The Value of Adaptability
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【要約】This paper examines a firm's ability to respond correctly to an unexpected change in the environment (i.e., its adaptability). We develop a model that allows for empirical examination of the impact of a firm's adaptability on its expected profits. The theory shows that a firm's adaptability can be estimated by the squared correlation between an unexpected change and the firm's reaction. The estimates show that adaptability has a large positive impact on the average profit rate and the market value of a firm. We also find that an increase in risk is correlated with a rise in adaptability.

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1 Introduction

Flexibility has recently been emphasized as an important feature that many firms wish to increase. Greater flexibility is expected to improve a firm’s adaptability to changes in consumer needs and technological developments. But how much has adaptability actually improved? More importantly, by how much does adaptability raise market value? This paper develops a model that can empirically analyze the impact of adaptability on expected profits.

We suggest that a firm’s adaptability is determined by its ability to recognize changes in its environment. When an unexpected change in productivity (or demand) occurs, the firm observes an indication of the change and infers the direction and magnitude of the change. If decision makers in the firm recognize the associated deviation from predicted productivity (or demand) before making production decisions, the firm will be able to respond appropriately to the change.

For example, multi-task production equipment can be interpreted as a means of allowing the firm to know customer needs without committing to certain products. Decentralization can be seen as an organizational system that enables decision makers to obtain precise information about a production process in order to correctly deal with change. We assess a firm’s ability to recognize, and therefore adapt to, an unexpected change, and estimate the impact of this adaptability on expected profits.

How do we measure adaptability?: Suppose that a firm maximizes its expected profits. Then we can identify theoretically the predicted input level that the firm would use if there were no unexpected change. Since in reality

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1 Milgrom and Roberts (1990) stress that programmable multi-task production equipment has replaced single-purpose equipment for mass production. Osterman (1994) concludes that substantial use of flexible work organization was found in about 35% of private sector establishments with 50 or more employees in 1992.
the firm always faces changes, this predicted input never materializes. The difference between actual and predicted input (the residual) can be considered as the firm’s reaction to changes. If the correlation between the changes and the residual is large, we can infer that the firm’s ability to adapt to change is high.

This intuition can be confirmed. Assuming that the production function is Cobb-Douglas and that an unpredicted productivity (or demand) shock is log-normally distributed, this theory shows that the firm’s adaptability can be estimated by the squared correlation between the unexpected change and the logarithm of the deviation of actual input from the theoretically-predicted input. This measure enables empirical investigation of the impact of adaptability on the average profit rate and on the market value of firms.

**Main Empirical Findings:** The main empirical findings are set out below.

*Robust and significant impact of adaptability on expected profits:* The theory predicts that expected profits are a function of productivity, adaptability, risk and size. Empirical testing of this prediction uses the COMPUSTAT dataset. Regression analysis shows that productivity and adaptability are two robust determinants of expected profits. The importance of adaptability depends on the level of risk that a firm faces. The estimates imply that if a firm fully recognizes previously unpredicted changes in its environment, its market value increases by 28% if the firm faces an average level of risk, by 48% if it faces the top 10% level of risk, and by 400% if it faces the top 1% level of risk. We argue that the real impact of adaptability is likely to be larger than these estimates.

*Risky environments demand adaptability:* Why are some firms more adaptable than others? We examine two hypotheses: (i) smaller firms are more adaptable; (ii) risky environments force firms to improve their adaptability. Managers
of large firms may be unable to observe all the local information. If local information is important for predicting changes in the environment, smaller firms will be more adaptable. On the other hand, if unexpected change occurs frequently, the benefits of appropriate reaction to change are large. Hence, a risky environment demands adaptability. A test of these two hypotheses reveals that a change in firm size is negatively correlated, and a shift in risk is positively correlated, with changes in adaptability. However, when risk is controlled for, the change in firm size has no impact on adaptability. This paper concludes that for firms that respond rationally to change, increased risk is the main contributory factor to greater firm adaptability.

