OSIPP Discussion Paper : DP-2003-E-010

Entrepreneurial Efficiency

October 1, 2003

Katsuya Takii

Associate Professor, Osaka School of International Public Policy (OSIPP)

【キーワード】E13,O40.

【要約】This paper examines a particular aspect of entrepreneurship, namely firms' ability to respond appropriately to unexpected changes in the environment (i.e., their adaptability). An increase in firms' adaptability improves allocative efficiency in a competitive economy, but can reduce it when opportunities are distorted. It is shown that adaptability can aggravate distortions in the presence of political risk. Because efficiency affects the total factor productivity (TFP) of an economy, the model can explain how entrepreneurship influences TFP. The quantitative effect of firms' adaptability on TFP is investigated using the Census of Manufacturing in Japan.

I would like to thank Carmen Li, Boyan Jovanovic and seminar participants at the University of Essex, the Kansai Macro Workshop, Osaka Prefecture University and OSIPP for helpful comments. I also thank Kenjiro Hirata for his excellent assistance. *Osaka School of International Public Policy, Osaka University, 1-31, Machikaneyama, Toyonaka, Osaka, 560-0043, Japan. Tel: 81-6-685-5631, E-mail: takii@osipp.osaka-u.ac.jp*

1 Introduction

It is widely known that entrepreneurship plays an essential role in several economic activities. When new technology is discovered, demographic change occurs or a government changes its policy, it is entrepreneurs who react to the changes and reallocate resources to seek opportunities. However, there have been only a few attempts to develop a formal model of entrepreneurship in macroeconomics. The absence of entrepreneurship from macroeconomics can be explained in part by the extensive use of an exogenously given aggregate production function. As the relation between output and inputs is given by the aggregate production function, there is no need for an economic agent who finds a productive use for inputs.

This paper develops a tractable macroeconomic model that enables a theoretical and empirical examination of a particular aspect of entrepreneurship, namely the ability to react appropriately to unexpected changes in the environment (i.e., adaptability). In this model, the aggregate production function is derived from firms' reactions to these changes. Therefore, we can analyze how firms' adaptability affects the macroeconomy, while retaining the advantages of the aggregate production function.

We assume that firms' adaptability is determined by entrepreneurs' ability to recognize changes in the environment. When an unexpected change in productivity occurs, the marginal products of inputs deviate from input prices, and this generates opportunities for entrepreneurs to exploit. If entrepreneurs recognize the change and react to it, the deviation of the marginal products of the inputs from input prices would be small. Hence, the improvement in firms' adaptability raises allocative efficiency, and therefore increases productivity in the economy. It is shown that the increased adaptability of firms raises the total factor productivity (TFP) of the aggregate production function in a competitive economy.

This result does not apply if opportunities are distorted. As the social marginal products of resources differ from the private marginal products, even if a talented entrepreneur can equate private marginal products to input prices, this may not improve allocative efficiency. This intuition is analyzed formally in the context of entrepreneurs seeking rent. Because the results of political negotiations are uncertain, entrepreneurs must respond to changes in the political environment. However, because rent-seeking activities simply redistribute incomes, adapting to political changes generates negative externalities: when an entrepreneur is good at taking opportunities, there are fewer opportunities for others. It is shown that increased firms' adaptability can lower TFP.

The effect of firms' adaptability on TFP is investigated empirically. This paper shows that the adaptability of firms can be estimated by the squared correlation between an unexpected shock and firms' reactions to the shock. Using data from the Census of Manufacturing in Japan, 1985–1999, we estimate firms' adaptability and examine the effect of adaptability on TFP by prefecture and industry. Our model predicts that, if political shocks are negligible, full recognition of previously unpredicted changes increases TFP by at least 14% in Japan. However, such recognition lowers TFP by the same amount if all changes are due to political shocks.

A similar view of entrepreneurs is emphasized by Kirzner (1973) and Schultz (1975). Kirzner (1973) emphasizes the essential role of entrepreneurial discovery in market processes. Schultz (1975) defines entrepreneurial ability as the ability to interpret new information and allocate resources to profitable opportunities. Although both researchers insist that equilibrium analysis is not suitable for understanding the importance of entrepreneurship, we suggest that there are benefits of using an equilibrium model. Hence, we represent similar aspects of entrepreneurship in a general equilibrium model, and it is hoped that this approach complements theirs.

This strategy has been adopted by Holmes and Schmitz (1990). Holmes and Schmitz (1990) incorporate a Schultzian entrepreneur into an equilibrium analysis, and emphasize the importance of the division of labor between entrepreneurs and managers.[1] This paper differs from theirs in three respects: (1) it assigns entrepreneurship a different social role, namely that of influencing allocative efficiency; (2) it incorporates this role into the neoclassical growth model; and (3) it examines the effect of entrepreneurship on the macroeconomy quantitatively.

This paper also contributes to a controversy in the literature on pressure groups. Becker (1983) shows that competition between pressure groups can lead to efficient allocation. On the other hand, Tullock (1967) argues that rent-seeking activities waste resources. The detrimental effect of entrepreneurs' rent-seeking activities has recently been re-emphasized in the context of economic growth [e.g., by Baumol (1990), Murphy et al. (1991) and Holmes and Schmitz (2001)]. In contrast to the previous literature, this paper shows that, even if rent-seeking activities are costless, competition between pressure groups may not lead to efficient allocation in the presence of political risk. Hence, the social welfare loss due to rent-seeking activities may have been underestimated by the existing literature.

Evidence obtained in this paper is related to that obtained from plant-level data. Much evidence from plant-level data suggests that the reallocation of resources towards more productive uses is an important component of productivity growth [see reviews by, e.g., Davis and Haltiwanger (1999) and Bartelsman and Doms (2000)]. This paper argues that the reallocation of resources might be affected by firms' adaptability.

This paper is organized as follows. The next section describes the model. Sec-

[1] The different roles of entrepreneurs are incorporated into equilibrium analysis by Kihlstrom and Laffont (1979) with respect to risk bearing, by Schmitz (1989) with respect to imitation, and by Aghion and Howitt (1992) with respect to innovation.

tion 3 defines the measure of adaptability and analyzes the macroeconomic effect of firms' adaptability. Section 4 extends the model to analyze the effect of political uncertainty. Section 5 shows how the theory is implemented empirically. Section 6 reports the empirical results. Section 7 concludes by summarizing the main results and discussing possible extensions.

2 The Model

We present a simple general equilibrium model, which is based on that of Lucas (1978). It lays the foundations for analyzing the macroeconomic effect of entrepreneurship in the next section.

An agent can be an entrepreneur or a worker. Each firm needs one entrepreneur to organize the firm. In this model, for simplicity, there is no distinction between entrepreneurs, managers and firms. This simplification is made to develop a tractable model that focuses on firms' adaptability.

Firms are continuously distributed on $[0, mN]$, where $m \in (0,1)$ is the proportion of entrepreneurs in the total population, N. Although the variables m and N can change over time, time subscripts are omitted throughout the paper unless they are necessary for clarity. The representative entrepreneur's problem is described first, and then the market equilibrium is defined.

The entrepreneur's problem: An entrepreneur establishes a firm, employs capital stock and workers, and produces output. The entrepreneur faces the following production function:

$$Y_i = z_i A \left[F\left(K_i, TL_i\right)\right]^{\alpha}, \ 0 < \alpha < 1$$

where z_i is a firm-specific productivity shock for the ith firm, and Y_i, K_i and L_i

are the amounts of the ith firm's output, capital stock and labor input, respectively. T measures the effectiveness of labor, which is increased by general factors such as the level of educational attainment and economy-wide technological progress. It is assumed that F exhibits constant returns to scale in K and L. By defining $f(k) = F(k, 1)$, where $k = \frac{K}{TL}$, we can express $F(K, TL)$ as a function of capital per unit of effective labor in production: $F(K, TL) = f(k) TL$. We assume that $f'(.) > 0$, $f''(.) < 0$, $\lim_{k \to 0} f(k) = 0$, $\lim_{k \to 0} f'(k) = \infty$ and $\lim_{k \to \infty} f'(k) = 0$.

The parameters α and A measure the span of control and the productivity of management, respectively. Because $\alpha \in (0, 1)$, the production function has decreasing returns to scale in K_i and L_i.

It is assumed that the productivity of management is a function of the effectiveness of the entrepreneur. Because agents are homogeneous, the effectiveness of the entrepreneur is the same as that of workers, which is given by T. Assuming that $A = T^{1-\alpha}$, the production function can be written as

$$Y_i = z_i \left[f(k_i) L_i \right]^{\alpha} T.$$

There are three advantages to assuming that $A = T^{1-\alpha}$. First, this assumption implies that the production function has constant returns to scale in capital stock, labor and managerial input. Hence, it can be shown that the firm's profit is equivalent to the returns to managerial input in a competitive environment [Mas-Colell et al. (1995), p.135]. Secondly, this production function, which has constant returns to scale, results from maximizing total output in a hierarchical organization, as Rosen (1982) has shown. Given Rosen's model, managerial input, T, is required to supervise different plants. Thirdly, when T grows at a constant rate, this assumption guarantees the existence of a balanced growth path, which is shown subsequently.

