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Limited Attention, Interaction and the Growth of a Firm

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【キーワード】Limited Attention, Complementarity and Substitutability, Investment, Tobin's Q.

【要約】A person cannot make many decisions at a time, but an organization needs millions of interrelated decisions. We incorporate this idea into investment theory and examine its influence on a firm's growth rate. Two assumptions are emphasized: an agent cannot optimize more than one input at a time, and there is interaction among inputs. Each investment is lumpy, but adjustment is gradual. Without an adjustment cost function and exogenous shocks, we derive the growth rate of a firm. The derived growth rate is independent of firm size and imperfectly correlated with Tobin's Q.

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1 Introduction

What determines the growth rate of a firm? The average growth rate of capital stock of firms that continuously appear in the COMPUSTAT data set between 1970 and 1999 was about 16% during the 1970s, 14% during the 1980s and 15% during the 1990s[1]. Major companies in the U.S. continued to grow for the 30 years covered by our data set.

In order to explain the continuous expansion of a firm, traditional investment theories typically assume the existence of a convex adjustment cost function [e.g., Lucas and Prescott (1971)]. This explicitly links the growth rate of capital to Tobin's Q. The derived growth rate is independent of the firm's size, which arguably serves as a first approximation for the growth of a large firm [e.g., Sutton (1997)]. The dynamics of a firm are described as a process of adjustment toward the desired capital stock.

However, this adjustment cost function is questioned by evidence that investment is lumpy at the plant level [e.g., Doms and Dunne (1998)]. Lumpy investment is more consistent with alternative theories that emphasize the importance of a non-convex adjustment cost function [e.g., Caballero (1999)]. Different from the Q theory of investment, adjustment is immediate once a firm decides to invest. Hence, continuous growth must be explained by expanding exogenous external investment opportunities[2].

In contrast to previous literature, this paper completely dismisses conventional adjustment cost functions from our model and examines an alternative friction: a

[1] The average growth rate of capital is the net value of property, plant and equipment (Compustat # 8) over the deflator for nonresidential investment. The deflator is taken from the Bureau of Economic Analysis.

[2] Abel and Eberly (1994) incorporate two frictions into one model and discuss its relation to Q. Cooper and Haltiwanger (2000) argue that a model that mixes both convex and nonconvex adjustment costs with irreversibility fits plant-level data well.

firm cannot optimize more than one input[3]. Hence, the immediate adjustment of more than one input is prohibited. It is shown that each investment is lumpy as in sunk cost models, but adjustment is gradual as in convex adjustment cost models.

The particular friction that this paper aims to capture is the limited capacity of human attention, the importance of which was originally emphasized by Simon (1947). It is a common observation that organizations demand millions of immediate decisions, but individual CEOs are not able to handle all of them at the same time. As it is difficult to delegate strategic decisions, such as finding investment projects, to other managers, CEOs must spend much of their time making and implementing an investment plan. Because their attention capacity is limited, CEOs must postpone other strategic decisions such as developing a trained management group. We investigate how the allocation of attention influences the growth rate of a firm.

In order to determine the growth rate, we need one more assumption: there is interaction among inputs, where the interaction means that inputs are strict complements or strict substitutes. When there is no interaction among inputs, adjustment is immediate: the firm immediately chooses the optimal level of inputs and maintains it. However, when interaction exists, the current optimal decision depends on the amount of other inputs. It is shown that adjustment then becomes gradual.

In this paper, we examine two-input cases, in which the inputs are called capital and labor. Two decisions are called investment and employment. To give an intuitive reason for gradual adjustment, consider the case where a firm's attention is alternately allocated to investment and employment. Suppose that capital and labor are complements and that the amounts of capital and labor are less than optimal. In this case, profits would be larger if the firm could increase both capital and labor at the same time. Hence, when the firm makes its investment decision, it invests more

[3] This assumption is the same as the one that a firm must incur infinite fixed costs when more than one decision is made. Hence, the model can be interpreted as a variant of a sunk cost model, though the firm never incurs sunk costs.

than needed for current production in order to increase the marginal product of labor and to enhance employment in the next period. Similarly, when the firm makes its employment decision, it employs more than needed for current production in order to increase the marginal product of capital and to enhance its investment in the next period. This capital and labor hoarding increases cost for current production. This static loss plays the role of adjustment cost.

Similar to the Q theory of investment, the adjustment cost derived has the property that the accumulated capital stock lowers the marginal cost of capital hoarding. Hence, it causes a positive feedback mechanism that enhances the continuous adjustment of a firm: an increase in capital stock lowers the marginal cost of capital hoarding, which gives more incentive to accumulate capital.

The following example is illustrative. Consider a firm that needs both an office and skilled managers as productive inputs. When the firm constructs a building, it will keep spare space. The spare space is not needed for current production, but it allows the firm to employ and train more managers in future. Similarly, excess trained managers are needed to set up a new office in a different region in the future. Although the spare spaces prepare for future expansion, it is costly to build a skyscraper when a firm currently employs only three managers. In this way, adjustment cost is endogenized. Note that once the firm employs 30 managers, it may be worth constructing a building for them. That is, the existence of many skilled managers as a result of the previous capital accumulation lowers the static loss due to capital hoarding and enhances further capital accumulation.

The above adjustment process may not occur when a firm can optimally allocate its attention. It may be optimal to stick to an employment decision. In this case, because a firm does not need to hoard labor, there is no growth. This might happen when the owner of a restaurant cares only about employing or training skilled chefs but not about investing in a new restaurant.

A natural question is: What might be the conditions under which the firm starts its adjustment? This paper derives necessary and sufficient conditions for the firm to adjust to the desired level under a constant-returns-to-scale production function. Intuitively, the firm must compare two types of rents: rent from changing its attention and that from sticking to one decision. The firm starts its adjustment if and only if the former rent is larger than the latter. We argue that when many workers quit a job and a firm cannot accumulate skills, the firm sticks to employment and stops its adjustment to the desired capital stock[4].

Assuming that the condition for the adjustment is satisfied, this paper derives the growth rate of a firm when the production function has constant returns to scale, there is no adjustment cost function and markets are competitive. The derived growth rate is shown to be a positive function of the modified user cost and the modified wage rate, both of which are correlated with Tobin's Q. It is shown that even if there is no adjustment cost function and the production function has constant returns to scale, Tobin's Q can deviate from one and contains information about investment opportunities.

The dynamics of investment have some empirical support. On the one hand, because there is no convex adjustment cost function in our model, each investment is lumpy, which is similar to a model with sunk cost. On the other hand, once decisions are aggregated over time, the investment–capital ratio is independent of the firm's size and correlated with Tobin's Q, which is similar to the Q theory of investment. In contrast to the Q theory of investment, the correlation between the derived investment–capital ratio and Tobin's Q is imperfect. Our calibration results show that the investment–capital ratio has a weak positive correlation with Q, but once we control for the cash flow–capital ratio, the correlation becomes negative. That is, the investment–capital ratio has a stronger positive relationship with the

[4] The results may interest readers for an alternative reason, because if a firm sticks to one decision, it cannot reach the standard optimal solutions. It implies that the standard profit maximization problem may not be a good approximation of reality if limited attention is important. The derived condition clarifies when the standard model is valid.

cash flow–capital ratio. The weak correlation between Q and investment is found in the literature [e.g., Chirinko (1993)], and the robust impact of cash flow on investment is also well known [e.g., Fazzari, Hubbard and Petersen (1988)].

Although we examine a model of limited attention, agents are perfectly rational, which is different from Simon (1947). In our model, attention is required to implement a plan. A similar strategy was previously used by Gifford (1992). She investigated the role of limited attention on the growth of a firm in a dynamic programming framework such as ours. However, Gifford (1992) does not investigate the interaction among inputs and, therefore, adjustment is immediate[5].

Investment decisions with complementary capital are investigated by Dixit (1997), Eberly and Mieghem (1997) and Jovanovic and Stolyarov (2000). In particular, Jovanovic and Stolyarov (2000) also produce spare spaces. However, their model does not deal with the limited capacity of attention and, therefore, the source of growth is exogenous movement of frontier technology.

This paper can be also interpreted as constructing a micro foundation for the adjustment cost of investment. To our knowledge, the literature that derives the adjustment cost of investment relies on gradual learning [e.g., Prescott and Visscher (1980)]. This paper does not include an information problem but still derives adjustment cost because hoarding behavior causes static losses.

Recently, Abel and Eberly (2003) proposed an investment model without an adjustment cost function. Similarly to ours, their model also predicts that Tobin's Q can be greater than one without adjustment cost, and the investment–capital ratio is correlated with Tobin's Q but not with marginal Q. In their model, investment

[5] More recently, some researchers investigated limited attention in a rational agent model to explain the stickiness of prices [Reis (2004)]. They describe limited attention by assuming information processing friction . In contrast, this paper limits the number of decisions an agent can take immediately. This strategy allows us to investigate interaction among inputs, which is the key to deriving a positive feedback mechanism for capital accumulation. We hope that our strategy nicely complements theirs.

opportunities are provided by exogenous rent due to monopoly power or decreasing-returns-to-scale technology. By contrast, our model does not need any exogenous shocks and is based on constant-returns-to-scale technology and a perfectly competitive market. Because the firm cannot optimize two inputs at the same time, there is a fixed input at each moment that creates quasi-rent, which provides an incentive for the firm to grow.

The paper is organized as follows. The next section examines the firm that allocates its attention alternately to investment and employment decisions. This exercise clarifies an intuitive mechanism to induce the dynamics of a firm. Section 3 analyzes the optimization of the allocation of attention. Section 4 investigates the conditions under which the firm adjusts to the desired level. Assuming that the firm satisfies the conditions for adjustment, Section 5 derives the growth rate of a firm. Section 6 calibrates the model and discusses quantitative relationships between investment, Q and cash flow in this model. Section 7 concludes and discusses possible extensions.

2 The Alternate Allocation of Attention

In this section, we examine a standard firm's profit maximization problem, except that a firm can optimize only one input at a time. In order to illustrate our idea, we assume that the firm alternates its attention. This assumption is relaxed in the next section.

As a benchmark, we first consider a stylized static problem. A firm maximizes its profits by choosing capital and labor given the wage rate, w, the user cost of capital, u, and the production function $F(K, L)$, where K is capital stock and L is labor measured in efficiency units. Although we call K and L capital and labor for the purpose of explanation, our mechanism works for any inputs.

Assuming that the production function is twice continuously differentiable, bounded, and strictly concave, and that the solutions are interior, it is known that the firm's decisions are characterized by two first-order conditions:

$$u = F_K (K^*, L^*), \ w = F_L (K^*, L^*),$$

where K^* and L^* are the solutions to this static optimization problem. Since the marginal products of capital and labor are equal to user cost and wage rates, the firm has no incentive to change its decision.

One of the important assumptions behind this result is that the firm can maximize labor and capital at the same time. However, this may not be true in reality. When a firm invests, it must consider thousands of matters: which machine to buy, where to set it, who should be assigned as an operator and so on. Our economic model is abstracted from these details. However, it is natural to think that planning and implementation are time consuming. More specifically, we assume that the firm can optimize only one input at a time.

When the firm optimizes an input, another input is assumed to be constant. This assumption aims to capture a role of routine operation. When the firm makes optimal decisions in investment, they presume that they maintain the same amount of labor as before[6]. After investment is made, the previous level of labor may not be optimal. However, as the previous level of labor is the result of past optimization, it would be one of the best alternatives given the restriction of limited attention. In this way, maintaining the same level can be interpreted as a very primitive description of "remembering by doing" which Nelson and Winter (1982, p.99) emphasize as an important function of a routine.

Given these assumptions, the standard problem must be rewritten as the following dynamic programming problem:

[6] Note that we measure labor in efficiency units. Hence, labor is considered to be the set of skills accumulated in a firm. The development of more skilled workers would demand time and attention. However, it would be an innocuous assumption that the firm can maintain the same level of human capital without attention.

$$
\begin{aligned}
V_l(K) &= \max_{L' \in R_+} \{F(K, L') - wL' - uK + \beta V_k(L')\}, \\
V_k(L) &= \max_{K' \in R_+} \{F(K', L) - wL - uK' + \beta V_l(K')\},
\end{aligned}
$$

where $\beta \in (0, 1)$ is a discount factor, and $V_l(K)$ and $V_k(L)$ are the value of the firm when it currently makes an employment decision and when it currently makes an investment decision, respectively. Because routinely chosen inputs are assumed to be constant, when the firm makes an investment decision, labor is considered a stock variable; when the firm makes an employment decision, capital is a stock variable. Two first-order conditions are

$$
\begin{aligned}
F_L(K, \lambda(K)) &= w - \beta V_k'(\lambda(K)), &(1) \\
F_K(\kappa(L), L) &= u - \beta V_l'(\kappa(L)), &(2)
\end{aligned}
$$

where $\lambda(K)$ and $\kappa(L)$ are optimal policy functions. Compared with the first-order conditions for simple static optimization, marginal cost deviates from input prices because of the additional terms, $\beta V_k'(\lambda(K))$ and $\beta V_l'(\kappa(L))$. Because the current investment decision (employment decision) can influence future profits, this possibility must be taken into account. Hence, the marginal products of capital and labor must be equal to input prices minus the effects on future profits. We call $w - \beta V_k'(\lambda(K))$ the modified wage rate and $u - \beta V_l'(\kappa(L))$ the modified user cost.

To illustrate the main idea, we assume that the value function is twice continuously differentiable. Then it is shown that

$$
\begin{aligned}
\lambda'(K) &= -\frac{F_{KL}(K, \lambda(K))}{F_{LL}(K, \lambda(K)) + \beta V_k''(\lambda(K))}, &(3) \\
\kappa'(L) &= -\frac{F_{KL}(\kappa(L), L)}{F_{KK}(\kappa(L), L) + \beta V_l''(\kappa(L))}. &(4)
\end{aligned}
$$

Because we assume a strict concave production function, the value functions are also strictly concave. Therefore, the denominators of the two equations are negative. This means that both policy functions are strictly increasing functions when two inputs are strict complements ($F_{KL}(K,L) > 0$ for all K and L) and that they are strictly decreasing functions when two inputs are strict substitutes ($F_{KL}(K,L) < 0$ for all K and L). We assume that the production function is either strictly complementary or strictly substitutionary. These assumptions are sufficient but not necessary for our results, but it makes our explanation clearer.

