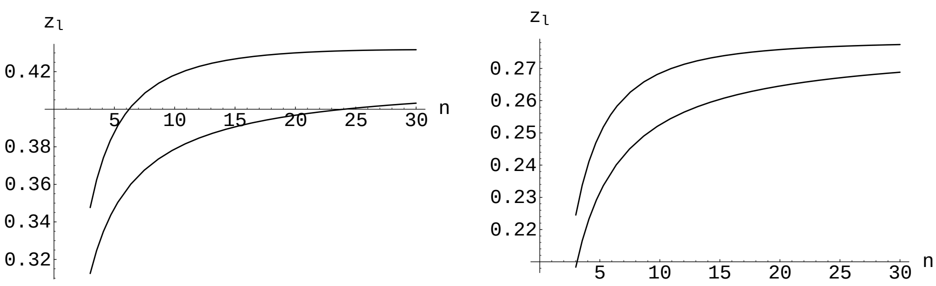


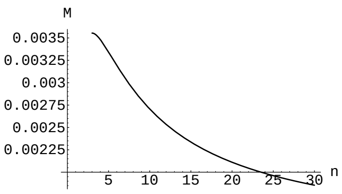
$(\gamma = 3/2)$ $(\gamma = 10)$

Figure 1: The range of z_1 ($\partial x_1/\partial n > 0$, $z_f = 1/2$)**.**

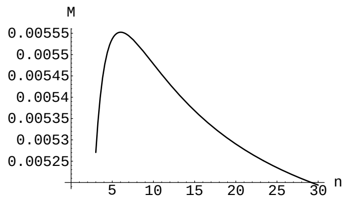


($\gamma = 6/5,\ z_f = 1/2$) ($\gamma = 9/5,\ z_f = 1/2$)

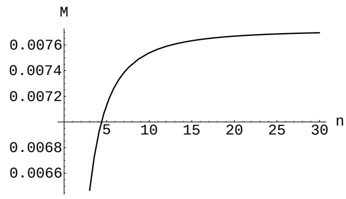
Figure 2: The range of z_1 ($\partial \Pi_1 / \partial n > 0$).



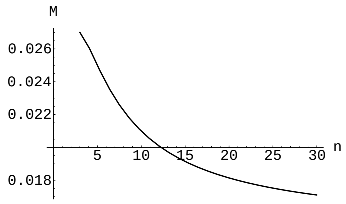
$(z_1 = 2/5,\ \gamma = 10)$



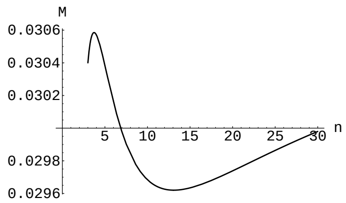
$(z_1 = 3/10,\ \gamma = 10)$



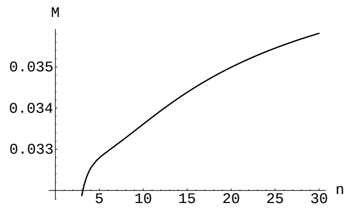
$(z_1 = 1/4,\ \gamma = 10)$



$(z_1 = 0.45,\ \gamma = 3/2)$



$(z_1 = 0.428,\ \gamma = 3/2)$



$(z_1 = 0.42,\ \gamma = 3/2)$

Figure 3: The relation between M and n ($z_f = 1/2$).