An entrepreneur has an important task other than the supervisory one. Because the movement of z_i is unpredictable *ex ante*, when z_i changes the entrepreneur must recognize the direction and magnitude of this change to respond appropriately. When the entrepreneur makes production decisions, she does not observe z_i, but does observe a noisy signal, s_i, from which the realization of z_i can be inferred. It is assumed that the entrepreneur's inference is based on a conditional distribution function, $Q^h(z|s)$, where h measures the entrepreneur's ability to recognize changes in z. The conditional distribution function is the same for all entrepreneurs. It implies that all entrepreneurs share the same knowledge about the relation between the productivity shock and the observable signal. A more detailed information structure is specified subsequently.

Note that z is assumed to be an idiosyncratic shock. Hence, the information required to infer z must be local information. That is, entrepreneurs must process their local information. However, as both z and s are idiosyncratic, prices in this model do not depend on them. Hence, prices are predictable without knowing what others observe. That is, entrepreneurs do not need to know all the local information in an economy because the price system summarizes the information they need. This is essentially the view of Hayek (1945). In this sense, this paper incorporates Hayek's (1945) arguments into the neoclassical growth model, and examines the social effect of local information.

It is assumed that the financial market is complete. Therefore, entrepreneurs can hedge against any idiosyncratic risk. Entrepreneurs maximize their firm's expected profits:

$$\pi(s_i) = \max_{k,L} \left\{ \int z_i \left[f(k_i) L_i\right]^{\alpha} T dQ^h(z_i|s_i) - wTL_i - rk_i TL_i \right\},$$

where w is the wage rate for effective labor and r is the rental price of capital. The first-order conditions are:

$$w = \alpha \int z_i dQ^h(z_i|s_i) \left[f(k(s_i)) L(s_i)\right]^{\alpha-1} f(k(s_i)) - rk(s_i), \tag{1}$$

$$r = \alpha \int z_i dQ^h (z_i|s_i) \left[f(k(s_i)) L(s_i)\right]^{\alpha-1} f'(k(s_i)), \tag{2}$$

for any s, where $k(s)$ and $L(s)$ are the optimal levels of k and L. Because the production function is strictly concave in k and L and satisfies the Inada conditions, there exists a unique interior solution, and the first-order conditions are necessary and sufficient for the maximization problem.

Note that the two first-order conditions imply that entrepreneurs equate the wage rate (rental price) to the expected marginal product of labor (capital), not to the actual marginal product of labor (capital). Unexpected idiosyncratic shocks cause marginal products to deviate from marginal costs, and these deviations provide opportunities for entrepreneurs to exploit. If entrepreneurs recognize the changes clearly, they can take the opportunities. This is the aspect of entrepreneurship that we emphasize in this model, and is also stressed by Kirzner (1973) and Schultz (1975).

The following is derived from the two first-order conditions:

$$\frac{w}{r} = \frac{f(k(s_i)) - f'(k(s_i))\, k(s_i)}{f'(k(s_i))} \tag{3}$$

This equation implies that the capital stock per unit of effective labor in production $k(s_i)$ does not depend on the realization of the signal s_i. Hence, we denote this by k. As the right-hand side of equation (3) is strictly increasing in k, k is uniquely determined by $\frac{w}{r}$.

Expected profits are derived by substituting the two first-order conditions into $\pi(s_i)$:

$$\int \pi(s)\, dQ_s^h(s) = \frac{(1-\alpha)}{\alpha} (w + rk)\, T \int L(s)\, dQ_s^h(s). \tag{4}$$

Equation (4) shows that expected profits are proportional to the costs of production.

The arbitrage condition: Because entrepreneurs can completely hedge their risks in the financial market, they do not bear risk. As agents are identical and can be entrepreneurs or workers, expected profits must be equal to the opportunity cost of being an entrepreneur, which is the wage rate in the labor market.

$$\int \pi(s)\, dQ_s^h(s) = wT. \tag{5}$$

Hence, equations (4) and (5) imply

$$w = \frac{(1-\alpha)}{\alpha}(w + rk)\int L(s)\, dQ_s^h(s). \tag{6}$$

Resource constraints: To close the model, the labor and capital markets must clear:

$$K^a = mNkT\int L(s)\, dQ_s^h(s), \tag{7}$$

$$(1-m)N = mN\int L(s)\, dQ_s^h(s), \tag{8}$$

where K^a is the aggregate capital stock. Equation (7) is the capital market clearing condition. The left-hand side is the supply of capital and the right-hand side is the demand for capital: mN is the number of firms and $kT\int L(s_t)\, dQ(s_t)$ is the average firm's demand for capital. Equation (8) is the labor market clearing condition. The left-hand side is the supply of labor and the right-hand side is the demand for labor.

Market equilibrium: Market equilibrium can be formally defined as follows:

Definition 1 *A market equilibrium is* $\{L(\cdot), k, w, r, m\}$ *that satisfies the following conditions.*

1. *The firm's profit maximization conditions: equations (1) and (3).*
2. *The arbitrage condition: equation (6).*
3. *The resource constraints: equations (7) and (8).*

Let us define $\theta(k) = \frac{f'(k)k}{f(k)}$. The following theorem proves the existence and uniqueness of the equilibrium. The proof is given in the Appendix.

Theorem 2 *Suppose that* $\lim_{k\to\infty} \theta(k) < 1$*. Then, for any* $k^a \equiv \frac{K^a}{TN} \in (0, \infty)$ *, a unique market equilibrium exists.*

When many agents become entrepreneurs, few are employees. This increases the demand for employees and reduces the supply of employees. The wage rate is determined so that demand equals supply, which guarantees the existence of an equilibrium. The assumption $\lim_{k\to\infty} \theta(k) < 1$ is a technical one. When fewer agents become employees, k is larger. This condition implies that as k becomes infinite, the labor share does not converge to 0. That is, the wage rate must increase at a faster rate than employment falls. This guarantees that m has a solution in $(0, 1)$.

The aggregate production function: Now we derive the aggregate production function. Let Y^a and y^a denote aggregate output and aggregate output per unit of effective labor in an economy, $y^a = \frac{Y^a}{TN}$, respectively. The following proposition shows that entrepreneurs' ability to recognize change can influence the TFP of the aggregate production function. The proof is given in the Appendix.

Proposition 3 *Suppose that* $\lim_{k\to\infty} \theta(k) < 1$*. Then for any* $k^a \in (0, \infty)$*, there exists an aggregate production function:*

$$
\begin{aligned}
y^a &= z(h)\,\phi(k^a) \qquad (9) \\
\textit{where } z(h) &\equiv \left[\int \left(\int z dQ^h(z|s) \right)^{\frac{1}{1-\alpha}} dQ_s^h(s) \right]^{(1-\alpha)},
\end{aligned}
$$

$$\phi\left(k^{a}\right) \equiv f\left(\frac{k^{a}}{1-m\left(k^{a}\right)}\right)^{\alpha} m\left(k^{a}\right)^{(1-\alpha)}\left[1-m\left(k^{a}\right)\right]^{\alpha},$$

and $m\left(k^{a}\right) \in (0,1)$ *is a solution of*

$$\frac{\alpha}{1-\alpha}\left[1-\theta\left(\frac{k^{a}}{1-m}\right)\right]=\frac{1-m}{m}.$$

This proposition shows that h affects the TFP of the aggregate production function, but says nothing about the direction of the effect. Before examining the effect of h on TFP, it is useful to show the properties of the aggregate production function. The following proposition shows that the derived aggregate production function satisfies the traditional assumptions of macroeconomics. The proof is given in the Appendix.

Proposition 4 *Suppose that* $\lim_{k\to\infty}\theta(k) < 1$ *and* $\lim_{k\to 0}\theta(k) < 1$. *Then the derived aggregate production function is increasing and concave in* $k^{a} \in (0,\infty)$*, and satisfies the Inada conditions:*

$$\begin{aligned}\phi'\left(k^{a}\right) &> 0,\ \phi''\left(k^{a}\right)<0. \\ \lim_{k^{a}\to 0}\phi\left(k^{a}\right) &= 0,\ \lim_{k^{a}\to\infty}\phi'\left(k^{a}\right)=0 \text{ and } \lim_{k^{a}\to 0}\phi'\left(k^{a}\right)=\infty\end{aligned}$$

Both concavity and satisfaction of the Inada conditions are essential to the existence of a globally stable unique steady state in the neoclassical growth model. That is, the derived aggregate production function satisfies all the important assumptions of the aggregate production function in the neoclassical growth model.

3 The Macroeconomic Effects of Adaptability

In this section, we specify the information structure that entrepreneurs can access and examine the macroeconomic effects of entrepreneurs' ability to recognize unexpected changes in the environment. TFP, $z(h)$, is shown to be an increasing function of entrepreneurial ability.