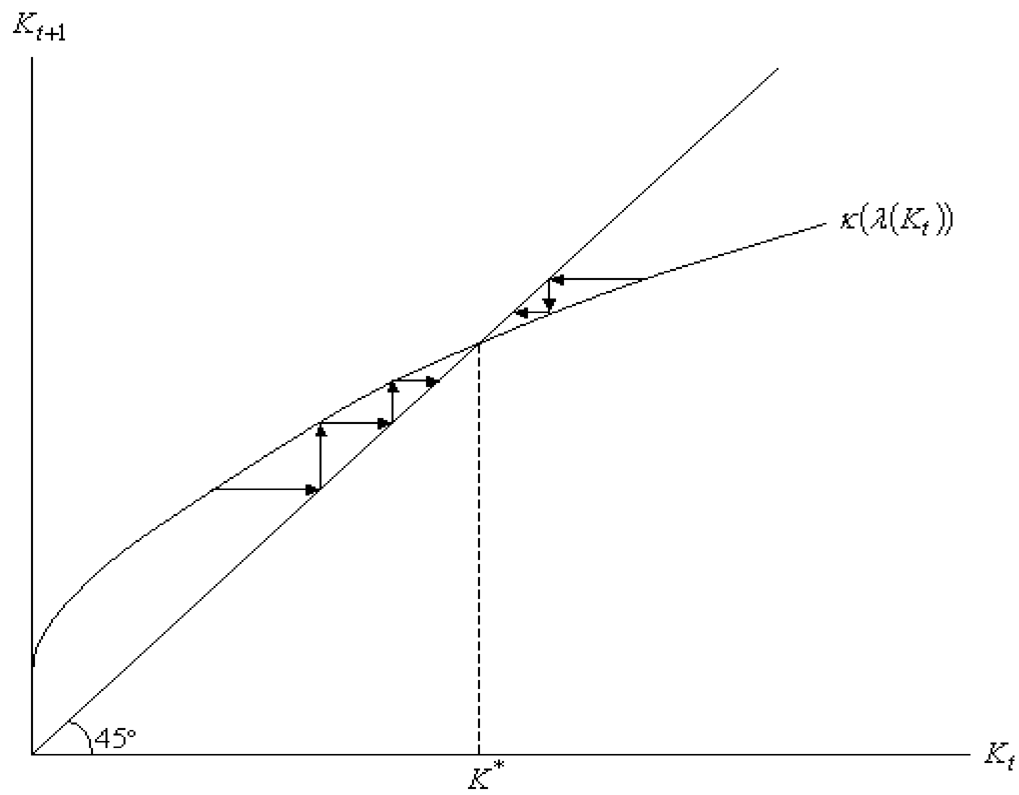

Figure 1: **The growth and decline of a firm**

We are interested in the following dynamics:

$$K_t = \kappa(\lambda(K_{t-1})).$$

Equations (3) and (4) imply that current capital stock is strictly increasing in the previous capital stock if two inputs are either strict complements or strict substitutes. Figure 1 depicts the dynamics of the firm, which are shown to be representative, below. If the initial capital stock is lower than K^*, it grows and converges to K^*. On the other hand, if the initial capital stock exceeds K^*, the firm declines and converges to K^*. It is shown below that K^* coincides with an optimal solution to the static problem. Hence, the process of growth can be seen as the adjustment process to the desired capital stock.

Note that if $F_{KL}(K, L) = 0$ for all K and L, the derivative of the function $\kappa(\lambda(\cdot))$ is 0. Hence, $\kappa(\lambda(K_{t-1}))$ in figure 1 is horizontal. This means that adjustment is immediate. It shows that interaction between two inputs for some K and L is necessary for gradual adjustment.

In order to understand the mechanism of the dynamics, two Euler equations are derived:

$$\begin{aligned}
MCLH(L_t, L_{t-1}) &= MRLH(L_t), \\
MCKH(K_t, K_{t-1}) &= MRKH(K_t), \ where \\
MCLH(L_t, L_{t-1}) &\equiv w - F_L(\kappa(L_{t-1}), L_t), \ MRLH(L_t) \equiv \beta[F_L(\kappa(L_t), L_t) - w], \\
MCKH(K_t, K_{t-1}) &\equiv u - F_K(K_t, \lambda(K_{t-1})), \ MRKH(K_t) \equiv \beta[F_K(K_t, \lambda(K_t)) - u].
\end{aligned}$$

$MCLH(L_t, L_{t-1})$ ($MCKH(K_t, K_{t-1})$) represents the marginal costs of labor hoarding (capital hoarding); $MRLH(L_t)$ ($MRKH(K_t)$) represents the marginal rent from labor hoarding (capital hoarding). Euler equations show that it is optimal to equate the marginal cost and marginal rent from hoarding. Since the intuitive logic is the same, we confine our discussion mainly to capital. However, the reader can apply the same logic to labor.

$MCKH(K_t, K_{t-1})$ is defined as the deviation of user cost from the marginal product of capital. When user costs are larger than the marginal productivity of capital, the firm employs more capital than is needed for current production. Employing one more unit of capital increases expenses by user cost and output by the marginal product of capital. Hence, the deviation of user cost from the marginal productivity can be considered the marginal cost of capital hoarding. Although we do not have any adjustment cost function, the cost of capital hoarding serves as adjustment cost in this paper.

On the other hand, hoarding capital generates a profit opportunity for the firm in the next period, because when the firm invests in capital, it knows that capital is fixed in the next period: it creates rent from capital. In order to increase this rent, the firm has an incentive to invest more than needed. Employing one more unit of capital increases output by the marginal product of capital and fixed cost by user cost. Hence, the marginal rent from capital hoarding is the present value of this difference, which is how $MRKH\left(K_t\right)$ is defined.

When the firm reaches the steady state, K^* and L^*, the firm keeps the same level of capital stock and labor. Hence, there is no reason to maintain capital hoarding. Substituting $K^* = K_t = K_{t-1}$ and $L^* = L_t = L_{t-1}$ into two Euler equations, it is shown that capital and labor in the steady state are the same as under static optimization:

$$F_K\left(K^*, \lambda\left(K^*\right)\right) = u,\ F_L\left(\kappa\left(L^*\right), L^*\right) = w.$$

By taking the derivative with respect to K around the steady state, the slope of $\kappa\left(\lambda\left(K\right)\right)$ at the steady state in Figure 1 is shown to be less than one:

$$\frac{d\kappa\left(\lambda\left(K\right)\right)}{dK}|_{K=K^*} = \frac{\left[F_{KL}\left(K^*, \lambda\left(K^*\right)\right)\right]^2}{F_{KK}\left(K^*, \lambda\left(K^*\right)\right) F_{LL}\left(K^*, \lambda\left(K^*\right)\right)} \in (0, 1)\,.$$

Since $\kappa\left(\lambda\left(K\right)\right)$ is a strictly increasing function, figure 1 shows that the dynamics must be globally stable. Hence, the dynamics depicted by figure 1 are representative.

In order to understand the mechanism of growth in detail, let us examine $MRKH\left(K_t\right)$ and $MCKH\left(K_t, K_{t-1}\right)$. Because of the strict concavity of the production function,

$MCKH(K_t, K_{t-1})$ is strictly increasing in K_t. Similarly, because $MRKH(K_t) = \beta V_l'(K_t)$ for all K_t and the value function is strictly concave, $MRKH(K_t)$ is strictly decreasing in K_t. Hence, the intersection is unique, which is depicted in Figure 2**.**

The dynamics occur because $MCKH(K_t, K_{t-1})$ depends on K_{t-1}. We can derive the following condition from the definition of $MCKH(K_t, K_{t-1})$:

$$\frac{\partial MCKH(K_t, K_{t-1})}{\partial K_{t-1}} = -F_{KL}(K_t, \lambda(K_{t-1}))\lambda'(K_{t-1}) < 0.$$

Note that when $F_{KL}(K, L) > 0$, $\lambda'(K) > 0$, while when $F_{KL}(K, L) < 0$, $\lambda'(K) < 0$. Hence, marginal cost declines as capital accumulates. This provides a nice positive feedback mechanism: an increase in capital stock reduces the marginal cost of capital hoarding, which generates incentive for the firm to make further investment. Figure 2 describes the feedback mechanism. If $MCKH(K_t, K_{t-1})$ and $MRKH(K_t)$ intersect at a positive value, capital accumulates. Larger capital lowers the marginal cost of capital hoarding and provides further incentive to accumulate capital. On the other hand, if $MCKH(K_t, K_{t-1})$ and $MRKH(K_t)$ intersect at a negative value, the firm disinvests. This increases the marginal cost of capital hoarding and forces the firm to give up more capital. Since $MCKH(K_t, K_{t-1})$ and $MRKH(K_t)$ are equal to 0 when $K_t = K^*$, the dynamics stop.

In the next section, we assume that the production function has constant returns to scale in K and L and analyze the optimal allocation of attention. In the case of constant returns to scale, the production function is not strictly concave, and therefore there is no steady state in general. Without any proof, the dynamics and mechanism of the growth of the firm with a constant-returns-to-scale production function are depicted in Figure 3 and 4. Figure 4 shows that $MRKH(K_t)$ is constant when the production function has constant returns to scale. Hence, as Figure 3 shows, the firm can grow as far as market conditions allow or can decline until it vanishes.