The components of $z(h)$: Assume that $\log z$ comprises a predictable component μ and an unpredictable component u:

$$\log z = \mu + u$$

where u is normally distributed with mean 0 and variance σ_u^2. It is assumed that the unpredictable component u summarizes an unexpected change in productivity. The entrepreneur cannot observe u before making production decisions, but she can observe the signal s:

$$s = u + \varepsilon$$

where ε is normally distributed with mean 0 and variance $\sigma_\varepsilon^2(h)$. This paper assumes that a firm's adaptability is determined by the entrepreneur's ability to recognize the unexpected change. This ability can be represented by the ability to predict u after observing s. Hence, we can apply Takii's (2003) notion of prediction ability in this context. Let $Q_u^h(u|s)$ denote the conditional distribution of u given s. The measure of an entrepreneur's ability to recognize the unexpected change, u, is defined as follows.

Definition 5 *The measure of an entrepreneur's ability to recognize the unexpected change u (the measure of the firm's adaptability) is defined by:*

$$h = 1 - \frac{\int Var(u|s)\, dQ_s^h(s)}{\sigma_u^2},$$

where $Var(u|s) = \int \left(u - \int u dQ_u^h(u|s) \right)^2 dQ_u^h(u|s)$.

This measure implies that the entrepreneur accurately recognizes u when she reduces on average the conditional variance having observed s. To compare ability in different environments, $\int Var(u|s)\, dQ_s^h(s)$ is divided by σ_u^2, which is the unconditional variance of u. The measure h ranges from 0 to 1. If the entrepreneur perfectly recognizes the change, $h = 1$, whereas if the entrepreneur does not recognize the change at all, $h = 0$.

Using the definition of h, the variance of the noise term is endogenously determined as follows:

$$\sigma_\varepsilon^2(h) = \frac{(1-h)\,\sigma_u^2}{h}. \tag{10}$$

As expected, when the entrepreneur more accurately recognizes an unexpected change, the variance of the noise term is smaller. When $h = 1$, the variance is 0, and when $h = 0$, the variance is infinite.

Macroeconomic effects of entrepreneurship: Using this measure, $z(h)$ can be decomposed into productivity, risk and adaptability. The following theorem summarizes one of the main results in this paper. The proof is given in the Appendix.

Theorem 6 *For any $k^a \in (0, \infty)$, a rise in h increases GDP per unit of effective labor in an economy, y^a:*

$$y^a = z^e \phi(k^a) \exp\left(\frac{\alpha\sigma_u^2 h}{2(1-\alpha)}\right). \tag{11}$$

When an unexpected change in productivity occurs, if entrepreneurs accurately recognize the change, the deviations of actual marginal productivities of inputs from input prices would be small. Hence, an improvement in a firm's adaptability raises allocative efficiency, and therefore increases the productivity of the economy. Theorem 6 shows that an increase in productivity is represented by a rise in the TFP of the derived aggregate production function and that an increase in the firms' adaptability raises GDP per unit of effective labor in an economy.

Steady state: It is easy to apply the derived aggregate production function to the neoclassical growth model. Assume that the productivity measure, T, and the population, N, grow at the constant rates g_T and n, respectively. Assume also that the utility function of the household is $\int \frac{\left((c_t^a T_t)^{1-\theta}-1\right)N_t}{1-\theta} e^{-\rho t} dt$, where c_t^a is consumption per unit of effective labor in an economy. The steady-state values of c_t^a and k_t^a in the neoclassical growth model must satisfy the following:

$$
\begin{aligned}
z(h)\,\phi'(k^a) &= \delta + \rho + \theta g_T, \\
c^a &= z(h)\,\phi(k^a) - (n + \delta + g_T)\,k^a,
\end{aligned}
$$

where $z(h) = z^e \exp \frac{\alpha \sigma_u^2 h}{2(1-\alpha)}$ and δ is the depreciation rate of the capital stock. These two equations yield the steady-state values of c^a and k^a. In the steady state, consumption per capita, GDP per capita and capital stock per capita grow at the constant rate g_T. Hence, an increase in h raises the level of k^a and c^a, but does not change the growth rate.

The lack of a growth effect may be perceived as a weakness of the model, as entrepreneurs are thought to develop new products and affect economic growth. Of course, if innovation were formally modeled, it would be possible to construct a model in which entrepreneurial ability affects the long-run growth rate. However, this paper separates adaptability from innovative ability. If only adaptability is considered, it is conceivable that entrepreneurial ability does not affect the long-run growth rate, although it would affect temporal changes in the growth rate.

4 Political Risk and Entrepreneurship

In the previous section, it was argued that entrepreneurship can improve allocative efficiency and the TFP of an economy, because when entrepreneurs recognize a change correctly, they can equate the marginal products of inputs to their prices. However, in this section, we use an example to show that if opportunities are distorted, adaptability can lower the TFP of an economy.

Assume that subsidies increase firm-specific productivity and that all subsidies are financed by income tax:

$$z_i = (1-\tau)(1+G_i)$$

where τ is the constant average and marginal income tax rate and G_i is the subsidy for the ith firm. It is assumed that in the absence of taxes and subsidies, the productivity of each firm is unity. The subsidies have two components: a predictable component $g(R_i)$ and an unpredictable component u_i.

$$\log(1+G_i) = g(R_i) + u_i$$

where R_i is the rent-seeking activity of the ith firm and u_i is normally distributed with mean 0 and variance σ_u^2. The random variable u_i can be interpreted as a political shock. Because political outcomes depend on the opinions, political tactics and negotiations of politicians, the results are difficult to predict. The random factor, u_i, represents this uncertainty. It is assumed that R_i is chosen before entrepreneurs observe signals. Then, because entrepreneurs are identical, all choose the same level of rent-seeking activity, $R_i = R$ and $g(R_i) = g(R)$. If we set $\mu = \log(1-\tau) + g(R)$, the analysis of the previous section is applicable in this context.

As rent-seeking activities do not change aggregate income, μ must be chosen endogenously to satisfy the following resource constraint:

$$0 = \int (1-z_i)\left[f(k)\,L(s_i)\right]^{\alpha} T dQ_{zs}^{h}(z_i, s_i)\, m \tag{12}$$

where $L(s)$, k and m are the market equilibrium solutions to the previous problem. Note that $\int \left[f(k)\,L(s_i)\right]^{\alpha} T dQ_{zs}^{h}(z_i, s_i)\, m$ and $\int z_i\left[f(k)\,L(s_i)\right]^{\alpha} T dQ_{zs}^{h}(z_i, s_i)\, m$ are the values of aggregate output before and after the transfer of income, respectively. Hence, equation (12) requires that the income transfer does not change aggregate output.

Note that $L(s)$ is chosen when entrepreneurs expect an income transfer. Hence, the value of aggregate output before the income transfer takes place is affected by entrepreneurs' predictions of the realization of the political shock. The following theorem shows that an increase in firms' adaptability lowers TFP when there is political risk. The proof is given in the Appendix.

Theorem 7 *Suppose that μ satisfies equation (12). Then, for any $k^a \in (0, \infty)$, a rise in h reduces GDP per unit of effective labor in an economy, y^a:*

$$y^a = \phi(k^a) \exp\left(\frac{-\alpha\sigma_u^2 h}{2(1-\alpha)}\right). \tag{13}$$

Political risk reduces TFP because political risk generates a negative externality: when an entrepreneur is good at taking opportunities, there are fewer opportunities for others. In fact, it is shown that when equation (12) is satisfied, μ is chosen to satisfy

$$\mu = \frac{-\alpha\sigma_u^2 h}{(1-\alpha)} - \frac{\sigma_u^2}{2}.$$

Although individual entrepreneurs react to the political shock given μ, since these reactions do not produce new value in the economy, adaptability lowers μ to satisfy equation (12). This generates a negative externality.

Two comments are warranted. First, equation (13) shows that, if there is no political risk, competition between pressure groups leads to efficient allocation, as suggested by Becker (1983). More importantly, equation (13) shows that if there is political risk, even if rent-seeking activities are costless, political uncertainty can confuse entrepreneurs and thereby lower productivity. That is, entrepreneurship might have a detrimental effect on the economy in the presence of political risk. Hence, the social welfare loss due to rent-seeking activities may have been underestimated by the existing literature.

5 A Framework for an Empirical Study

The objective is to investigate the effect of firms' adaptability on TFP. The growth rate of TFP, g_{TFP}, is usually defined as $g_{TFP} \equiv g_{\frac{Y}{N}} - \frac{\phi'(k^a)k^a}{\phi(k^a)} g_{\frac{K}{N}}$, where $g_{\frac{Y}{N}}$ and $g_{\frac{K}{N}}$ are the growth rates of GDP per capita and capital stock per capita, respectively. Theorem 6 implies that

$$g_{TFP} \approx g_{z^e} + \left(1 - \frac{\phi'(k^a)\,k^a}{\phi(k^a)}\right) g_T + \frac{\alpha}{2(1-\alpha)} \left[h d\sigma_u^2 + \sigma_u^2 dh \right]. \tag{14}$$

To test the implication of equation (14), the following empirical equation is examined:

$$\Delta \log TFP = \psi_0 + \psi_z \Delta \log z^e + \psi_T \Delta \log T + \psi_\sigma h_t \Delta \sigma_u^2 + \psi_h \sigma_{ut-1}^2 \Delta h + \varepsilon, \tag{15}$$

where ψ_0, ψ_T, ψ_σ and ϕ_h are constant parameters. The growth rate of T, $\Delta \log T$, represents aggregate productivity growth. The growth rate of z^e, $\Delta \log z^e$, can be interpreted as firm-specific productivity growth. After controlling for these two effects, our theory predicts that a change in both risk and adaptability has a positive effect on the growth rate of TFP. Note that even if political risk is important, once we control for z^e, adaptability should have a positive effect on TFP.

The estimation of this equation requires estimates of the variables, $\Delta \log TFP$, $\Delta \log z^e$, $\Delta \log T$, σ_u^2 and h, which are described below.

Data description: Proxies for Y, K, wTL, wT and r were constructed mainly from the Census of Manufacturing in Japan for 1985–1999, which was provided by I-N Information Systems, LTD. The census covers all establishments in which four or more persons work as employers or employees, and the data are aggregated by city and industry for each year. This enables panel data analysis of the behavior of the typical establishment by city and industry. Details of the data and the construction of variables are given in the Appendix.

The data are split into two periods: 1985–1991 and 1992–1999. These periods roughly correspond to before and after the bursting of the bubble in Japan. We estimate, σ_u^2 and h using the constructed Y, wTL, wT, r and K, by prefecture, industry and period. Then, $\Delta \log TFP$, $\Delta \log z^e$, $\Delta \log T$, $\Delta\sigma_u^2$ and Δh are estimated by prefecture and industry. We treated 1988 and 1996 as representative years for each period. That is, we estimated the aggregate production function's TFP growth by prefecture and industry between 1988 and 1996. These estimated variables are used in our regression analyses. The estimation method is explained below.

Estimation of $\Delta \log TFP$, $\Delta \log z^e$ and $\Delta \log T$: First, we derive the equations that relate $\Delta \log TFP$, $\Delta \log z^e$ and $\Delta \log T$ to observable variables, and then provide an interpretation of each equation. Details of the estimation method are given in the Appendix.

The following proposition explains the estimation of $\Delta \log TFP$, $\Delta \log z^e$and $\Delta \log T$. The proof is given in the Appendix.

Proposition 8 *If w is constant, the growth rate of TFP, $\Delta \log TFP$, aggregate productivity growth, $\Delta \log T$, and firm-specific productivity growth, $\Delta \log z^e$, can be estimated as follows:*

$$\begin{aligned} \Delta \log TFP &= \Delta \log \frac{Y}{N} - \frac{\phi'(k^a)\,k^a}{\phi(k^a)} \Delta \log \frac{K}{N} & (16) \\ \Delta \log T &= \Delta \log wT, & (17) \\ \Delta \log z^e &= \Delta \log \int \frac{Y(z,s)}{(wTL(s))^{\alpha}\, wT^{(1-\alpha)}} Q^h_{zs}(z,s) - \alpha\theta(k)\, \Delta \log \frac{K}{wTL} & (18) \end{aligned}$$

where $\frac{\phi'(k^a)k^a}{\phi(k^a)}$, $\alpha\theta(k)$ *and* α *are estimated by*

$$\begin{aligned} \frac{\phi'(k^a)k^a}{\phi(k^a)} &= \alpha\theta(k) = \frac{1}{\int \left[\frac{Y(z,s)}{rK(s)}\right] dQ^h_{zs}(z,s)}, && (19) \\ \alpha &= \frac{1}{\int \left[\frac{Y(z,s)}{rK(s)+wTL(s)}\right] dQ^h_{zs}(z,s)}, && (20) \\ Y(z,s) &\equiv z\left[f(k)L(s)\right]^{\alpha} T,\ K(s) \equiv kTL(s). \end{aligned}$$

Equation (16) is the usual definition of TFP growth, except that the method of estimating the elasticity of output with respect to capital is unusual. Equation (19) shows that this can be estimated by the average capital share. Note that the definition of the average capital share corresponds to the usual one when there is no random component.

Equation (17) shows that aggregate productivity growth can be estimated by the growth rate of the average wage. When productivity growth is economy-wide, competition in the labor market pushes up workers' wage rates. Equation (17) reflects this intuition.

To interpret equation (18), we consider the case in which z is predictable. If z is predictable, equation (18) can be re-written as:

$$\Delta\log z = \Delta\log Y - \alpha\left[1-\theta(k)\right]\Delta\log L - \alpha\theta(k)\Delta\log K - \left[1-\alpha\theta(k)\right]\Delta\log wT.$$

This equation shows that $\Delta\log z$ represents the growth rate of value added that cannot be explained by the growth rate of labor input, capital input or wage rates. Hence, this is TFP growth excluding the contribution represented by the growth rate of wage rates. That is, the estimated $\Delta\log z$ excludes aggregate shocks in productivity changes. An aggregate shock raises wage rates and causes the effect to deviate from the components of $\Delta\log z$. Firm-specific productivity growth is the component of $\Delta\log z$.

It may be objected that a constant w is not consistent with our theory, as shifts in z^e, h and σ_u^2 change w. However, as we are using regional data, workers can move between regions. Hence, if the labor market is competitive, w is approximately the same in all regions and industries. To the extent that z^e, h and σ_u^2 do not change on average, a constant w is justified. In fact, the data support this assumption, as is shown subsequently. Hence, this assumption is innocuous.

Estimation of h and σ_u^2: Next, we explain the estimation of h and σ_u^2. It is shown subsequently that h can be estimated by the correlation between the unexpected shock and the reaction to the shock. If a firm's response to the shock is appropriate, this correlation must be high. To confirm this intuition, we must define the reaction to the shock.

Definition 9 *The firm's reaction to the shock $R(L(s))$, is defined as the logarithm of the deviation of actual labor input, $L(s)$, from predicted labor input, L^*:*

$$R(L(s)) = \log L(s) - \log L^*,$$

where L^* is estimated from the input level in the absence of an unexpected shock:

$$L^* = \left\{\frac{z^e}{w}\alpha\left[f(k^a) - f'(k^a)k^a\right]\right\}^{\frac{1}{1-\alpha}} f(k^a). \tag{21}$$

Equation (21) is derived by substituting z^e into the first-order conditions (1) and (2) for $\int z_i dQ^h(z_i|s_i)$. Using the definition of the firm's reaction to the shock, the following theorem is proved in the Appendix.

Theorem 10 *The entrepreneurs' adaptability, h, can be estimated by the correlation between u and $R(L(s))$:*

$$h = \left[\rho_{uR(L(s))}\right]^2, \ \rho_{uR(L(s))} > 0,$$

where

$$\rho_{uR(L(s))} = \frac{\int u \left(R(L(s)) - \int R(L(s))\, dQ_s^h(s) \right) dQ_{us}^h(u,s)}{\sqrt{\int u^2 dQ_u(u) \int \left(R(L(s)) - \int R(L(s))\, dQ_s^h(s) \right)^2 dQ_s^h(s)}}.$$

Theorem 10 shows that h can be estimated by the squared correlation between the unexpected shock and the firms' reaction to the shock. The proof is based on the first-order condition (1). The entrepreneur employs more than the predicted level of labor input when she believes that a positive productivity shock has been realized, and employs less than the predicted level when she believes that a negative one has occurred. When the entrepreneur's belief is accurate, the correlation must be larger.

To implement this idea, we need to estimate L^*. This involves the estimation of an unknown function $f(\cdot)$. However, if the economy is in the steady state, the correlation between the unexpected shock and the reaction to the shock can be estimated without using the function $f(\cdot)$. In the steady state, k and w are constant. Because the correlation coefficient is invariant to an affine transformation of a variable ($\rho_{XY} = \rho_{X(\eta Y+\iota)}$, where η and ι are constant), the following corollary can be easily proven from the definition of u and $R(L(s))$.

Corollary 11 *In the steady state, the correlation between the unexpected shock and the reaction to the shock can be estimated by the correlation between* $\log Y - \alpha \log wTL - (1-\alpha) \log wT$ *and* $\log wTL - \alpha \log wTL - (1-\alpha) \log wT$*.*

Corollary 11 shows that the main factor affecting h is the correlation between value added and labor expenses. Obviously, this is a fairly crude measure of adaptability. If we could explicitly model the information that entrepreneurs observe, a more accurate measure might be obtained. However, observable data are less likely to reflect the ideas of Hayek (1945) and Kirzner (1973), who emphasize the importance of unobservable local information. Moreover, Davis and Haltiwanger (1999) insist that unobserved idiosyncratic factors play a dominant role in explaining the redistribution of workers. Genda (1998) confirms this for Japan.

The correlation measure reflects the value of local information. To understand this, it is helpful to modify the equations in corollary 11.

$$\log Y - \alpha \log wTL - (1-\alpha)\log wT = \log Y - \alpha \log L - \log wT, \quad (22)$$

$$\log wTL - \alpha \log wTL - (1-\alpha)\log wT = (1-\alpha)\log L. \quad (23)$$

Equation (22) measures firm-specific productivity, since an aggregate productivity shock must also increase wT. The two equations show that h can be measured by the correlation between a firm-specific shock and labor input.[2] Hence, we suggest that, despite its potential problems, the correlation measure contains useful information about the ability of firms to process local information.