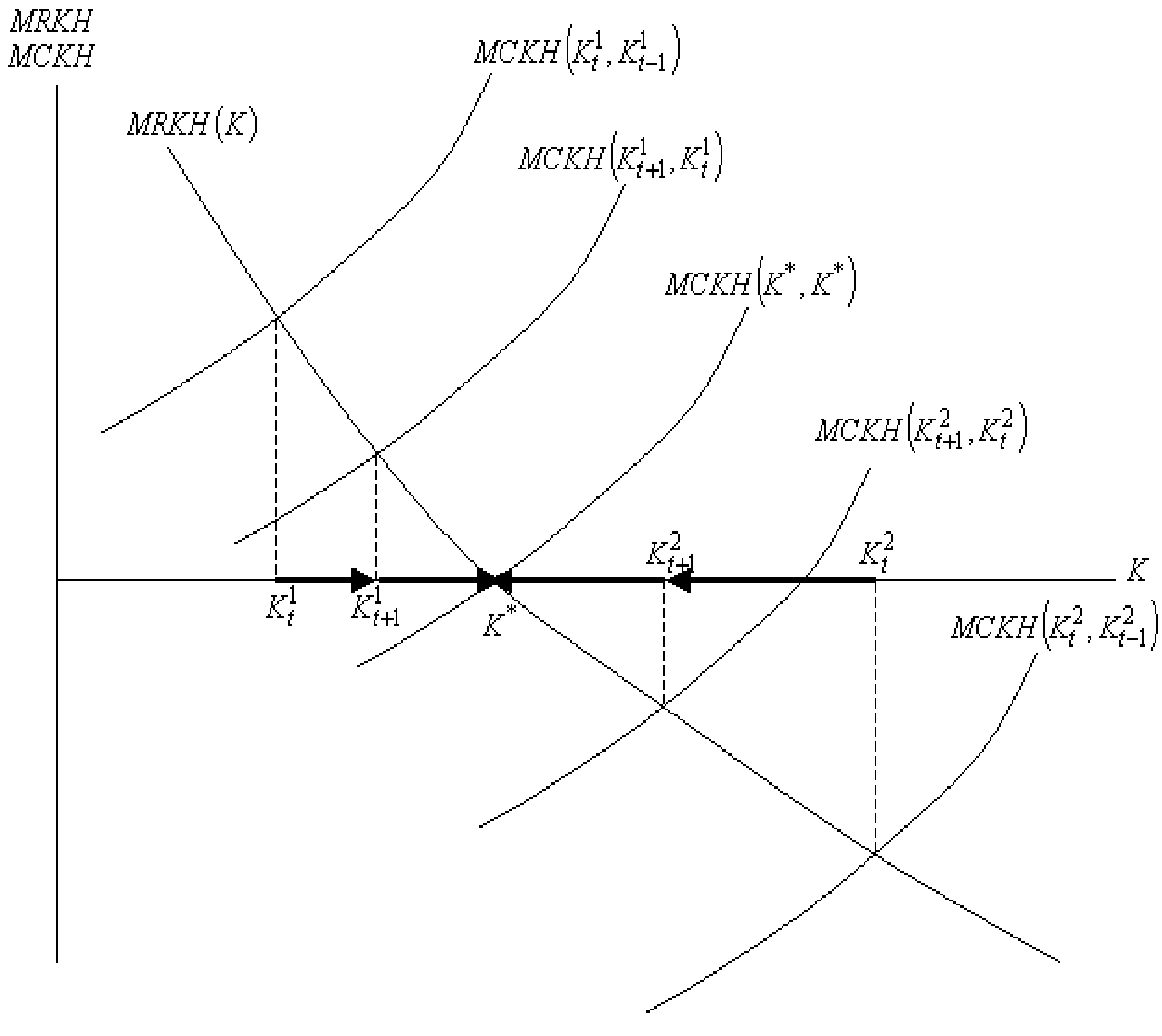

Figure 2: **The mechanism of growth and decline of a firm**

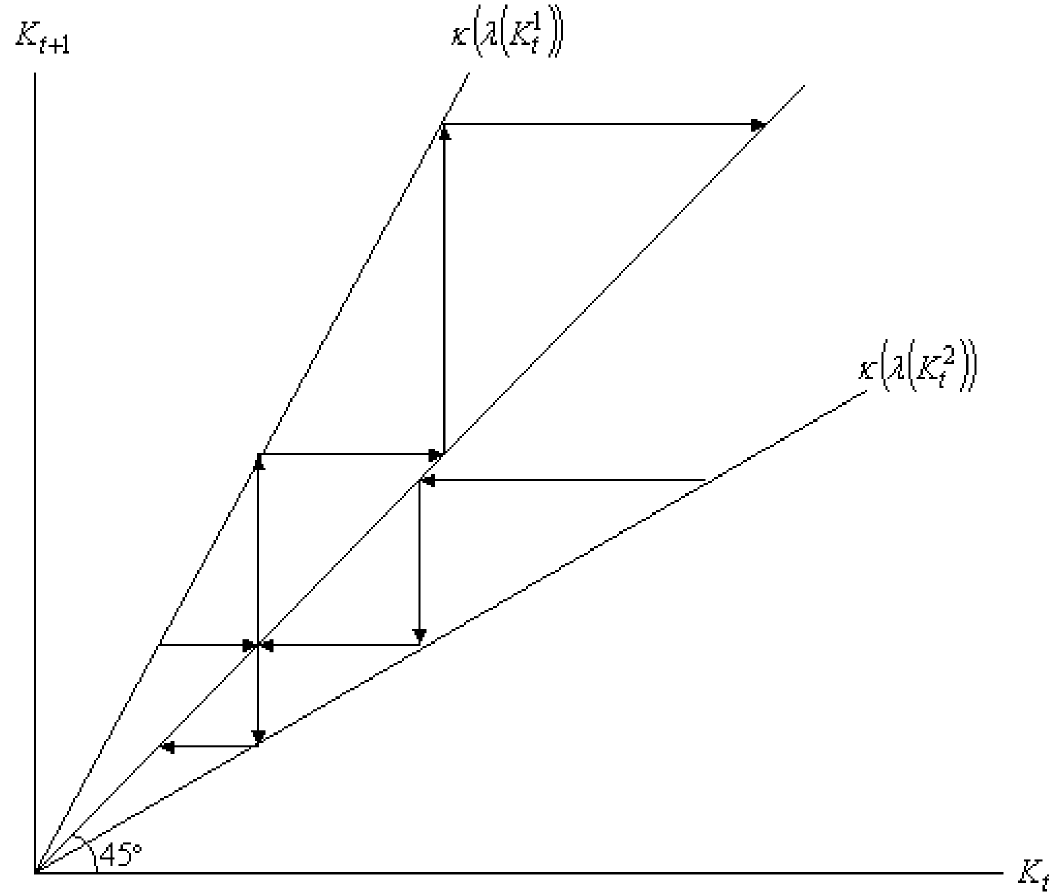

Figure 3: **The growth and decline of a firm when the production function has constant returns to scale**

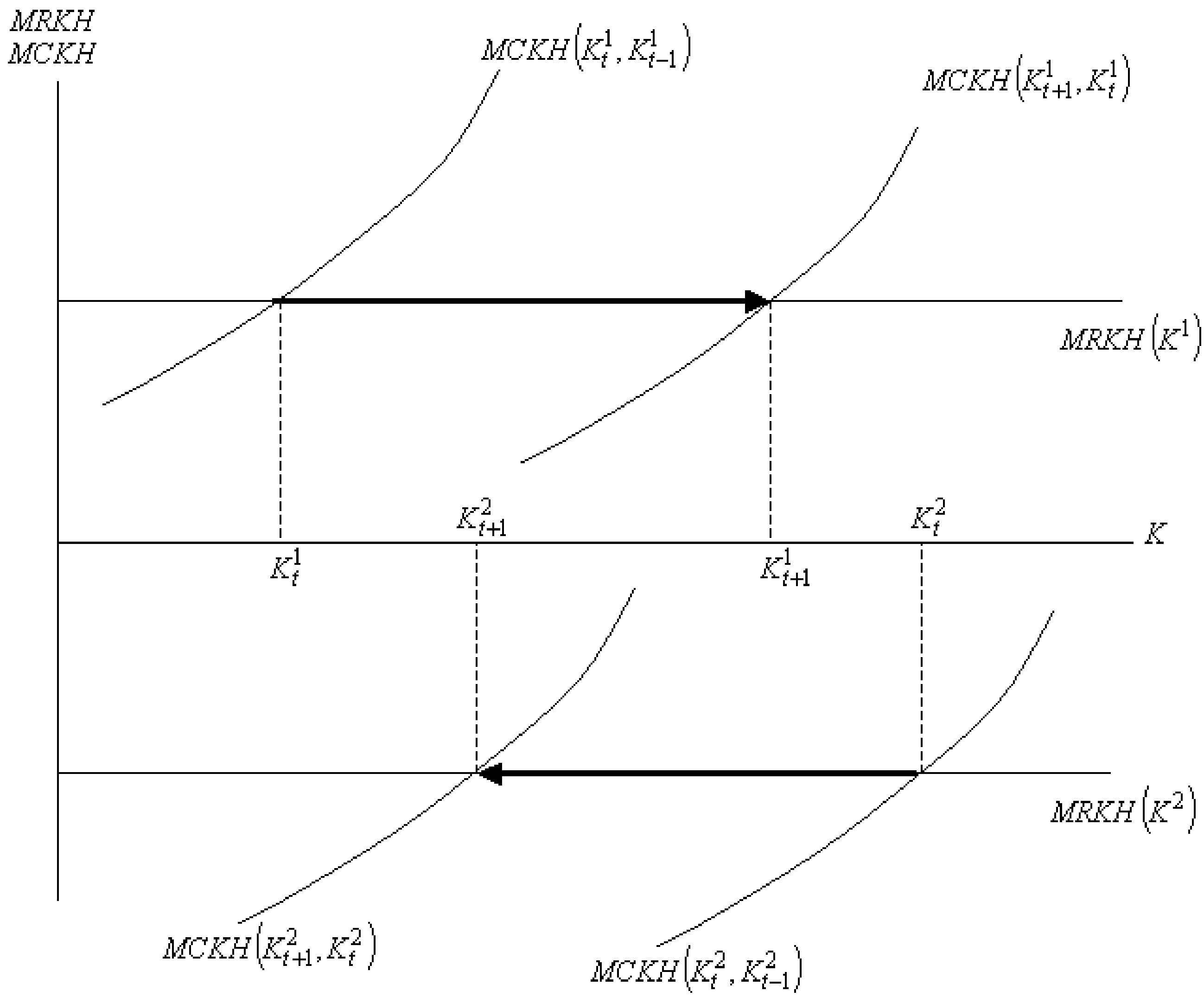

Figure 4: **The mechanism of the growth and decline of a firm when the production function has constant returns to scale**

3 The Optimal Allocation of Attention

In the previous section, the order of decisions is given. However, the alternate allocation of attention may not be optimal. Some firms may concentrate on training and employing workers and not care much about new physical investment. In this section, we relax this restriction and assume that the firm optimally chooses investment or employment. This makes our analysis more complicated. In order to provide a tractable model, we assume that $F(K, L)$ exhibits constant returns to scale in K and L. This assumption has other advantages, too. First, it allows us to link our theory to Tobin's Q. Second, since constant returns to scale and a competitive economy eliminate any economic rent, the only source of rent in this model becomes the existence of quasi-fixed input due to routine operation. In this section we show that the rent from the quasi- fixed factor causes Tobin's Q to deviate from 1 [7].

Let us first set up our model:

$$
\begin{aligned}
V(K, L) &= \max\left\{V_l(K), V_k(L; K)\right\}, && (5)\\
V_l(K) &= \beta \max_{L' \in \left[0, \frac{\alpha K}{1-\delta_l}\right]} \left\{F(K, L') - wL' + V\left((1-\delta_k)K, (1-\delta_l)L'\right)\right\}, && (6)\\
V_k(L; K) &= \max_{K' \in \left[0, \frac{\alpha L}{1-\delta_k}\right]} \left\{\beta\left[F(K', L) - wL + V\left((1-\delta_k)K', (1-\delta_l)L\right)\right] - p_k I\right\} && (7)\\
s.t.\ I &= K' - K,
\end{aligned}
$$

where $\alpha < \beta = \frac{1}{1+i}$ and i is an interest rate. The parameters δ_k and δ_l are the depreciation rate of capital and effective labor, respectively. The parameter p_k is the price of investment goods, and the price of output is normalized to 1. Equation (5) describes the optimal choice of projects, where the projects are investment or employment, and the value function $V(K, L)$ represents the value of the firm when

[7] There is also a cost to assuming constant-returns-to-scale technology. As the production function is constant returns to scale in K and L, we must restrict our attention to the case of complementarity, below.

the firm currently selects projects. The function $V_l(K)$ represents the value of the firm when the firm currently makes an employment decision, which is defined by Equation (6) and the function $V_k(L;K)$ represents the value of the firm when the firm currently makes an investment decision, which is defined by Equation (7).

There are three technical differences from the previous section. First, we explicitly model investment decisions in Equation (7) and exclude rental expenses of capital from the model. The explicit expression of an investment decision clarifies the link between investment and Tobin's Q. The timing of discounting and depreciation rate is arranged so we can later derive Jorgenson's user cost. Second, we allow the depreciations of capital and labor. The depreciation of labor is unusual. Since we consider L as human capital rather than the number of workers, it has to be interpreted as the depreciation rate of human capital in a firm. Finally, there are explicit upper bounds on choice variables, which are influenced by $\alpha < \beta$. This guarantees that the solution does not explode and that there exists a unique value function $V(K,L)$. The proof is an application of Stokey and Lucas (1989, p87) and we omit it[8].

Since the production function has constant returns to scale, the value function is expected to be linear in K, $V(K,L) = Q(l)K$ where $l = \frac{L}{K}$. Define the deviation of Tobin's Q from 1 by $D(l) \equiv \left[\frac{Q(l)}{p_k} - 1\right] p_k$. The standard guess and verify method proves that $D(l)$ must recursively satisfy the following equation:.

$$
\begin{aligned}
D(l) &= \beta \max\{\pi_k l, \pi_l\} \\
\pi_k &\equiv \max_{k' \in [0,\alpha_k]} \left\{ f(k') - uk' - w + D\left(\frac{1-\delta_l}{(1-\delta_k)k'}\right)(1-\delta_k)k' \right\}, \qquad (8) \\
\pi_l &\equiv \max_{l' \in [0,\alpha_l]} \left\{ \left[f\left(\frac{1}{l'}\right) - w \right] l' - u + D\left(\frac{(1-\delta_l)l'}{1-\delta_k}\right)(1-\delta_k) \right\}, \qquad (9)
\end{aligned}
$$

where $k' = \frac{K'}{L}$, $l' = \frac{L'}{K}$, $\alpha_k = \frac{\alpha}{1-\delta_k}$, $\alpha_l = \frac{\alpha}{1-\delta_l}$ and $f(\cdot) = F(\cdot, 1)$. The variable u is Jorgenson's (1963) user cost, $u \equiv (i + \delta_k) p_k$. Variable, π_k (π_l) is the present

[8] Technically, it is also assumed that there is $B_F \in (0,\infty)$, which satisfies $|F(K,L)| \leq B_F(|K| + |L|)$ for any $K \in R_+$ and $L \in R_+$.

value of the stream of the discounted rent per unit labor (per unit capital) when the firm currently makes an investment decision (employment decision). The rent exists because there are quasi-fixed factors in the model. We call π_k and π_l the average rent when the firm makes an investment decision and an employment decision, respectively.

Substituting the definition of $D(l)$ into the definition of π_k and π_l, we can express π_k and π_l as the solutions to the following two equations. The proof of the following proposition is given in the Appendix.

Proposition 1 *The deviation of Tobin's Q from* 1, $D(l) = \left[\frac{Q(l)}{p_k} - 1\right] p_k$, *is a function of two measures of average rent,* π_k *and* π_l *:*

$$D(l) = \beta \max\{\pi_k l, \pi_l\},$$

where π_k *and* π_l *are the solutions to the following two equations:*

$$\begin{aligned}
\pi_k &= \max\{\Pi_k(u - \beta_k \pi_l), \Pi_k(u) + \beta_l \pi_k\} && (10)\\
\pi_l &= \max\{\Pi_l(w - \beta_l \pi_k), \Pi_l(w) + \beta_k \pi_l\}. && (11)\\
\Pi_k(x) &= \max_{k' \in [0, \alpha_k]} [f(k') - xk'] - w && (12)\\
\Pi_l(x) &= \max_{l' \in [0, \alpha_l]} \left[f\left(\frac{1}{l'}\right) - x\right] l' - u && (13)
\end{aligned}$$

where $\beta_k = (1 - \delta_k)\beta$ *and* $\beta_l = (1 - \delta_l)\beta$*.*

The function $\Pi_k(x)$ ($\Pi_l(x)$) is the average instantaneous profit function when the firm invests (employs) and maps from the prices for the variable inputs to instantaneous profits per unit labor (per unit capital).

The proposition shows that Tobin's Q can deviate from 1 when $\max\{\pi_k l, \pi_l\}$ is not equal to 0 and that π_k and π_l are the maximums of different types of the average rent. Let us explain the meaning of equation (10). We can apply the same arguments to Equation (11). When the firm makes an investment decision and the next decision is employment, the firm invests more than is needed for current production to increase the marginal product of labor at the next period. Because of this additional benefit, the relevant user cost for this investment is the modified user cost, $u-\beta_k\pi_l$, which is explained in Section 2. Hence, the average rent from changing its attention to employment is $\Pi_k\left(u-\beta_k\pi_l\right)$. However, if the firm continues to invest during the next period, there is no benefit from capital hoarding. Hence, the relevant user cost is equal to Jorgenson's. After maximizing profits, given an input price of u, the firm expects to receive π_k again at the next period. Hence, the average rent from holding to an investment decision is $\Pi_k\left(u\right)+\beta_l\pi_k$. Equation (10) shows that π_k is the maximum of the two types of the average rent.

The next section solves Equations (10) and (11), and derives the conditions under which the firm alternates its attention and adjusts to the desired level.

4 When Does a Firm Adjust to the Desired Level?

This section investigates the condition under which it is optimal to alternate its attention. Firstly, exploiting the benefits of stationarity, the sequential problem expressed in Equations (10) and (11) is modified into a one shot problem. Let us define $G_l\left(\pi_k\right)$, $G_k\left(\pi_l\right)$, R_l, and R_k as

$$\begin{aligned}
G_l\left(\pi_k\right) &= \Pi_l\left(w-\beta_l\pi_k\right),\ G_k\left(\pi_l\right)=\Pi_k\left(u-\beta_k\pi_l\right),\\
R_l &= \frac{\Pi_l\left(w\right)}{1-\beta_k},\ R_k=\frac{\Pi_k\left(u\right)}{1-\beta_l}.
\end{aligned}$$

New functions $G_l\left(\pi_k\right)$ and $G_k\left(\pi_l\right)$ are mapping from π_k to π_l and π_l to π_k, respectively. New variables R_l (R_k) are the present discounted value of the stream of rent per unit capital (per unit labor) when the firm sticks to employment (investment). We call R_l and R_k the reservation value of average rents when the firm sticks to employment and investment, respectively.

We define the following new problem:

$$\pi_k = \max\left\{G_k\left(\pi_l\right),R_k\right\}, \tag{14}$$

$$\pi_l = \max\{G_l(\pi_k), R_l\}. \quad (15)$$

The new problem says that the firm adjusts to the desired level when the two average rents from changing attention are larger than their reservation values. Since the environment is stationary, this one shot problem is expected to be equivalent to the original problem. Let $\pi_k^{\#}$ and $\pi_l^{\#}$ denote the solutions to Equations (10) and (11), and let π_k^* and π_l^* denote the solutions to Equations (14) and (15). The following lemma shows that the new problem is equivalent to the original one. The proof is established in the Appendix.

Lemma 2 *The solutions to Equations (10) and (11) are equivalent to the solutions to Equations (14) and (15).*

$$\pi_l^* = \pi_l^{\#},\ \pi_k^* = \pi_k^{\#}.$$

The lemma permits us to work with Equations (14) and (15). In order to characterize the conditions under which adjustment takes place, it is convenient to define mappings $H_l(\pi_l)$ and $H_k(\pi_k)$:

$$H_k(\pi_k) = \pi_k - G_k(G_l(\pi_k)), \quad (16)$$

$$H_l(\pi_l) = \pi_l - G_l(G_k(\pi_l)). \quad (17)$$

Let π_k^{**} and π_l^{**} denote the solutions to $\pi_k^{**} = G_k(\pi_l^{**})$ and $\pi_l^{**} = G_l(\pi_k^{**})$. Then

$$H_l(\pi_l^{**}) = 0, H_k(\pi_k^{**}) = 0. \quad (18)$$

That is, the solutions to Equations (18) characterize the average rents when the firm adjusts to the desired level. We first state the property of H - functions. The proof is given in the Appendix.

Lemma 3 *The functions $H_l(\cdot)$ and $H_k(\cdot)$ are strictly increasing and concave:*

$$H_l'(\pi_l) > 0, H_k'(\pi_k) > 0 \quad (19)$$

$$H_l''(\pi_l) \leq 0, H_k''(\pi_k) \leq 0. \quad (20)$$

Since Lemma 3 shows that $H_l(\cdot)$ and $H_k(\cdot)$ are continuous and strictly increasing functions, the existence and uniqueness of π_l^{**} (π_k^{**}) can be proved by showing that for some small π_l (π_k), $H_l(\pi_l) < 0$ ($H_k(\pi_k) < 0$) and for other large π_l (π_k), $H_l(\pi_l) > 0$ ($H_k(\pi_k) > 0$). The formal proof of the following theorem is given in the Appendix.

Theorem 4 *There exists a unique π_l^{**} and π_k^{**}.*

Now, we are ready to provide conditions under which the firm alternates its attention. The following theorem achieves this. The proof is established in the Appendix.

Theorem 5 *The firm alternates its attention and adjusts to the desired level, if and only if $H_l(R_l) < 0$ and $H_k(R_k) < 0$:*

$$H_l(R_l) < 0,\ H_k(R_k) < 0\ iff\ \pi_l^* = \pi_l^{**} > R_l, \pi_k^* = \pi_k^{**} > R_k. \tag{21}$$

Note that $H_l(R_l)$ and $H_k(R_k)$ are not influenced by endogenous variables. This means that Theorem 5 characterizes technological and market conditions when the firm adjusts to the desired level. The intuition behind this condition is explained as follows. Note that

$$H_l(R_l) < 0,\ iff\ R_l < G_l(G_k(R_l)), \tag{22}$$

$$H_k(R_k) < 0,\ iff\ R_k < G_k(G_l(R_k)). \tag{23}$$

The left-hand side of Equation (22), R_l, is the average rent when the firm holds to employment decision from now on. The right-hand side of Equation (22), $G_l(G_k(R_l))$, is the average rent when the firm holds to employment after the firm changes its attention once. Since the environment is stationary, if the firm changes its attention once, it will continue to do so. A similar interpretation is applied to Equation (23). Hence, Theorem 5 says that when the benefit from a one-time change in its attention

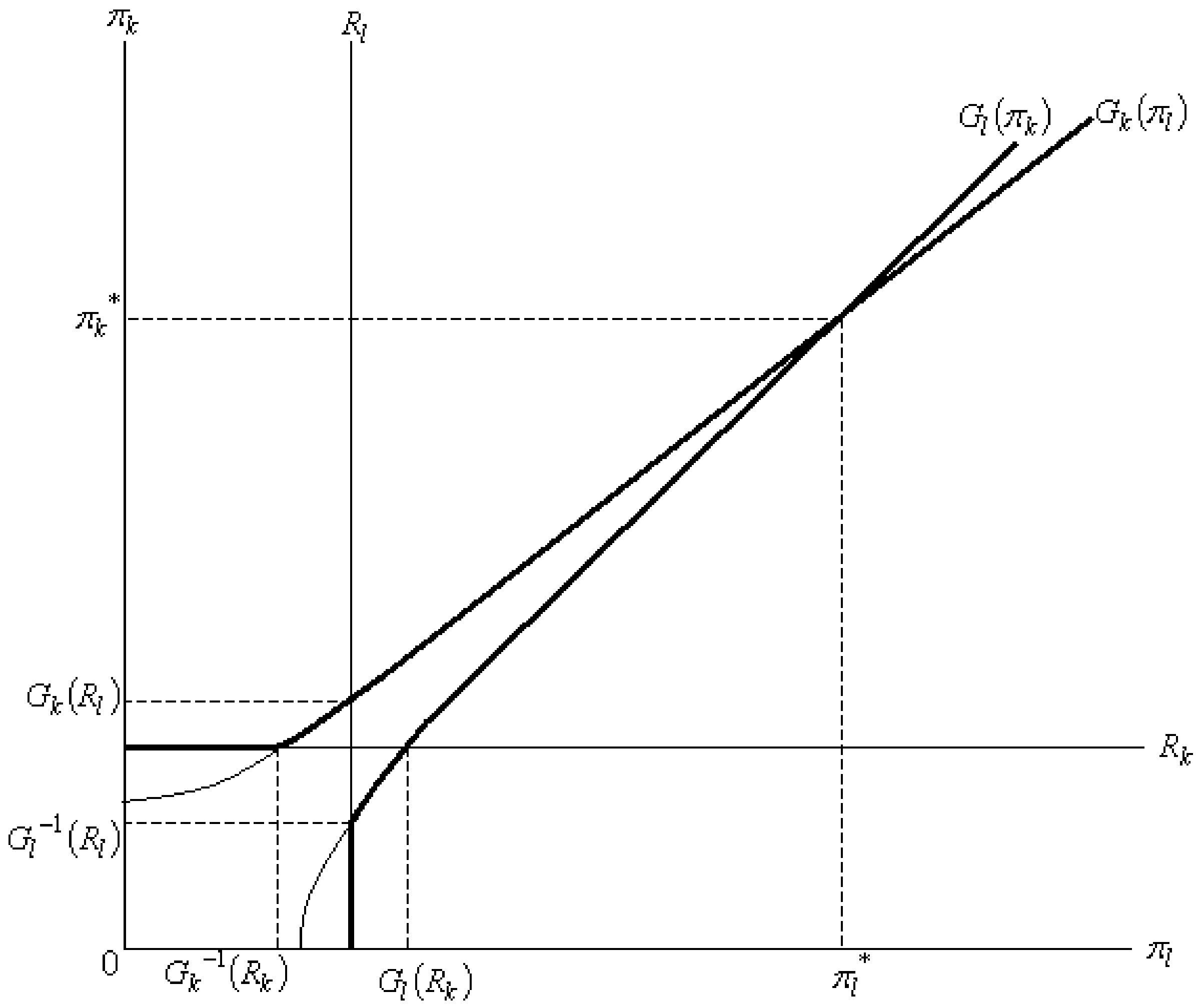

Figure 5: $\mathbf{H}_l(R_l) < \mathbf{0}, \mathbf{H}_k(R_k) < \mathbf{0}$**:** In this case, the firm alternates its attention between investment and employment and adjusts to the desired level. The solid line represents $\max\{G_k(\pi_l), R_k\}$ and $\max\{G_l(\pi_k), R_l\}$ and the intersection of the two solid lines is the solution.

is greater than sticking to the same decision for both investment and employment, it is optimal for the firm to allocate its attention and make its adjustment.

A graphical explanation of this result is depicted in Figure 5. Note that Theorem (5) is restated as

$$G_l^{-1}(R_l) < G_k(R_l), \; G_k^{-1}(R_k) < G_l(R_k), \; iff \; \pi_l^* > R_l, \; \pi_k^* > R_k.$$

The conditions, $G_l^{-1}(R_l) < G_k(R_l)$ and $G_k^{-1}(R_k) < G_l(R_k)$, are satisfied in Figure 5. The two solid lines represent $\max\{G_k(\pi_l), R_k\}$ and $\max\{G_l(\pi_k), R_l\}$. Hence, the intersection of the two solid lines is the solution to the original problem. As you can see, $\pi_k^* > R_k$ and $\pi_l^* > R_l$.

How do economic parameters influence the conditions for adjustment? Unfortunately, changes in most of the parameters do not bring clear results. We compare the average rent from sticking to one decision and that from changing attention. Most of the parameters, such as w and u, influence both rents in the same direction. Hence, the effect of w and u on the conditions for adjustment depends on which rents are more influenced by w and u. However, a relatively simple condition is obtained for the effect of the depreciation of labor.

Proposition 6 *Suppose that the employment decision is solved by an interior solution. Then*

$$\frac{dH_l\left(R^l;\delta_l\right)}{d\delta_l} > 0.$$

The condition implies that the larger the depreciation, the more likely the condition for adjustment is violated. As argued below, when the depreciation rate of labor is large, the firm is likely to hold to the employment decision. A company that cannot maintain human capital does not expect high profits from investment. Hence, it must spend much time recruiting and training workers.

Note that there is an asymmetry between the depreciation of labor and capital because the depreciation of capital also influences user cost, u. This additional effect obscures the result. If we can keep u constant, the effect of δ_k is similar to that of δ_l.

When the firm holds to one decision, there are three possible cases: the firm holds to investment, the firm holds to employment, or the firm's attention depends on the initial capital–labor ratio. The following theorem establishes the necessary and sufficient conditions under which each case occurs. The proof is established in the Appendix.

Theorem 7 *Suppose $H_l(R_l) \geq 0$ or $H_k(R_k) \geq 0$.*

$$G_l(R_k) > R_l, G_k(R_l) \leq R_k, \ iff \ \pi_l^* = G_l(R_k) > R_l, \ \pi_k^* = R_k, \tag{24}$$

$$G_l(R_k) \leq R_l, G_k(R_l) > R_k, \ iff \ \pi_l^* = R_l, \ \pi_k^* = G_k(R_l) > R_k, \tag{25}$$

$$G_l(R_k) \leq R_l, G_k(R_l) \leq R_k, \ iff \ \pi_l^* = R_l, \ \pi_k^* = R_k. \tag{26}$$

Equation (24) shows the condition under which the firm holds to investment, Equation (25) shows the condition under which the firm holds to employment and Equation (26) shows the condition under which the allocation of attention depends on the initial capital–labor ratio.

Figure 6, 7 and 8 provide the examples for each case. In figure 6, $H_k(R_k) \geq 0$ ($G_k^{-1}(R_k) \geq G_l(R_k)$), $G_l(R_k) > R_l$, and $G_k(R_l) \leq R_k$ are satisfied. It shows that $\pi_l^* = G_l(R_k) > R_l$ and $\pi_k^* = R_k$. Since $G_l(R_k) > R_l$, when a firm makes an employment decision, the next decision is investment. However, because $G_k(R_l) \leq R_k$, once the firm makes an investment decision, it does not change its attention. Hence, the firm holds to investment decisions.

In Figure 7, $H_l(R_l) \geq 0$ ($G_k^{-1}(R_k) \geq G_l(R_k)$), $H_k(R_k) \geq 0$ ($G_l^{-1}(R_l) \geq G_k(R_l)$), $G_l(R_k) \leq R_l$ and $G_k(R_l) > R_k$ are satisfied. It shows that $\pi_l^* = R_l$ and $\pi_k^* = G_k(R_l) > R_k$. The condition $G_k(R_l) > R_k$ means that when the firm makes an investment decision, the next decision is employment; $G_l(R_k) \leq R_l$ means that once the firm makes an employment decision, it does not change its attention. Hence, the firm holds to employment.