The correlation measure can be affected by various factors, including talent levels in management groups, education, personal networks, population density, regional transportation costs, and communication costs within organizations. In the absence of a theory of what determines adaptability, identifying the factors that enhance entrepreneurship is beyond the scope of this paper. However, we doubt that the correlation measure would be greatly affected by factors affecting adjustment costs, as adjustment costs lower not only the covariance of value added and labor expenses, but also their variances.

Corollary 11 uses labor expenses as a proxy for labor input for the estimation of h. There are two justifications for this proxy. First, the data set used for the empirical study does not include data on the number of employees, but does include data on the sum of employees and employers. Hence, labor expenses are the best available measure of labor input. Second, many unobserved inputs cannot be represented by

[2] In fact, we can show that the correlation between the unexpected shock and the reaction to the shock is equivalent to the correlation between $\log z$ and $\log L$ in the steady state.

the number of employees. Because it is difficult to fire employees, firms are likely to react to unexpected changes by varying working hours or workers' effort levels. Expenses best reflect these unobserved inputs.

If the economy is in the steady state, σ_u^2 can be estimated as follows:

$$\sigma_u^2 = Var\left[\log Y(z,s) - \alpha \log wTL(s) - (1-\alpha)\log wT\right],$$

where $Var(x)$ is the variance of x. As discussed above, equation (22) implies that σ_u^2 can be interpreted as a measure of changes in firm-specific productivity.

Strictly speaking, the steady-state assumption is not consistent with regression equation (2), because if the economy is in the steady state, $g_{TFP} \approx \quad 1 - \frac{\phi'(k^a)k^a}{\phi(k^a)} \quad g_T$. The economy is assumed to be in the steady state between 1985 and 1991, and between 1992 and 1999. A large shock is assumed to occur around 1991, which caused the economy to move from one steady state to the other. The values of h and σ_u^2 during the transition period are approximated by the steady-state values of h and σ_u^2. Because the Japanese bubble burst in 1991, it is not unreasonable to assume that the steady state changed around this time.

Alternative method of estimation: If the steady-state assumption is restrictive, an alternative method of estimating h is to specify the production function. Assume that $f(k) = Bk^{\beta}$. Then the following corollary can be proven in the same way as corollary 11 was proven.

Corollary 12 *Suppose that $f(k) = Bk^{\beta}$ and w are constant. Then, the unexpected shock and the reaction to the shock can be estimated by the correlation between* $\log Y - \alpha(1-\beta)\log wTL - \alpha\beta \log K - (1-\alpha)\log wT$ *and* $\log wTL - \alpha(1-\beta)\log wTL - \alpha\beta\log K - (1-\alpha)\log wT$. *Moreover, the measure of risk, σ_u^2, can be estimated by the variance of* $\log Y - \alpha(1-\beta)\log wTL - \alpha\beta\log K - (1-\alpha)\log wT$, *where* $\alpha(1-\beta)$

and $\alpha\beta$ can be estimated respectively by

$$\alpha(1-\beta) = \frac{1}{\int \left[\frac{Y(z,s)}{wTL(s)}\right] dQ^h_{zs}(z,s)},$$
$$\alpha\beta = \frac{1}{\int \left[\frac{Y(z,s)}{rK(s)}\right] dQ^h_{zs}(z,s)}.$$

When $f(k) = Bk^{\beta}$ and w are constant, the estimation of $\Delta \log z^e$ is modified as follows:

$$\Delta \log z^e = \Delta \log \int \frac{Y(z,s)}{[wTL(s)]^{\alpha(1-\beta)} K(s)^{\alpha\beta} wT^{(1-\alpha)}} dQ^h_{zs}(z,s).$$

The robustness of the results that follow can be checked by using the two alternative measures.

6 Empirical Results

This section reports the empirical results. First, summary statistics are reported. Then the regression results are reported.

Summary statistics: Table 1 reports the summary statistics of our estimates. The annual TFP growth rate is about 2 and aggregate productivity growth is 1.7%. This means that aggregate productivity growth accounts for most of the growth in TFP, which is broadly consistent with the steady-state assumption.

On average, firm-specific productivity declined slightly (-0.39% $\sim$ -0.45%), and the level of idiosyncratic risk remained constant. Note that we split the sample at approximately the point when Japan's bubble burst. Barseghyan (2003) and Nishimura et al (2003) argue that many unproductive firms survived in Japan during the 1990s. The fall in average firm-specific productivity might reflect the survival of weak firms in economy.

Variable	Obs.	Mean (1988-1996)	Std. Dev.	Mean (annual)
$\Delta \log TFP$	800	0.156*	0.301	1.95
$\Delta \log z^e \left(f(k) = Bk^\beta\right)$	800	-0.031*	0.376	-0.39
$\Delta \log z^e \, (steady)$	800	-0.036*	0.380	-0.45
$\Delta \log T$	800	0.139*	0.088	1.74
$\Delta \rho_{uR(L(s))} \left(f(k) = Bk^\beta\right)$	800	0.053*	0.322	0.66
$\Delta \rho_{uR(L(s))} \, (steady)$	800	0.024*	0.315	0.30
$\Delta h \left(f(k) = Bk^\beta\right)$	587	0.028*	0.152	0.35
$\Delta h \, (steady)$	592	0.023*	0.153	0.29
$\Delta \sigma_u^2 \left(f(k) = Bk^\beta\right)$	800	-0.003	0.070	-0.04
$\Delta \sigma_u^2 \, (steady)$	800	-0.003	0.070	-0.04

Table 1: Summary statistics (1988–1995)

"*steady*" means that the steady state is assumed and "$f(k) = Bk^\beta$ means that $f(k)$ is assumed to be Cobb-Douglas for the purposes of estimation. $*$ indicates significance at the 5% level.

The average of the measure of firms' adaptability increased modestly (0.29% $\sim$ 0.35%). The modest increase in adaptability is confirmed by the simple correlation between an unexpected shock and firms' reactions to the shock (0.3% $\sim$ 0.66%). The increase in the adaptability measure during the 1990s is interesting. Although adjustment is expected to be more difficult during a recession, the data show that, on average, Japanese firms improved their adaptability. This suggests that the measure of adaptability is not greatly affected by recession. As already discussed, the use of the correlation measure broadly corresponds to excluding the effect of adjustment costs. The data lend some support to this argument.

Small and opposing changes in firm-specific productivity and adaptability and no movement in risk imply that w would not be greatly affected by changes in these variables. This provides empirical justification for our assumption that w is constant. Moreover, opposing changes in firm-specific productivity and adaptability are also consistent with our theory: an increase in adaptability lowers z^e in the presence of political risk. Because the government's share of output increased during the 1990s in Japan [e.g., Hayashi and Prescott (2002)], our theory suggests that firms might have responded more to political shocks during the 1990s.

Note here that there are fewer observations on Δh than on $\Delta\rho_{uR(L(s))}$. To estimate Δh, we require a positive correlation in both periods. Twenty-six percent of our observations do not satisfy this condition. To check the robustness of our results, we also investigate regression analysis with the simple correlation below.

Regression results: Table 2 reports our regression results. All regressions show that a change in firms' adaptability increases TFP. This is consistent with the prediction of our theory: adaptability increases TFP once z^e has been controlled for. This result is robust. It passes several robustness checks.[3]

However, our theory is not consistent with risk having a negative effect on TFP growth: our theory predicts a positive effect. This suggests that other factors should be incorporated into our model. For example, if risk cannot be hedged in a financial market, then the behavior of risk-averse entrepreneurs may explain the perverse finding. Irreversible investment constitutes another possible explanation. These possibilities suggest further research.

[3] We included employment or the number of establishments to control for scale effects. We also added the growth rate of employment to check whether the high correlation picks up the effect of growing firms. Including these variables does not change our results. We also used weighted least squares estimation using the square root of the number of cities as weights. This did not change our results either.

$$\Delta \log TFP = \psi_0 + \psi_z \Delta \log z^e + \psi_T \Delta \log T + \psi_\sigma h_t \Delta \sigma_u^2 + \psi_h \sigma_{ut-1}^2 \Delta h + \varepsilon$$

	$f(k) = Bk^\beta$	$f(k) = Bk^\beta$	*steady*	*steady*
$\Delta \log T$	0.752***	0.882***	0.787***	0.902***
	(0.084)	(0.104)	(0.083)	(0.099)
$\Delta \log z^e$	0.522***	0.547***	0.510***	0.546***
	(0.020)	(0.023)	(0.019)	(0.022)
$\rho_{uR(L(s))t} \Delta \sigma_u^2$	-0.593*	———	-0.787***	———
	(0.298)		(0.272)	
$\sigma_{ut-1}^2 \Delta \rho_{uR(L(s))}$	0.781 **	———	1.139***	———
	(0.294)		(0.289)	
$h_t \Delta \sigma_u^2$	———	-1.199*	———	-1.877***
		(0.607)		(0.479)
$\sigma_{ut-1}^2 \Delta h$	———	2.415***	———	4.136***
		(0.587)		(0.836)
constant	0.068***	0.051***	0.065***	0.051***
	(0.014)	(0.016)	(0.014)	(0.016)
Adj R-squared	0.526	0.550	0.539	0.571
Obs.	800	587	800	592

Table 2: The effect of firms' adaptability on TFP

"*steady*" means that the steady state is assumed and "$f(k) = Bk^\beta$ means that $f(k)$ is assumed to be Cobb-Douglas for the purposes of estimation. * indicates significance at the 5% level. ** indicates significance at the 1% level. *** indicates significance at the 0.5% level. Standard errors are in parentheses.