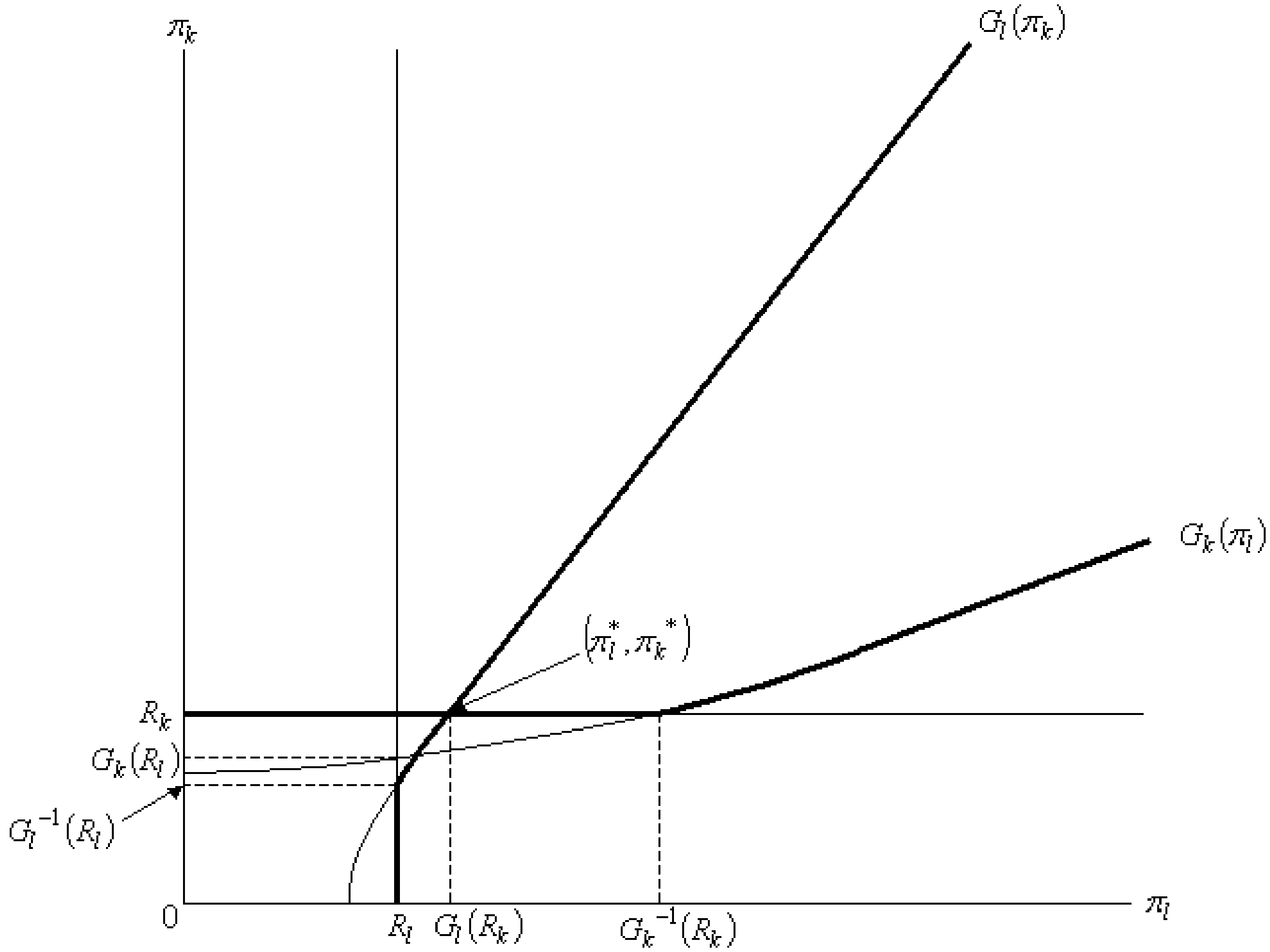

Figure 6: $\mathbf{H}_k(R_k) \geq \mathbf{0}, \mathbf{G}_l(R_k) > \mathbf{R}_l, \mathbf{G}_k(R_l) \leq \mathbf{R}_k$**:** In this case, the firm holds to investment. The solid line represents $\max\{G_k(\pi_l), R_k\}$ and $\max\{G_l(\pi_k), R_l\}$ and the intersection of the solid lines is the solution.

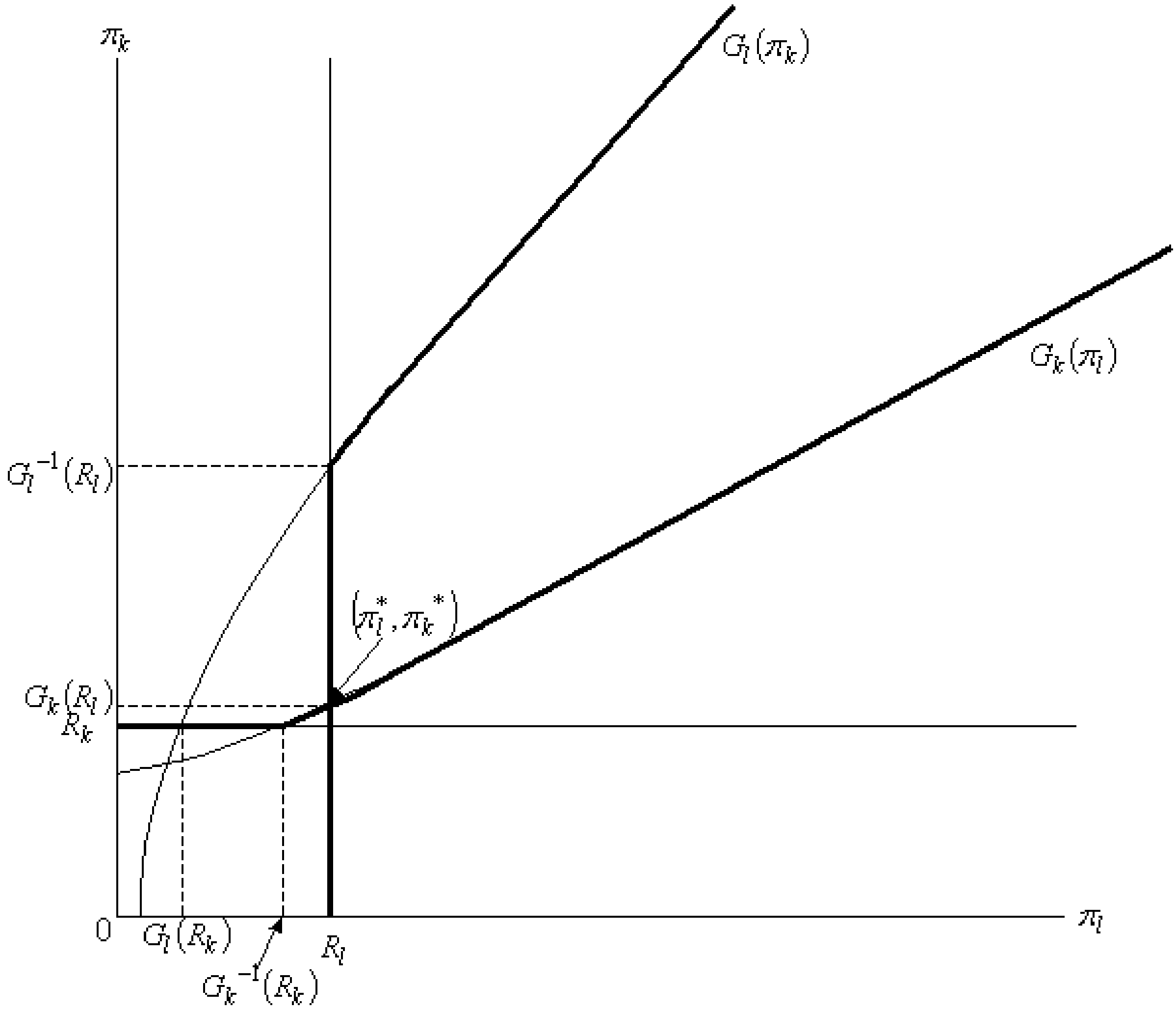

Figure 7: $\mathbf{H}_l(R_l) \geq \mathbf{0}, \mathbf{H}_k(R_k) \geq \mathbf{0}, \mathbf{G}_l(R_k) \leq \mathbf{R}_l, \mathbf{G}_k(R_l) > \mathbf{R}_k$**:** In this case, the firm sticks to employment. The solid line represents $\max\{G_k(\pi_l), R_k\}$ and $\max\{G_l(\pi_k), R_l\}$ and the intersection of the solid lines is the solution.

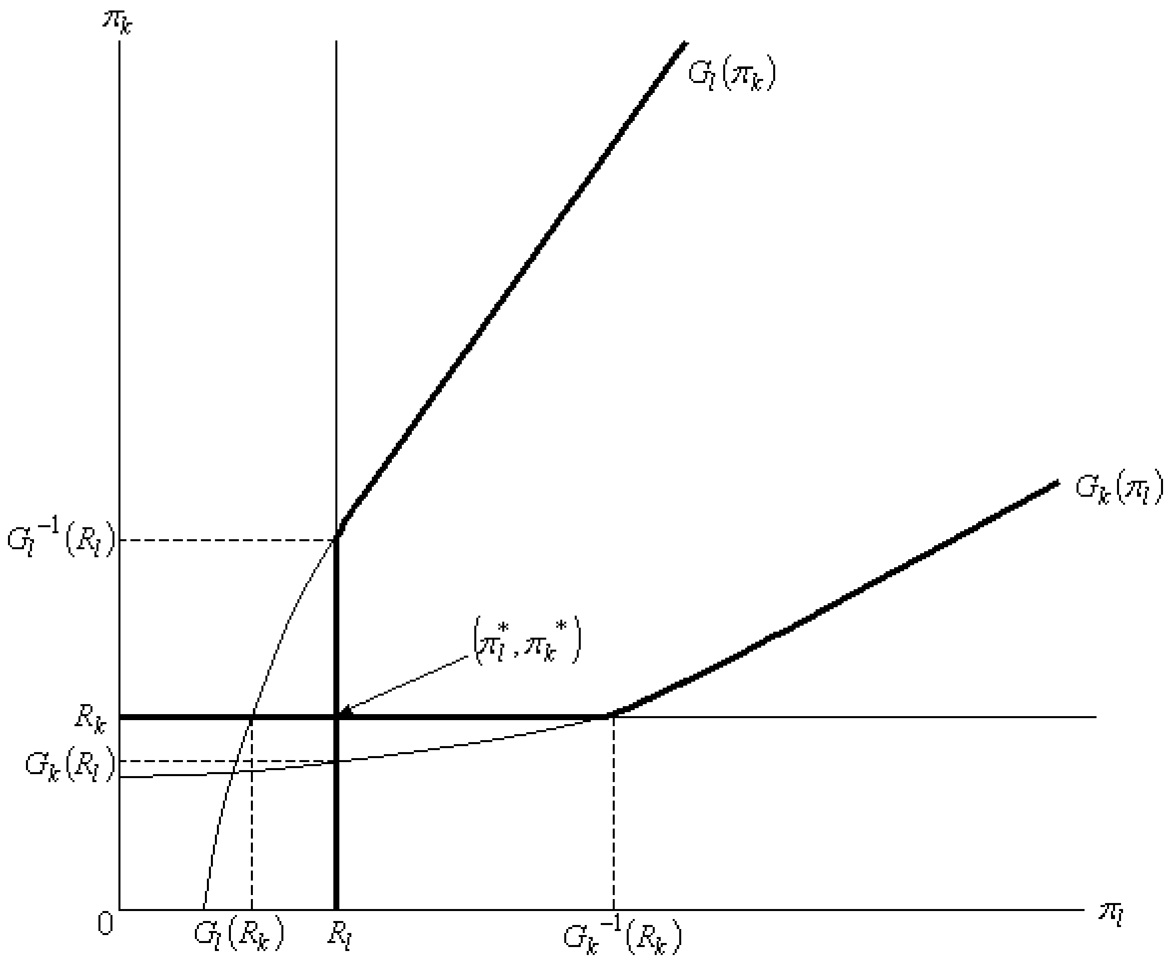

Figure 8: $\mathbf{H}_l(R_l) \geq \mathbf{0}, \mathbf{H}_k(R_k) \geq \mathbf{0}, \mathbf{G}_l(R_k) \leq \mathbf{R}_l, \mathbf{G}_k(R_l) \leq \mathbf{R}_k$**:** In this case, an initial capital–labor ratio determines the allocation of the firm's attention. The solid line represents $\max\{G_k(\pi_l), R_k\}$ and $\max\{G_l(\pi_k), R_l\}$ and the intersection of the solid lines is the solution.

Note that when the employment decision is solved by an interior solution, an increase in the depreciation rate of labor lowers $G_l(R_k)$ and R_k, while it keeps $G_k(R_l)$ and R_l the same. Hence, it is likely that the large depreciation rate forces the firm to hold to employment. This confirms our previous argument about the effect of a change in the depreciation rate of labor.

In figure 8, $H_l(R_l) \geq 0$ ($G_k^{-1}(R_k) \geq G_l(R_k)$), $H_k(R_k) \geq 0$ ($G_l^{-1}(R_l) \geq G_k(R_l)$), $G_l(R_k) \leq R_l$ and $G_k(R_l) \leq R_k$ are satisfied. Because $G_l(R_k) \leq R_l$ and $G_k(R_l) \leq R_k$ are satisfied, once the firm allocates its attention either to investment or to employment, it never changes its attention. The allocation of the firm's attention depends on its initial capital–labor ratio. If its initial capital–labor ratio is large enough, the marginal productivity of labor is larger than the marginal productivity of capital and the firm holds to employment. If an initial capital–labor ratio is small enough, the opposite is true: the firm holds to investment.

5 Investment and the Growth of a Firm

Suppose that $H_l(R_l) < 0$ and $H_k(R_k) < 0$. The firm alternates its attention. This section derives the growth rate of the capital stock and discusses its property. Note that

$$\begin{aligned}
\frac{I}{K} &= \frac{K'}{(1-\delta_l)L'}\frac{(1-\delta_l)L'}{K} - 1, \\
&= \kappa\left(u - \beta_k \pi_l^*\right)(1-\delta_l)\lambda\left(w - \beta_l \pi_k^*\right) - 1,
\end{aligned}$$

where $\kappa(\cdot)$ and $\lambda(\cdot)$ are the optimal policy functions corresponding to Equations (12) and (13), respectively. Using Hotelling's lemma, the optimal policy functions are related to the profit functions:

$$\begin{aligned}
-\kappa\left(u - \beta_k \pi_l^*\right) &= \Pi_k'\left(u - \beta_k \pi_l^*\right), \\
-\lambda\left(w - \beta_l \pi_k^*\right) &= \Pi_l'\left(w - \beta_l \pi_k^*\right).
\end{aligned}$$

The following theorem summarizes the above arguments.

Theorem 8 *Suppose that $H_l(R_l) < 0$ and $H_k(R_k) < 0$. Then the average investment is a decreasing function of the modified user cost and the modified wage rate:*

$$\frac{I}{K} = (1-\delta_l)\,\Pi_k'\left(u - \beta_k \pi_l^*\right)\Pi_l'\left(w - \beta_l \pi_k^*\right) - 1. \tag{27}$$

The theorem shows that the firm's investment is fully determined by the modified user cost and modified wage rate when $\delta_l = 0$. Recall that these two measures of average rents, π_l^* and π_k^*, have a clear relationship with Tobin's Q:

$$\frac{Q(l)}{p_k} = \frac{\beta}{p_k}\max\left\{\pi_k^* l, \pi_l^*\right\} + 1. \tag{28}$$

Hence, Tobin's Q is correlated with investment. However, the correlation between growth rate and Tobin's Q is indirect. This might explain the weak correlation between investment and Tobin's Q found in the literature. This point is examined quantitatively in the next section.

Similarly to Abel and Eberly (2003), marginal Q is not correlated with investment. Since the marginal cost of investment is p_k in this model, if the solution is interior, the marginal benefit of investment has to be p_k. Hence, the marginal Q does not have any connection to the investment decision. Although the marginal Q is not informative, Tobin's Q is still informative since it contains information about future rent. This point is emphasized by Abel and Eberly (2003).

Note that investment is periodic. When the firm makes its employment decision, there is no investment; when the firm makes its investment decision, the investment is lumpy. Lumpy investment is consistent with evidence in Doms and Dunne (1998). Equation (27) appears when we aggregate them over time.

Since Equation (27) shows gross investment, the net growth rate must take into account the depreciation rate of capital stock. Note that

$$\begin{aligned}
g_k &\equiv \frac{(1-\delta_k)\,K'}{K} - 1 \\
&= (1-\delta_k)(1-\delta_l)\,\kappa\left(u - \beta_k \pi_l^*\right)\lambda\left(u - \beta_k \pi_l^*\right) - 1. \\
&= \frac{(1-\delta_l)\,L'}{L} - 1 \equiv g_l.
\end{aligned}$$

Hence, the following corollary is immediate.

Corollary 9 *Suppose that $H_l(R_l) < 0$ and $H_k(R_k) < 0$. The net growth rate of a firm is*

$$g \equiv g_l = g_k = (1-\delta_k)(1-\delta_l)\,\Pi_k'(u - \beta_k \pi_l^*)\,\Pi_l'(w - \beta_l \pi_k^*) - 1.$$

Corollary 9 shows that the growth rate is independent of firm size. We would like to examine what influences the growth rate. Applying the implicit function theorem to Equations (18), the following propositions are immediate.

Proposition 10 *Suppose that $H_l(R_l) < 0$ and $H_k(R_k) < 0$. Then there exist functions $\pi^l(w,u)$, $\pi^k(w,u)$ and $g(w,u)$ such that*

$$\begin{aligned}
\pi_l^* &= \pi^l(w,u), \pi_1^l(w,u) < 0, \pi_2^l(w,u) < 0, \\
\pi_k^* &= \pi^k(w,u), \pi_1^k(w,u) < 0, \pi_2^k(w,u) < 0, \\
g &= g(w,u), g_1(w,u) \leq 0, g_2(w,u) \leq 0.
\end{aligned}$$

Moreover, assume that $\kappa(u - \beta_k \pi_l^*)$ and $\lambda(w - \beta_l \pi_k^*)$ are interior solutions. Then there exist functions, $\pi^{l\delta}(\delta_l, \delta_k)$, $\pi^{k\delta}(\delta_l, \delta_k)$ and $g^\delta(\delta_l, \delta_k)$ such that

$$\begin{aligned}
\pi_l^* &= \pi^{l\delta}(\delta_l, \delta_k),\ \pi_1^{l\delta}(\delta_l, \delta_k) < 0,\ \pi_2^{l\delta}(\delta_l, \delta_k) < 0, \\
\pi_k^* &= \pi^{k\delta}(\delta_l, \delta_k),\ \pi_1^{k\delta}(\delta_l, \delta_k) < 0,\ \pi_2^{k\delta}(\delta_l, \delta_k) < 0, \\
g &= g^\delta(\delta_l, \delta_k), g_1^\delta(\delta_l, \delta_k) < 0, g_2^\delta(\delta_l, \delta_k) < 0.
\end{aligned}$$

The results in the proposition are intuitive. Since an increase in wage rate, user cost, the depreciation rate of capital stock and labor input all lower the average rents, they lower the growth rate of the firm.

6 Calibration

In this section, we specify the production function and calibrate the model. This exercise is aimed at examining the quantitative relationship between investment, Tobin's Q and cash flow. We assume that $F(K,L) = z\left[\theta K^\rho + (1-\theta) L^\rho\right]^{\frac{1}{\rho}}$, where

$\rho < 1$. Before showing our calibration results, it may be instructive to look at analytical solutions given this specification. For this purpose, we assume for the moment that the solutions, $\kappa(u - \beta_k \pi_l^*)$, $\lambda(w - \beta_l \pi_k^*)$, $\kappa(u)$ and $\lambda(w)$ are interior. It is shown from Equations (14) and (15) that

$$\begin{aligned}
\pi_l^* &= \max\left\{ \frac{\theta}{(1-\theta)} [w - \beta_l \pi_k^*] [\lambda(w - \beta_l \pi_k^*)]^{1-\rho} - u, \frac{\frac{\theta}{(1-\theta)} w [\lambda(w)]^{1-\rho} - u}{1 - \beta_k} \right\} \\
\pi_k^* &= \max\left\{ \frac{1-\theta}{\theta} [u - \beta_k \pi_l^*] [\kappa(u - \beta_k \pi_l^*)]^{1-\rho} - w, \frac{\frac{1-\theta}{\theta} u [\kappa(u)]^{1-\rho} - w}{1 - \beta_l} \right\}
\end{aligned}$$

where$\lambda(p) = \left[\frac{\theta}{\left(\frac{p}{z(1-\theta)}\right)^{\frac{\rho}{1-\rho}} - (1-\theta)} \right]^{\frac{1}{\rho}}$ and $\kappa(p) = \left[\frac{1-\theta}{\left[\frac{p}{z\theta}\right]^{\frac{\rho}{1-\rho}} - \theta} \right]^{\frac{1}{\rho}}$ for any p. The variables π_l^* and π_k^* are the solutions of these two complex equations. If the conditions for adjustment are satisfied, the following investment function is derived:

$$\frac{I}{K} = (1 - \delta_l) \left[\frac{1-\theta}{\left[\frac{u - \beta_k \pi_l}{z\theta}\right]^{\frac{\rho}{1-\rho}} - \theta} \right]^{\frac{1}{\rho}} \left[\frac{\theta}{\left(\frac{w - \beta_l \pi_k}{z(1-\theta)}\right)^{\frac{\rho}{1-\rho}} - (1-\theta)} \right]^{\frac{1}{\rho}} - 1.$$

Note that after controlling $u - \beta_k \pi_l$ and $w - \beta_l \pi_k$, z still has a positive impact on the growth rate. This suggests that cash flow may positively influence investment after controlling Q, which is examined quantitatively later.

Calibration: We assume that a firm makes decisions twice a year. Hence, the derived investment is considered to be an annual value.

We assume that the price of investment goods, p_k, is equal to 1 as a benchmark, the depreciation rate of capital, δ_k, is 0.05 and the interest rate, i, is 0.08. This means that user cost, $u = 0.13$ and $\beta = 0.93$. The interest rate, $i = 0.08$, implies that the annual interest rate is equal to 0.1664, which is higher than the riskless rate of return. This number corresponds roughly to an annual interest rate of 0.16, which was assumed in Abel and Eberly (2002). They argued that the firm uses risk-adjusted hurdle rates of return, which correspond to their number. We follow their argument. Since we define L as the efficiency unit of labor, it is not clear what

Table 1: Benchmark Parameters

w	p_k	i	δ_k	θ	δ_l	α
1	1	0.08	0.05	0.3	0.1	1.07

might be a reasonable w. As a benchmark, we assume that $w = 1$. We also assume that $\theta = 0.3$, which means the capital share of 0.3 when the production function is approximated by Cobb–Douglas ($\rho = 0$). For the depreciation of human capital, δ_l, is assumed to be 0.1, which corresponds to an annual value of 19 percent[9]. We assume that $\alpha = 1.07$, which means $\alpha = \frac{1}{\beta} - 0.01$. Table 1 summarizes our benchmark parameters.

We have generated the investment–capital ratio, Tobin's Q and cash flows for several values of z and ρ, which largely influence investment behavior. For this purpose, we first discuss what is the reasonable range of z and ρ. Since w is assumed to be 1, we can estimate z from $z = \left[\theta\left(\frac{K}{Y}\right)^{\rho} + (1-\theta)\left(\frac{wL}{Y}\right)^{\rho}\right]^{-\frac{1}{\rho}}$. Hence, we need to know the reasonable range for ρ to find a reasonable range for z.

Berndt (1991, p455) reviews the literature and claims that empirical findings from two-digit cross-sectional studies support a Cobb–Douglas production function ($\rho = 0$), though time series estimates of the elasticity show a range of 0.3 to 0.5, which roughly corresponds to the range of $\rho = -1$ to $\rho = -2.3$. However, these estimates are not based on the efficiency unit. Krusell, Ohanian Ríos-Rull and Violante (2000) estimate the elasticity of substitution between equipment and skilled labor as 0.67, which corresponds to $\rho = -0.5$. From this evidence, we expect that a plausible range of ρ is roughly 0 to -2.5.

[9] It is not clear what might be a reasonable number. One possible source of information about the depreciation rate can be found from the separation rate between a firm and a worker. Yashiv (2000) assumes that the separation rate is 1.7 percent a month, which corresponds to an annual value of 19 percent. Christensen, Lentz, Mortensen, Neumann and Werwatz (2005) report that the separation rate of managers in the largest firms is 22 percent, where managers are the group who accumulate the most firm-specific human capital. These numbers provide a justification for our assumption. In any case, our results do not change much by using other numbers.

Using data from the National Income and Product Accounts, the capital–output ratio in the U.S., K/Y, is roughly 2.11 in 2004. Assuming that the labor share, wL/Y, is 0.7, the estimated value of z is 1.03 when $\rho = 0$ and 1.25 when $\rho = -2.5$. Hence, we consider that a reasonable range of z is between 1 and 1.25.

For given ρ, we have generated 100 observations that correspond to different z that increase by 0.0025 starting from 1. Hence, the maximum value of z is 1.2475. The parameter ρ ranges from -7 to 0 in 0.5 intervals. Hence, we have 15 different values of ρ. We have 1500 observations[10]. As we have argued, the plausible value of ρ is expected to be greater than -2.5. However, we find that it is instructive to report the results when ρ is less than -2.5.

Figures 9, 10 and 11 show our calibration results. Figure 9 shows the relationship between the investment–capital ratio (I/K) and the productivity parameter (z) for the production functions with different elasticity of substitution between capital and labor, ($\frac{1}{1-\rho}$). When the production function is Cobb–Douglas ($\rho = 0$), I/K is constant at 0 or 0.205. The zero I/K means that the firm holds to an employment decision in that range of z, while a positive constant value of I/K implies that the firm grows and both investment decisions and employment decisions are constrained by their upper bounds. In other words, when the production function is Cobb–Douglas and the firm grows, there is no interior solution.

When $\rho = -2$, negative investment appears. The lower bound of I/K is -1, which means that either the investment or employment decision is constrained by the lower bound of 0[11]. In this case, the firm immediately disappears. There is a range of z that supports interior solutions with negative investment. However, interior solutions with positive investment still do not appear. When $\rho = -4$, there is a

[10] Of course, strictly speaking, ρ and z must satisfy a certain relationship. For the purpose of this exercise, the strict regulation is not particularly important. We allow the variation of ρ and z within a certain range, which results in the heterogeneity of firms.

[11] Technically, we cannot allow 0 inputs. Hence, the lower bound is assumed to be 1×10^{-17}, which is approximately 0.