Despite this inconsistency, the significantly positive effect of adaptability is interesting. Suppose the coefficient for adaptability is 4.1 for example. Since average risk is 0.035, a coefficient of 4.1 implies that if a firm fully recognizes previously unpredicted changes in its environment, TFP increases by 14%, having controlled for the negative externality. This means that if political risk is negligible, full recognition of previously unpredicted changes raises TFP by 14%. An obvious problem is that we cannot distinguish political shocks from others. However, another extreme example is informative. Suppose that political shocks are the only source of risk. When a firm fully recognizes previously unpredicted political shocks, equation (13) predicts that TFP falls by 14%.

Of course, this is a rough estimate. Although we implicitly assume that aggregation eliminates measurement errors, this assumption might be questionable. However, measurement error typically causes effects to be underestimated. Hence, our estimates probably understate the real effect. Similarly, if h is heterogeneous in an industry and a prefecture, the estimated relation between aggregate productivity and the average value of h would understate the real relation. Again, our estimates would understate the real relation. Hence, the estimate obtained in this paper is conservative. A more accurate estimate requires more disaggregated data.

7 Conclusions and Extensions

This paper has presented a tractable macroeconomic model that enables a theoretical and empirical investigation of a particular aspect of entrepreneurship—reacting appropriately to unexpected changes in the environment. We have shown that greater adaptability raises total factor productivity (TFP) in a competitive economy. However, greater adaptability may lower TFP if opportunities are distorted.

Several extensions are being considered. First, although it has been assumed for simplicity that all shocks are idiosyncratic and not persistent, this assumption leads us to underestimate the importance of entrepreneurship. An election or a revolution represents an aggregate political shock. The introduction of new technology probably generates persistent shocks. Incorporating these shocks into the model would be an interesting extension.

Second, the source of shocks must be empirically identified. Although we find empirical evidence of a positive direct effect of entrepreneurship on TFP, the overall effect is ambiguous. Identifying the source of shocks would enable estimation of the overall effect of entrepreneurship on TFP.

Third, investigating factors that enhance entrepreneurship would be interesting. Although average adaptability is estimated by industry and prefecture in this paper, we have said nothing about why adaptability differs between industries and prefectures. Adaptability may be affected by various factors, including inherited ability, education, social networks, connections, the density of a region, regional communication systems, and organizations. Since we have a well-defined measure of adaptability, it may be possible to identify empirically the factors that influence entrepreneurship.

Finally, it would be interesting to examine the extent to which a distorted reward structure for entrepreneurs might reduce TFP. Macroeconomists have recently found that differences in unexplained exogenous productivity in the aggregate production function explain a high proportion of income differences between countries [e.g., Hall and Jones (1999) and Prescott (1998)]. Hayashi and Prescott (2002) argue that part of the 'lost decade' in Japan can be explained by the slowdown in TFP growth. Because the model in this paper relates entrepreneurs' rent-seeking activities to TFP, it may provide a suitable empirical framework within which to examine the extent to which rent-seeking activities explain these problems.

8 Appendix

Mathematical Appendix:

Proof of Theorem 2: From the two resource constraints, (7) and (8), k can be expressed as a function of the aggregate capital stock per unit of effective labor in an economy:

$$k = \frac{k^a}{1-m}, \tag{24}$$

where $k^a = \frac{K^a}{TN}$. From equations (3), (6) and (8), we derive that

$$\frac{1-m}{m} = \frac{\alpha}{1-\alpha}\left[1-\theta\left(k\right)\right], \tag{25}$$

where $\theta\left(k\right) \equiv \frac{f'(k)k}{f(k)} \in (0,1)$. Define a function $G\left(m,k\right)$:

$$G\left(m,k^a\right) \equiv \frac{\alpha}{1-\alpha}\left[1-\theta\left(\frac{k^a}{1-m}\right)\right] - \frac{1-m}{m} \tag{26}$$

We must show that for any $k^a \in (0,\infty)$, there exists an $m \in (0,1)$ that satisfies

$$G\left(m,k^a\right) = 0. \tag{27}$$

It can be shown that $\lim_{m\to 0} G\left(m,k^a\right) < 0$ since when m converges to 0, the second term of equation (26) goes to ∞ and the first term is finite. When m converges to 1, the second term of equation (26) goes to 0 and the first term is positive, since $\lim_{k\to\infty}\theta\left(k\right) < 1$ by assumption. This proves that there exists an $m^* \in (0,1)$ that satisfies equation (27). Moreover, the solution m^* is unique, since it can be shown that

$$\begin{aligned}
& G_1\left(m,k^a\right)|_{G(m,k^a)=0} \\
= \; & \frac{1}{m\left(1-m\right)}\left[\frac{1-m}{m} - \frac{\alpha m}{1-\alpha}\theta'\left(k\right)k\right] \\
= \; & \frac{1}{m\left(1-m\right)}\frac{\alpha}{1-\alpha}\left[1-\theta\left(k\right)-m\theta'\left(k\right)k\right] \\
= \; & \frac{1}{m\left(1-m\right)}\frac{\alpha}{1-\alpha}\left\{\left[1-\theta\left(k\right)\right]\left[1-m\theta\left(k\right)\right] - m\frac{f''\left(k\right)k}{f'\left(k\right)}\theta\left(k\right)\right\} > 0.
\end{aligned} \tag{28}$$

The derivation of the second equation uses $G(m, k^a) = 0$ and the derivation of the third equation uses the definition of $\theta(k)$. Given m, equation (24) uniquely solves for k, and, given a unique k, equation (3) uniquely solves for $\frac{w}{r}$. Equations (1), (7) and (8) imply that

$$\frac{1-m}{m} = \left[\int \left(\int z dQ^h(z|s)\right)^{\frac{1}{1-\alpha}} dQ^h_s(s)\right] \left(\frac{\alpha f(k)^\alpha}{w+rk}\right)^{\frac{1}{1-\alpha}}, \tag{29}$$

Given unique values of $\frac{w}{r}$ and k, this equation uniquely solves for r and, therefore, also uniquely solves for w. Finally, given unique values of w, r and k, equation (1) uniquely solves for $L(s)$. **Q.E.D.**

Proof of Proposition 3: Aggregate output per unit of effective labor in an economy, y^a, can be expressed as follows:

$$y^a = \frac{mNyT \int L(s) dQ^h_s(s)}{TN}, \tag{30}$$

where $y = \int \frac{Y(zs)}{TL(s)} dQ^h_{zs}(z,s)$. Since it can be shown that $y = \frac{w+rk}{\alpha}$, substituting this equation and equation (8) into equation (30) yields

$$y^a = \frac{w+rk}{\alpha}(1-m). \tag{31}$$

Rearranging equation (29) yields

$$\frac{w+rk}{\alpha} = \left[\int \left(\int z dQ^h(z|s)\right)^{\frac{1}{1-\alpha}} dQ^h_s(s)\right]^{(1-\alpha)} f\left(\frac{k}{(1-m)}\right)^\alpha \left(\frac{m}{1-m}\right)^{(1-\alpha)}.$$

Applying this equation to equation (31) yields the desired result. **Q.E.D.**

Proof of Proposition 4: The definition of $\phi(k^a)$ implies that

$$\begin{aligned} \phi'(k^a) &= \alpha f(k)^{\alpha-1}\left(\frac{m}{1-m}\right)^{(1-\alpha)} f'(k) D, \\ \textit{where } D &\equiv \left[1 + k\frac{dm}{dk^a}\right] + \frac{f(k)}{f'(k)}\left[\frac{(1-\alpha)(1-m)}{\alpha m} - 1\right]\frac{dm}{dk^a}. \end{aligned} \tag{32}$$

Equation (27) implies $D = 1$. Hence $\phi'(k^a) > 0$.

To examine the second derivative, note that the first derivative can be rewritten as

$$\phi'(k^a) = \alpha \frac{\theta(k)\,\phi(k^a)}{k^a}. \tag{33}$$

Hence the second derivative can be written as

$$\begin{aligned} \phi''(k^a) &= M \frac{\phi'(k^a)}{\theta(k)\,k^a} \\ \textit{where } M &= \theta'(k)\frac{dk}{dk^a}k^a - \theta(k)\left[1 - \alpha\theta(k)\right] \end{aligned}$$

Hence, the sign of $\phi''(k^a)$ is the same as that of M. To determine the sign of M, two lemmas are required.