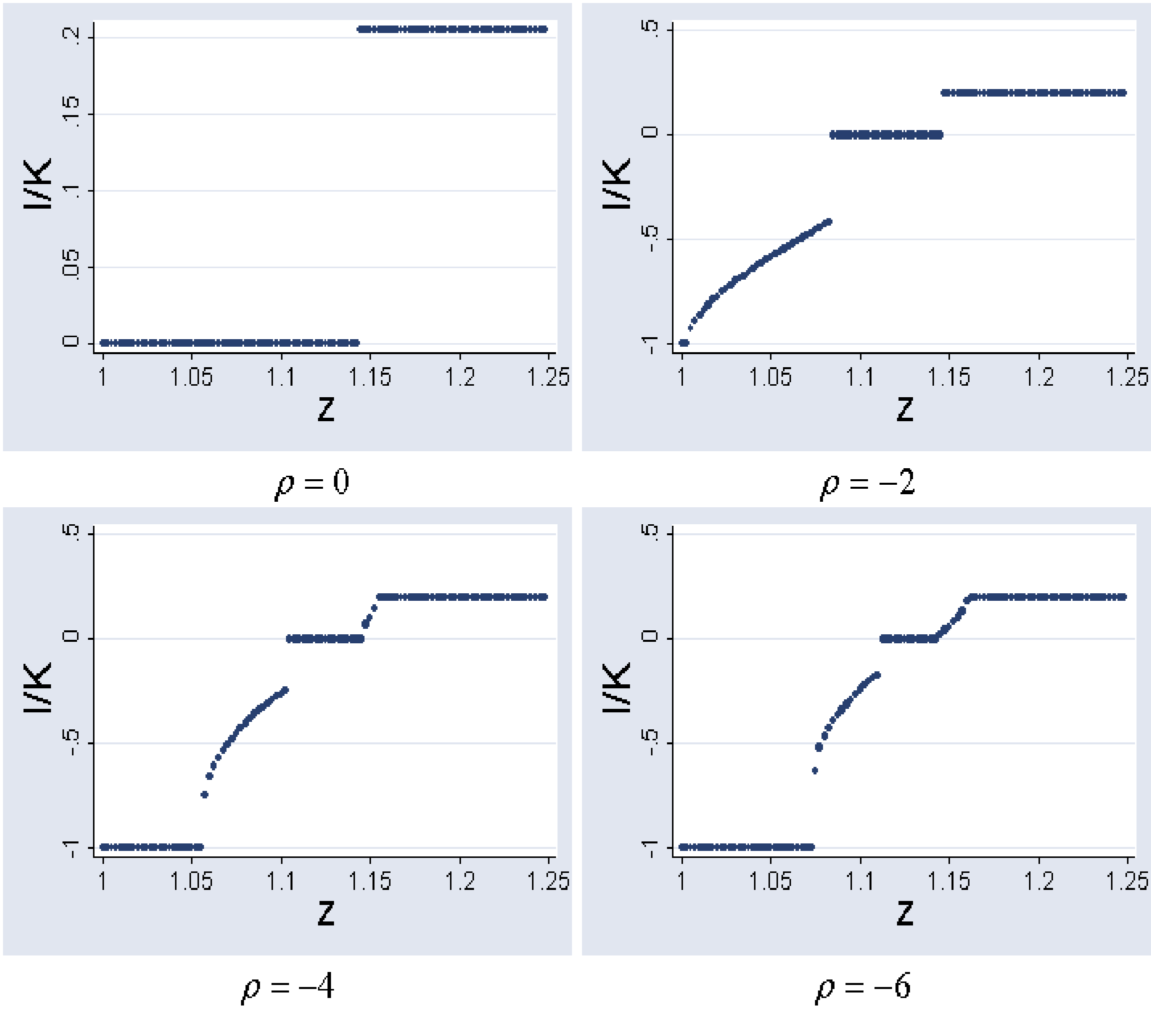

Figure 9: **The relationship between investment–capital ratio (I/K) and productivity (z) for production functions with different elasticities of substitutions between capital and labor ($\frac{1}{1-\rho}$).**

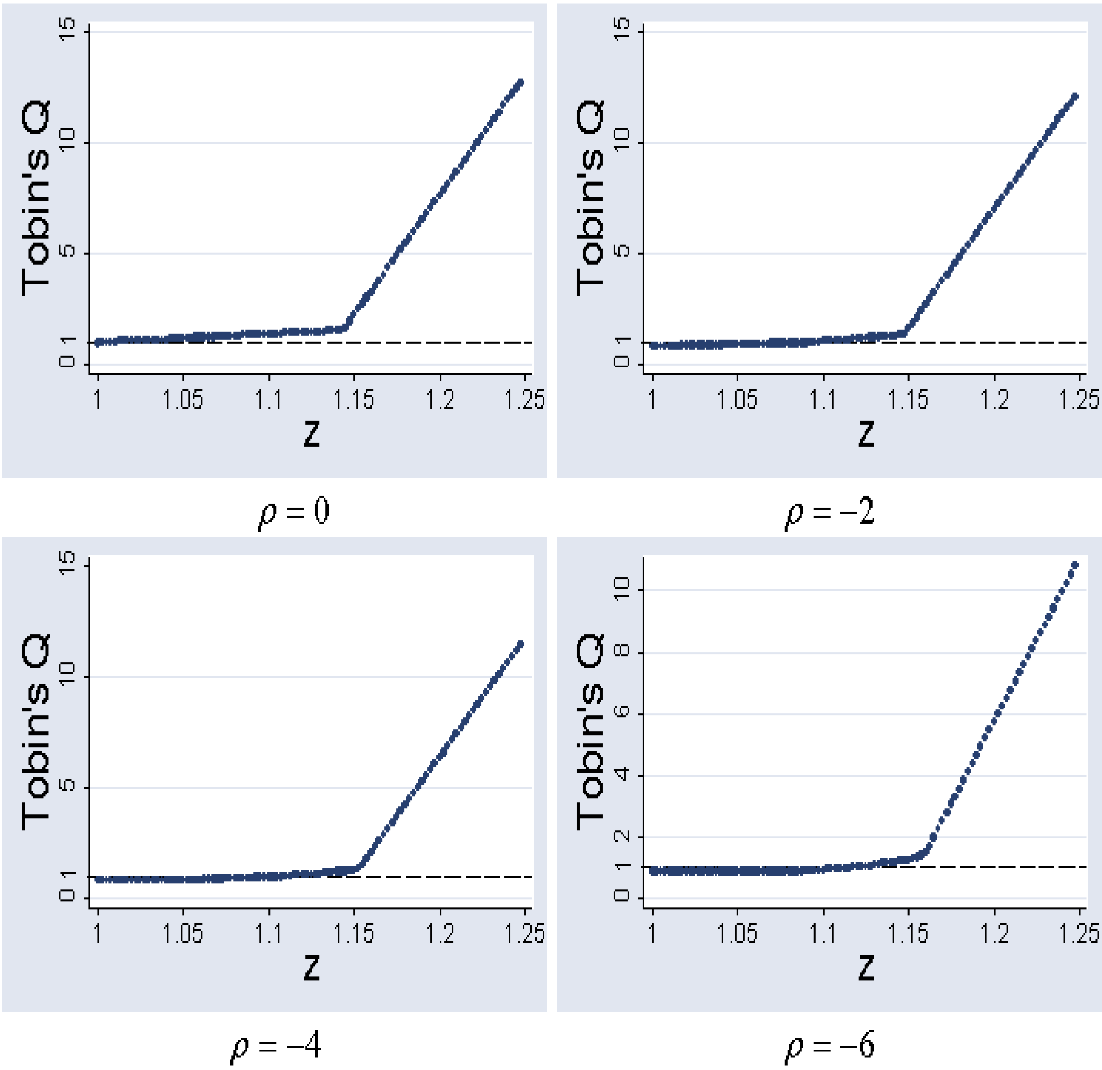

Figure 10: **The relationship between Tobin's Q and productivity (z) for production functions with different elasticities of substitutions between capital and labor ($\frac{1}{1-\rho}$):** Tobin's Q is calibrated by $\frac{\pi_i^*}{1+i} + 1$.

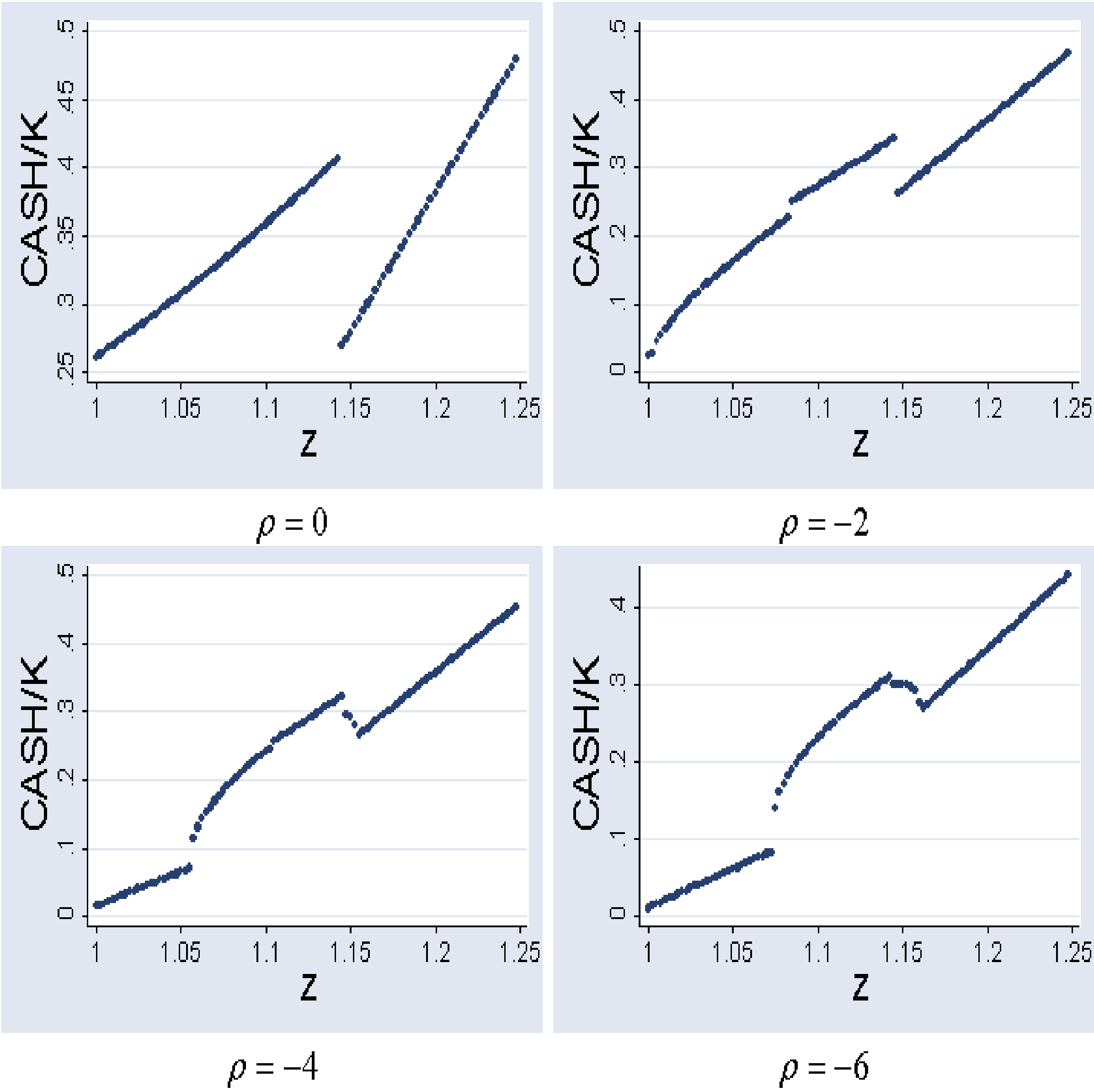

Figure 11: **The relationship between cash flow–capital ratio and productivity (z) for production functions with different elasticities of substitutions between capital and labor ($\frac{1}{1-\rho}$):** cash flow–capital ratio is calibrated by the summation of $\frac{z[\theta K^{\rho}+(1-\theta)(L)^{\rho}]^{\frac{1}{\rho}}-wL}{K}$ for two consecutive periods.

range of z that does support interior solutions that bring positive investment. When $\rho = -6$, the range of z that supports interior solutions with positive investment becomes larger. To summarize, we can generate interior solutions with positive investment, but capital and labor must be more complementary than the estimates within our plausible range. Possible extensions to mitigate this problem are discussed in our conclusion.

Figure 10 demonstrates the relationship between Tobin's Q and z. We report Tobin's Q when a firm makes an employment decision, which is denoted by $\frac{Q^l(l)}{p_k}$ and calibrated from $\frac{\pi_l^*}{1+i}+1$. For our range of parameters, there is no case for which the firm will hold to an investment decision. Hence, every observation generates $\frac{Q^l(l)}{p_k}$. For all ρ, Tobin's Q is close to 1 until z reaches a threshold. Above the threshold level of z, Tobin's Q expands greatly for a slight increase in z and deviates far from 1. Note that the threshold level of z coincides with that of z above which I/K hits its upper bound in Figure 9. Once I/K hits its upper bound, an increase in z cannot increase capital stock further, although it increases market value. That is why Tobin's Q becomes sensitive after I/K hits its upper bound.

Figure 11 shows the relationship between the cash flow–capital ratio and z for the production functions with different ρ. The cash flow–capital ratio is calibrated by the summation of $\frac{z[\theta K^\rho+(1-\theta)(L)^\rho]^{\frac{1}{\rho}}-wL}{K}$ for two consecutive periods. For all ρ, the cash flow–capital ratio increases as z increases except for the region in which either an employment decision or an investment decision has interior solutions that bring positive investment. When the firm expands and has an interior solution, adjustment cost is generated by static losses as explained before. These static losses make the cash flow–capital ratio smaller. Once both decisions hit upper bounds, the firm cannot increase investment any further. Hence an increase in z simply increases output and therefore increases the cash flow–capital ratio.