Lemma 13 *The sign of $m'(k^a)$ is determined by the sign of $\theta'(k)$:*

$$m'(k^a) = \frac{\theta'(k)\,m}{1 - \theta(k) - \theta'(k)\,km}.$$

Proof. *Equation (27) implies that*

$$\begin{aligned} m'(k^a) &= \frac{\frac{\alpha}{1-\alpha}\theta'(k)}{\frac{1}{m}\left(\frac{1-m}{m}\right) - \frac{\alpha}{1-\alpha}\theta'(k)\,k}, \\ &= \frac{\theta'(k)\,m}{1 - \theta(k) - \theta'(k)\,km}. \end{aligned}$$

The derivation of the second equation uses the fact that $\frac{\alpha}{1-\alpha}\left[1 - \theta\left(\frac{k^a}{1-m}\right)\right] = \frac{1-m}{m}$. *Equation (28) shows that* $1 - \theta(k) - \theta'(k)\,km$ *is positive.* ■

Lemma 14 *A rise in k^a increases k:*

$$\frac{dk}{dk^a} = \frac{1 - \theta(k)}{(1-m)\left(1 - \theta(k) - \theta'(k)\,km\right)} > 0$$

Proof. *Equation (24) implies that*

$$\frac{dk}{dk^a} = \frac{1 + km'(k^a)}{1 - m}.$$

The desired result follows from Lemma 13. ■

Applying Lemma 14 to the definition of M, the following equation is derived:

$$M = \frac{\theta'(k)[1-\theta(k)]k - \theta(k)[1-\alpha\theta(k)](1-\theta(k)-\theta'(k)km)}{1-\theta(k)-\theta'(k)km}.$$

Since $\theta(k) = \frac{f'(k)k}{f(k)}$, the numerator becomes:

$$\begin{aligned} & \theta'(k)[1-\theta(k)]k - \theta(k)[1-\alpha\theta(k)](1-\theta(k)-\theta'(k)km) \\ = \; & \theta(k)^2[1-\theta(k)][\alpha+m-1-\alpha m\theta(k)] \\ & +\theta(k)\frac{f''(k)k}{f'(k)}[1-\theta(k)+(1-\alpha\theta(k))m\theta(k)] \end{aligned}$$

Equation (27) implies that $\alpha + m - 1 - \alpha m\theta(k) = 0$. Hence $M < 0$, and therefore, $\phi''(k^a) < 0$.

Equation (27) also implies that $\lim_{k\to 0}\theta(k) < 1$ guarantees that $\lim_{k^a\to 0} m(k^a) \in (0,1)$. Hence, $\lim_{k^a\to 0}\phi(k^a) = 0$ since $\lim_{k\to 0} f(k) = 0$, and $\lim_{k\to 0}\phi'(k) = \infty$ since equation (32) implies that $\lim_{k\to 0} f(k) = 0$ and $\lim_{k\to 0} f'(k) = \infty$ guarantee this. Equation (27) also implies that $\lim_{k\to\infty}\theta(k) < 1$ guarantees that $\lim_{k^a\to\infty} m(k^a) \in (0,1)$. Hence, equation (32) proves $\lim_{k\to\infty}\phi'(k) = 0$ since $\lim_{k\to\infty} f'(k) = 0$. **Q.E.D.**

Proof of Theorem 6: Applying the standard Bayesian updating technique, it can be shown that

$$\begin{aligned} \int u\,dQ_u^h(u|s) &= hs, \\ Var(u|s) &= (1-h)\sigma_u^2. \end{aligned}$$

Using these results, $\int z\,dF(z|s)$ can be expressed as follows:

$$\int z\,dQ^h(z|s) = z^e \exp\left(hs - \frac{\sigma_u^2 h}{2}\right), \tag{34}$$

where $z^e = \exp\left(\mu + \frac{\sigma_u^2}{2}\right)$. Since the variance of s is $\sigma_u^2 + \sigma_\varepsilon^2$, this can be written as

$\frac{\sigma_u^2}{h}$. Using this result, it is easy to show that

$$\left[\int \left(\int z dQ^h(z|s)\right)^{\frac{1}{1-\alpha}} dQ_s^h(s)\right]^{(1-\alpha)} = z^e \exp\frac{\alpha\sigma_u^2 h}{2(1-\alpha)}.$$

Hence, the desired result follows. **Q.E.D.**

Proof of Theorem 7: To prove the theorem, the following lemma is needed.

Lemma 15 *Equation (12) is equivalent to*

$$\int \left(\int z_i dQ^h(z_i|s_i)\right)^{\frac{\alpha}{1-\alpha}} dQ_s^h(s) = \int \left(\int z_i dQ^h(z|s)\right)^{\frac{1}{1-\alpha}} dQ_s^h(s)$$

Proof. *Since $L(s)$ satisfies equation (1),*

$$\begin{aligned} &\int (1-z_i)\left[f(k)L(s)\right]^{\alpha} T dQ_{zs}^h(z,s)\, m \\ = &\int (1-z_i)\left(\int z_i dQ^h(z_i|s_i)\right)^{\frac{\alpha}{1-\alpha}} dQ_{zs}^h(z,s)\left(\frac{\alpha f(k)}{w+rk}\right)^{\frac{\alpha}{1-\alpha}} Tm \end{aligned}$$

Hence, equation (12) implies

$$0 = \int (1-z_i)\left(\int z_i dQ^h(z_i|s_i)\right)^{\frac{\alpha}{1-\alpha}} dQ_{zs}^h(z,s).$$

The desired result follows. ■

Since $\log z = \mu + u$, equation (34) implies that

$$\int \left(\int z_i dQ^h(z_i|s_i)\right)^{\frac{1}{1-\alpha}} dQ_s^h(s_i) = (z^e)^{\frac{1}{1-\alpha}} \exp\frac{\alpha\sigma_u^2 h}{(1-\alpha)^2},$$

and

$$\int \left(\int z_i dQ^h(z_i|s_i)\right)^{\frac{\alpha}{1-\alpha}} dQ_s^h(s_i) = (z^e)^{\frac{\alpha}{1-\alpha}} \exp\frac{\alpha(2\alpha-1)\sigma_u^2 h}{2(1-\alpha)^2},$$

where $z^e = \exp\left(\mu + \frac{\sigma_u^2}{2}\right)$. Using lemma 15, z^e can be solved for as a function of h and σ_u^2:

$$z^e = \exp\frac{-\alpha\sigma_u^2 h}{(1-\alpha)}.$$

The desired result follows from theorem 6. **Q.E.D.**

Proof of Proposition 8: The derivation of equation (17) is straightforward. Equation (18) is derived from the definition of the firm's production function. We utilize the fact that random variables do not affect k. Equation (20) is derived from the two first-order conditions (1) and (2). We now explain the derivation of equation (19). Equation (33) implies that

$$\frac{\phi'(k^a)\,k^a}{\phi(k^a)} = \alpha\theta(k).$$

In addition, the first-order condition (2) implies that

$$\alpha\theta(k) = \frac{1}{\int \frac{z[f(k)L(s)]^{\alpha}T}{rkTL(s)}dQ_z(z)}.$$

Q.E.D.

Proof of Theorem 10: Applying equation (34) to equation (21), it can be shown that $L^* = \left\{\frac{z^e}{w}\alpha\left[f(k^a) - f'(k^a)\,k^a\right]\right\}^{\frac{1}{1-\alpha}} f(k^a)$

$$L(s) = L^* \exp\left(hs - \frac{\sigma_u^2 h}{2}\right)^{\frac{1}{1-\alpha}}.$$

Hence, the firm's reaction to the shock $R(L(s))$ is given by $\frac{1}{(1-\alpha)}\left[hs - \frac{\sigma_u^2 h}{2}\right]$. Hence, the definition of the correlation coefficient implies that

$$\rho_{uR(L(s))} = \sqrt{h}.$$

Q.E.D.

Data Appendix:

Data description: Every year the Japanese Ministry of the Economy, Trade and Industry releases the Census of Manufacturing by city and industry. However, because of minor changes in the classification of industries and the integration and division of cities, the data that are released must be modified for use in panel data analysis. I-N Information Systems, Ltd undertakes this modification and thereby enables panel data analysis of the behavior of the average establishment by city and industry.

Although the census covers all establishments in which four or more persons work as employers or employees, if there are fewer than three establishments for an industry in a city in any given year, data are not reported by the census, to maintain the privacy of establishments. To improve estimation of the correlation, we exclude entities on which there are missing variables in any period.

The industries covered by the census are Food, Drink/tobacco/feed, Textiles, Apparels, Lumber/wood products, Furniture/fixtures, Pulp/paper, Printing, Chemicals, Petroleum/coal products, Plastic products, Rubber, Leather/leather products, Pottery/glass products, Iron/steel, Non-ferrous metals, Metal products, General machinery, Electrical machinery, Transportation machinery, Precision tools, Weapons and Other industries.

Y **and** wTL : Gross value added and labor expenses are divided by the number of establishments. These values are then deflated by the GDP price deflator.