In order to examine the effect of Tobin's Q and cash flow on investment, we conduct a simple regression using data generated from our calibration. Table 2

Table 2: Regression on Investment–Capital Ratio:

	$\rho \in [-7, 0]$	$\rho \in [-7, 0]$	$\rho \in [-2.5, 0]$	$\rho \in [-2.5, 0]$
Q	0.079	-0.031	0.056	-0.020
	(0.003)	(0.001)	(0.003)	(0.003)
$CASH/K$		4.041		3.579
		(0.040)		(0.088)
$Constant$	-0.429	-1.126	-0.270	-1.059
	(0.013)	(0.008)	(0.017)	(0.021)
$Adjusted - R^2$	0.333	0.915	0.316	0.820
$\#\ of\ observation$	1500	1500	600	600

Every coefficient is significant at a 0.5 % level.

reports our results. The first column shows that the investment–capital ratio is positively correlated with Tobin's Q. However, the coefficient and R^2 is small. This is consistent with evidence [e.g., Chirinko (1993)]. Moreover, once we include cash flow, the coefficient on Tobin's Q is negative and cash flow has a strong positive effect on investment[the second column]. The robust impacts of cash flow, after controlling for Tobin's Q, is found in the investment literature [e.g., Fazzari, Hubbard and Petersen (1988)]. The results do not change even when we restrict our attention to the observations with $\rho \geq -2.5$. The calibration results show that our model can generate stylized facts found in the literature.

7 Conclusion and Extensions

This paper applies a simple idea to investment theory: a person cannot make many decisions at a time, but an organization needs millions of interrelated decisions. The growth rate of the firm in our model is derived when the production function displays constant returns to scale, there is no adjustment cost function and markets are competitive. We show that each investment is lumpy, but adjustment is not immediate.

Furthermore, the growth rate of a firm is independent of firm size and imperfectly correlated with Tobin's Q.

One drawback found in our calibration exercises is that in order to support an interior solution that has positive investment, the model demands more complementarity than the estimates, which are expected to be plausible. This indicates that we need some modifications for the purpose of empirical research. The current model is designed to distinguish a novel mechanism for the growth of a firm from other models in the literature. For this purpose, we make two extreme assumptions. First, we completely dismiss any conventional adjustment costs. However, it is natural to think that not only physical capital accumulation but also human capital accumulation is likely to involve a certain adjustment cost. Incorporating these additional adjustment costs makes it easier to support interior solutions. Second, we assume that there is no price adjustment and that the production function has constant returns to scale. When the production function displays constant returns to scale and there is no price adjustment, an optimal decision should be on the boundary in a standard static optimization. Hence, it is understandable to find many boundary solutions in our model. This reasoning suggests that extension to a general equilibrium model will make the model support interior solutions with more reasonable parameters. Alternatively, the assumption of a decreasing-returns-to-scale production function will also mitigate problems. These are interesting future extensions.

One may also object to our assumption on routine operation. We assume that a firm maintains its previous level of input when it does not give any attention to optimizing the input. However, it is clear that an alternative decision rule might be possible. We also assume that a firm can costlessly change its routine if it wishes to do so. Common observation tells us that this is not the case. Our assumptions should be considered as simplifications to clarify the messages of our paper. However, we believe that we point out important aspects of a routine: limited attention demands routine operation, and routine operation causes rent, which can be both the source of growth and a barrier to change.

Developing more reasonable models of a routine is beyond the scope of this paper, but it is also an interesting research agenda. We hope that our model can be extended to incorporate more realistic features of a routine.

8 Appendix

The Proof of Proposition 1: We derive only Equation (10). Since the derivation of Equation (11) is similar, we omit the proof. Substitute $D(l) = \beta \max\{\pi_k l,\ \pi_l\}$ into Equation (8),

$$\pi_k = \max_{k' \in [0, \alpha_k]} \left\{ f(k') - uk' - w + \max\{\beta_l \pi_k,\ \beta_k \pi_l k'\} \right\}. \tag{29}$$

Define π_k^1 and π_k^2,

$$\begin{aligned} \pi_k^1 &= \max_{k' \in [0, \alpha_k]} \left[f(k') - (u - \beta_k \pi_l) k' - w \right], \\ \pi_k^2 &= \max_{k' \in [0, \alpha_k],} \left[f(k') - uk' - w \right] + \beta_l \pi_k. \end{aligned}$$

Note that if $\beta_l \pi_k \leq \beta_k \pi_l k$, $\pi_k = \pi_k^1$ and if $\beta_l \pi_k \geq \beta_k \pi_l k$, $\pi_k = \pi_k^2$, where k is a solution of Equation (29).

Suppose that $\arg\max_{k' \in [0, \alpha_k]} \left[f(k') - [u - \beta_k \pi_l] k' - w \right] = k^*$ and that $\beta_l \pi_k \geq \beta_k \pi_l k^*$. Then

$$\pi_k^1 \leq (k^*) - [u - \beta_k \pi_l] k^* - w \leq f(k^*) - uk^* - w + \beta_l \pi_k \leq \pi_k^2.$$

Similarly, suppose that $\arg\max_{k' \in [0, \alpha_k]} \left[f(k') - uk' - w \right] + \beta_l \pi_k = k^*$ and that $\beta_l \pi_k \leq \beta_k \pi_l k^*$. Then

$$\pi_k^2 \leq f(k^*) - uk^* - w + \beta_l \pi_k \leq f(k^*) - [u - \beta_k \pi_l] k^* - w \leq \pi_k^1.$$

Hence,

$$\pi_k = \max\left\{ \pi_k^1, \pi_k^2 \right\}.$$

Q.E.D.

The Proof of Lemma 2: We only prove $\pi_l^* = \pi_l^{\#}$. Since $\pi_k^* = \pi_k^{\#}$ can be proved by the same method, we omit its proof. Define $R_l^* (\pi_k)$ such that

$$G_l (\pi_k) = \Pi_l (w) + \beta_k R_l^* (\pi_k) . \tag{30}$$

Note that $\pi_l \geq R_l^* (\pi_k)$, if and only if $\Pi_l (w) + \beta_k \pi_l \geq G_l (\pi_k)$. Rearranging Equation (30),

$$R_l^* (\pi_k) = \frac{G_l (\pi_k) - (1 - \beta_k) R_l}{\beta_k}.$$

Necessity: Suppose that $\pi_l = \max \{G_l (\pi_k), \Pi_l (w) + \beta_k \pi_l\}$. Suppose that $\pi_l \geq R_l^* (\pi_k)$. Then $\pi_l = R_l$. Hence, it is shown that

$$\pi_l - R_l^* (\pi_k) = \frac{R_l - G_l (\pi_k)}{\beta_k}.$$

This means that $\pi_l \geq R_l^* (\pi_k)$ and $\pi_l = R_l$ imply $R_l \geq G_l (\pi_k)$ and $\pi_l = R_l$. Similarly, suppose that $\pi_l < R_l^* (\pi_k)$. Then $\pi_l = G_l (\pi_k)$. Hence, it is shown that

$$\pi_l - R_l^* (\pi_k) = \frac{(1 - \beta_k) \{R_l - G_l (\pi_k)\}}{\beta_k}.$$

This means that $\pi_l < R_l^* (\pi_k)$ and $\pi_l = G_l (\pi_k)$ imply $R_l < G_l (\pi_k)$ and $\pi_l = G_l (\pi_k)$. Therefore, $\pi_l = \max \{G_l (\pi_k), R_l\}$.

Sufficiency: Suppose that $\pi_l = \max \{G_l (\pi_k), R_l\}$. Suppose that $G_l (\pi_k) \leq R_l$. Then $\pi_l = R_l$. Hence, it is shown that

$$R_l - G_l (\pi_k) = \beta_k [\pi_l - R_l^* (\pi_k)] .$$

This means that $G_l (\pi_k) \leq R_l$ and $\pi_l = R_l$ imply $\pi_l \geq R_l^* (\pi_k)$ and $\pi_l = R_l$. Similarly, suppose that $G_l (\pi_k) > R_l$. Then $\pi_l = G_l (\pi_k)$. Hence, it is shown that

$$R_l - G_l (\pi_k) = \frac{\beta_k}{1 - \beta_k} [\pi_l - R_l^* (\pi_k)] .$$

This means that $G_l (\pi_k) > R_l$ and $\pi_l = G_l (\pi_k)$ imply $\pi_l \geq R_l^* (\pi_k)$ and $\pi_l = G_l (\pi_k)$. Therefore, $\pi_l = \max \{G_l (\pi_k), \Pi_l (w) + \beta_k \pi_l\}$. **Q.E.D.**

The Proof of Lemma 3: We prove $H_l'(\pi_l) > 0$ and $H_l''(\pi_l) \leq 0$. Since $H_k'(\pi_k) > 0$ and $H_k''(\pi_k) \leq 0$ is proved by the same method, we omit it.

$$H_l'(\pi_l) = 1 - \beta_k \beta_l \lambda [w - \beta_l G_k(\pi_l)] \kappa [r - \beta_k \pi_l] \geq 1 - \beta^2 \alpha^2 > 0,$$

where $\lambda[\cdot]$ is an optimal policy function of Equation (13) and $\kappa[\cdot]$ is an optimal policy function of Equation (12).

$$H_l''(\pi_l) = -\left[G_l''(G_k(\pi_l)) [G_l'(G_k(\pi_l))]^2 + G_l'(G_K(\pi_l)) G_k''(\pi_l) \right] \leq 0.$$

The second inequality comes from the fact that $G_l(\cdot)$ and $G_k(\cdot)$ are strictly increasing and convex functions.

The Proof of Theorem 4: We prove the existence and uniqueness of π_l^{**}. Since the proof of the existence and uniqueness of π_k^{**} is the same as that of π_l^{**}, we omit it. Note that $H_l(\pi_l)$ can be written as follows:

$$\begin{aligned} H_l(\pi_l) &= (1 - \beta_l \beta_k \kappa(u - \beta_k \pi_l) \lambda(w - \beta_l G_K(\pi_l))) \pi_l + D(\pi_l) \\ D(\pi_l) &\equiv u - \left\{ \begin{array}{c} pf\left(\frac{1}{\lambda(w - \beta_l G_K(\pi_l))}\right) \\ -[(1+\beta_l) w - \beta_l (pf(\kappa(u - \beta_k \pi_l)) - u\kappa(u - \beta_k \pi_l))] \end{array} \right\} \lambda(w - \beta_l G_K(\pi_l)). \end{aligned}$$

Note that since $\lambda(w - \beta_l G_K(\pi_l))$ and $\kappa(u - \beta_k \pi_l)$ are bounded, $D(\pi_l)$ is also bounded for any π_l. Note also that $1 > \beta_l \beta_k \kappa(u - \beta_k \pi_l) \lambda(w - \beta_l G_K(\pi_l))$ for any π_l because of the constraints on choice variables. Hence, there exists large $\pi_l \in R$ such that $H_l(\pi_l) > 0$, and there also exists small $\pi_l \in R$ such that $H_l(\pi_l) < 0$. Since $H_l(\pi_l)$ is continuous and strictly increasing in π_l, there exists an unique π_l^{**}. **Q.E.D.**

The Proof of Theorem 5:

Necessity: Suppose that $H_l(R_l) < 0$ and $H_k(R_k) < 0$. Suppose that $G_l(\pi_k^*) \leq R_l$. Then $\pi_l^* = R_l$. Hence, $\pi_k^* = \max\{G_k(R_l), R_k\}$ and $G_l(\max\{G_k(R_l), R_k\}) \leq R_l$. $0 > R_l - G_l(G_k(R_l)) \geq G_l(\max\{G_k(R_l), R_k\}) - G_l(G_k(R_l))$. Hence, $G_k(R_l) >$

$\max\{G_k(R_l), R_k\}$. Contradiction. Similarly, suppose that $G_k(\pi_l^*) \leq R_k$. Then $\pi_k^* = R_k$. Hence, $\pi_l^* = \max\{G_l(R_k), R_l\}$ and $G_k(\max\{G_l(R_k), R_l\}) \leq R_k$. $0 > R_k - G_k(G_l(R_k)) \geq G_k(\max\{G_l(R_k), R_l\}) - G_k(G_l(R_k))$. Hence, $G_l(R_k) > \max\{G_l(R_k), R_l\}$. Contradiction.

Sufficiency: Suppose that $\pi_l^* = \pi_l^{**} > R_l, \pi_k^* = \pi_k^{**} > R_k$. The result is immediate from the following lemma.

Lemma 11

$$\begin{aligned} H_l(R_l) &\geq 0\, iff\ \pi_l^{**} \leq R_l, \\ H_k(R_k) &\geq 0\, iff\ \pi_k^{**} \leq R_k. \end{aligned}$$

Proof. *Note that since $H_l(\cdot)$ and $H_k(\cdot)$ are strictly increasing functions, the result is obvious.* ■

Q.E.D.

The Proof of Theorem 7:

Necessity: Note that from theorem 5, either $\pi_k^* = R_k$ or $\pi_l^* = R_l$.

Suppose $G_l(R_k) > R_l$ and $G_k(R_l) \leq R_k$. Suppose that $\pi_k^* > R_k$. Then $\pi_l^* = G_l(\pi_k^*) > G_l(R_k) > R_l$. Contradiction. Hence, $\pi_k^* = R_k$. Then $\pi_l^* = G_l(R_k) > R_l$.

Suppose that $G_l(R_k) \leq R_l$ and $G_k(R_l) > R_k$. Suppose that $\pi_l^* > R_l$. Then $\pi_k^* = G_k(\pi_l^*) > G_k(R_l) > R_k$. Contradiction. Hence, $\pi_l^* = R_l$. Then $\pi_k^* = G_k(R_l) > R_k$.

Suppose that $G_l(R_k) \leq R_l$ and $G_k(R_l) \leq R_k$. Suppose that $\pi_l^* = R_l$. Then $\pi_k^* = \max\{G_k(R_l), R_k\} = R_k$. Suppose $\pi_k^* = R_k$. $\pi_l^* = \max\{G_l(R_k), R_l\} = R_l$.

Sufficiency:

Suppose $\pi_l^* = G_l(R_k) > R_l, \pi_k^* = R_k$. Then $G_k(R_l) < G_k(\pi_l^*) \leq R_k$.

Suppose $\pi_l^* = R_l, \pi_k^* = G_k(R_l) > R_k$. Then $G_l(R_k) < G_l(\pi_k^*) \leq R_l$.

Suppose $\pi_l^* = R_l, \pi_k^* = R_k$. Then the result is obvious. **Q.E.D.**

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