K : Fixed tangible assets are divided by the number of establishments. The replacement cost of the capital stock is then estimated from the following equation:

$$\begin{aligned} K_{it+1} &= K_{it} + \left(F_{it+1} - F_{it}\right)/p_{It+1},\ if\ F_{it+1} > F_{it}, \\ &= K_{it} + F_{it+1} - F_{it},\ if\ F_{it+1} \leq F_{it}, \end{aligned}$$

where $K_{i1} = F_{i1}/p_{I1}$ and F_{it} is average fixed tangible assets per establishment and

p_{It} is a price deflator for investment goods, which is taken from Keizai Tokei Nenkan (2002) (Annual Economic Statistics 2002) by Toyo Keizai Shinpo Sya. As we do not have data on investment, this simplified estimation method is used as an approximation, which is the approach taken by Nishimura, Nakajima and Kiyota (2003).

The Census of Manufacturing only reports fixed tangible assets for establishments in which the number of employers or employees is at least 10 [size group 2]. Hence, the capital stock of establishments with between four and nine employees or employers [size group 1] is estimated as follows. First, average labor expenses per establishment of size groups 1 and 2 are estimated by city, industry and year. Average fixed assets per establishment are then regressed on average labor expenses per establishments in size group 2 for each industry. A fixed-effects regression is used for this purpose. The parameters of this regression are used to estimate the capital stock in size group 1.

To estimate average labor expenses per establishment in size groups 1 and 2, the following estimation method is used. The Census of Manufacturing reports labor expenses and the number of establishments for size groups 1 and 2 by industry and year. Using these data, average labor expenses per establishment are estimated by size group, industry and year. Assuming that the ratio of average labor expenses per establishment in size group 1 to those in size group 2 is the same for each industry and year, average labor expenses per establishment are estimated by city, industry and year.

wT : wT is estimated by prefecture, industry and year by the weighted average of deflated labor expenses over the number of employees. The number of establishments is the weight. Because the number of employees is not reported, it is estimated. The Census of Manufacturing reports the number of employees, the sum of employees and employers, and the number of establishments by industry and year. Assuming that the ratio of employees per establishment to the sum of employers and employees per establishment is the same for each industry and year, the number of employees is estimated by city, industry and year.

r : The return to the capital stock is estimated by using

$$r_t = p_{It}\left(i_t + \delta\right),$$

where i_t is the yield on 10-year government bonds and δ is the average depreciation rate by industry over average fixed tangible assets by industry. The yield data are from the homepage of Bank of Japan. Average depreciation and average fixed tangible assets are taken from the Census of Manufacturing. As in Nishimura et al (2003), changes in the price deflator for investment goods are ignored, since this index increased so much during the bubble in Japan that the user cost of capital became negative. Because r_t is only used to estimate the average capital share over time, this simplification is unlikely to affect our results. To check for robustness, we also used the return to the capital stock, as in Hayashi and Prescott (2002). As this did not change the results, we do not report them in this paper.

$\mathbf{\Delta} \log \mathbf{TFP}$: We estimate the average capital share of each firm from the sample average of $\frac{Y}{rK}$ over time by city and industry. Then the weighted average of the capital share is estimated by prefecture, industry and period, with the number of establishments in 1988 and 1996 as weights. Unless otherwise stated, the same weight is used to estimate the prefecture average. The average of the capital share over the period is chosen to estimate $\frac{\phi'(k^a)k^a}{\phi(k^a)}$. Value added, capital stock and the sum of employees and employers are aggregated by industry and prefecture in 1988 and 1996. Then $\Delta \log TFP$ is estimated as defined.

$\mathbf{\Delta} \log \mathbf{wT}$: wT in 1988 and 1996 is chosen for this estimation.

$\mathbf{\Delta} \log \mathbf{z}^e$: We estimate α, $\alpha\left(1-\beta\right)$ and $\alpha\beta$ from the sample averages of $\frac{Y}{rK+wTL}$, $\frac{Y}{wTL}$

and $\frac{Y}{rK}$ over time by city and industry, respectively. Using the estimated α, $\alpha(1-\beta)$ and $\alpha\beta$, $\int \left[\frac{Y(z,s)}{(wTL(s))^{\alpha} wT^{(1-\alpha)}} \right] Q^{h}_{zs}(z,s)$ and $\int \frac{Y(z,s)}{[wTL(s)]^{\alpha(1-\beta)} K(s)^{\alpha\beta} wT^{(1-\alpha)}} dQ^{h}_{zs}(z,s)$ are estimated by their sample averages over time by city, industry and period. The weighted averages of these values are calculated by prefecture, industry and period. The weighted average of the capital-labor expense ratio, $\frac{K}{wTL}$, in 1988 and 1996 is also estimated by prefecture and industry. Using these values, $\Delta \log z^{e}$ is then estimated by prefecture and industry.

h : To implement corollaries 11 and 12, the correlations are estimated from the sample averages over time by city, industry and period. Then, weighted averages of the correlations are estimated by prefecture, industry and period. The squared correlations are then calculated when they are positive.

$\boldsymbol{\sigma}_{u}^{2}$: To estimate the variance, the standard deviation is estimated by city, industry and period. Then the weighted average of the standard deviation is estimated by prefecture, industry and period. The square of the average standard deviation is then estimated.

References

[1] Aghion, Philippe and Howitt, Peter (1992), "A Model of Growth Through Creative Destruction," Econometrica, vol. 60, 323-351.

[2] Barseghyan, Levon (2003), "Non-Performing Loans, Prospective Bailouts, and Japan's Slowdown," unpublished.

[3] Bartelsman, Eric J. and Doms, Mark (2000), "Understanding Productivity: Lessons from Longitudinal Microdata," Journal of Economic Literature, 569-594.

[4] Baumol, William J. (1990), "Entrepreneurship: Productive, Unproductive, and Destructive," Journal of Political Economy, 98, 893-921.

[5] Becker, Gary S.,"A Theory of Competition Among Pressure Groups for Political Influence," Quarterly Journal of Economics, 98 (3), 371-400.

[6] Davis, Steven J. and Haltiwanger, John (1999), "Gross Job Flows," *In Handbook of Labor Economics, Volume 3,* edited by Ashenfelter, Orley and Card, David, 2711-2805.

[7] Genda, Yuji (1998), "Job Creation and Destruction in Japan, 1991-1995," Journal of the Japanese and International Economies, 1-23.

[8] Hall, Robert E. and Jones, Charles I. (1999), " Why do Some Countries Produce So Much More Output per Worker than Others?," Quarterly Journal of Economics, 83-116.

[9] Hayashi, Fumio and Prescott, Edward C. (2002), "The 1990s in Japan: A Lost Decade," Review of Economic Dynamics, 5, 206-235.

[10] Hayek (1945), "The Use of Knowledge in Society," American Economic Review, 519-530

[11] Holmes, T. J., and Schmitz, J. A. (1990), "A Theory of Entrepreneurship and its Application to the Study of Business Transfers," Journal of Political Economy 98, 265-294.

[12] Holmes, Thomas J, and Schmitz, James A. Jr. (2001), "A Gain from Trade: From Unproductive to Productive Entrepreneurship," Journal of Monetary Economics, 47, 417-446.

[13] Kihlstrom, Richard E. and Laffont, Jean-Jacques (1979), "A General Equilibrium Entrepreneurial Theory of Firm Formation Based on Risk Aversion," Journal of Political Economy, 87, 719-748.

[14] Kirzner (1973), *Competition and Entrepreneurship*, Chicago, University of Chicago Press.

[15] Lucas, Robert E. (1978),"On the Size Distribution of Business Firms," Bell Journal of Economics, 9, 508-523.

[16] Murphy, Kevin M., Shleifer, Andrei and Vishny, Robert W. (1991), "The Allocation of Talent: Implications for Growth," Quarterly Journal of Economics, 503-530.

[17] Mas-Colell, Andreu, Whinston, Michael and Green, Jerry R. (1995), *Microeconomic Theory*, New York, Oxford University Press.

[18] Nishimura, Kiyohiko, Nakajima, Takanobu, and Kiyota, Kozo (2003), "Ushinawareta 1990 nendai, Nihon Sangyo ni Naniga Okottanoka? - Kigyo no Sannyu Taisyutsu to Zenyouso Seisansei-," RIETI Discussion Paper Series 03-J-002.

[19] Prescott Edward C. (1998), "Lawrence R. Klein Lecture 1997 Needed: A Theory of Total Factor Productivity," International Economic Review, 39, 525-551.

[20] Rosen, Sherwin (1982), " Authority, Control and the Distribution of Earnings," Bell Journal of Economics, 311-323.

[21] Schultz, Theodore W. (1975), "The Value of the Ability to Deal with Disequilibria," Journal of Economic Literature, 13, 827-846.

[22] Schmitz, James A (1989), "Imitation, Entrepreneurship, and Long-Run Growth," Journal of Political Economy, 1989, vol. 97, 721-739.

[23] Takii, Katsuya (2003), "Prediction Ability," Review of Economic Dynamics, 6, 80-98.

[24] Tullock, G. (1967), "The Welfare Cost of Tariffs, Monopolies, and Theft," Western Economic Journal 5, 224-232.