**Related papers:** To our knowledge, this is the first attempt to identify the value of adaptability in a firm. However, this paper draws on previous research. Stigler (1939), Mills and Schumann (1985) and Thesmar and Thoenig (2000) investigate the trade-off between static efficiency and flexibility. Mills and Schumann (1985) find that the variability of sales is inversely related to firm size within industries. Flexibility improves profits only when firms know how decisions should be changed. Hence, the results in this paper complement this finding.

The results of this paper can also be interpreted as the comparison of entrepreneurial ability and managerial ability at the firm level. Lucas (1978) describes managerial ability as the ability to increase productivity in a firm, and shows that this ability is valued more in large firms. Schultz (1975) characterizes entrepreneurial ability as the ability to interpret new information and allocate resources to profitable opportunities, and Takii (2003) argues that this ability is more valuable in risky environments. If it can be assumed that a firm’s productivity and adaptability reflects these two different abilities, this paper confirms both the above hypotheses.²

²Holmes and Schmitz (1990) also emphasize that it is important to distinguish entrepre-
This paper also contributes to discussion of organizational change. Gittleman, Horrigan and Joyce (1998) and Osterman (2000) report that many firms have introduced flexible work practices. However, they do not examine by how much firms have improved their adaptability. Interestingly, this paper’s estimates of adaptability do not demonstrate a trend. Since COMPUSTAT provides only firm-level data, we cannot examine reactions within firms. Hence, while the results in this paper are not conclusive, the evidence indicates that organizational change may not improve a firm’s ability to react appropriately to a firm-level shock.

Organization of this paper: The next section presents the theoretical model and shows that expected profits are a function of productivity, adaptability, risk and the capital stock. Section 3 discusses the estimation of productivity, adaptability and risk. Section 4 reports the regression results. This section shows that adaptability has a significant and robust impact on a firm’s average profit rate and the market value of a firm. Section 5 examines the relationship between risk, size, adaptability and productivity. Section 6 concludes the paper.

2 The Model

This section develops the theoretical model. It is shown that the firm’s expected profits depend on its productivity, adaptability, risk and size.

The firm’s production function is assumed to be Cobb-Douglas:

\[ Y = zX^\alpha K^\beta, \]

where \( Y \) is sales, \( K \) is capital stock, \( X \) is the composition of all other inputs, \( z \) is a random shock and \( \alpha \) and \( \beta \) are parameters. Although each variable

neural ability from managerial ability.
differs between firms and over time, time and firm subscripts are omitted (unless omission causes confusion).

\( K \) is assumed fixed. Hence the firm can only determine \( X \). It is also assumed that changes in the environment that affect the firm’s profits are summarized by changes in \( z \). The natural interpretation of the random shock is a productivity shock. However, since empirically \( Y \) is measured as nominal sales deflated by the implicit GDP deflator, \( z \) can be affected by changes in taste.

The firm does not know the realization of the random shock. However, the firm observes a signal \( s \), which is used to predict \( z \). The firm’s profit maximization problem is:

\[
\Pi(s, K) = \max_X Z \cdot \frac{\alpha}{1 - \alpha} z dF(z|s) X^\alpha K^\beta - X^{\gamma / \alpha},
\]

where \( F(z|s) \) is the conditional distribution of \( z \) given \( s \). The price of the input is normalized to unity.

It is easy to derive the optimal amount of input \( X(s) \) from the first order condition:

\[
X(s) = \frac{\alpha}{1 - \alpha} Z \cdot \frac{1}{z dF(z|s)} K^{\beta / \alpha},
\]

(1)

This shows that the optimal amount of the input depends on \( z dF(z|s) \). If a firm believes that \( z \) is large, it is profitable for the firm to produce more goods.

By substituting equation (1) into the profit function, \( \Pi(s, K) \), one derives the expected profits of the firm:

\[
\Pi^e = \frac{\alpha}{Z \cdot Z} \cdot \frac{1}{z dF(z|s)} \left( 1 - \alpha \right) (\alpha) z^* K^{\beta / \alpha},
\]

(2)

where \( z^* = \frac{z dF(z|s)}{dF_s(s)} \), and \( \Pi^e = \int \Pi(s, K) dF_s(s) \). The distribution \( F_s(s) \) is the marginal distribution of the signal \( s \). Equation (2) shows that expected profits depend on the capital stock, \( K \), and the variable, \( z^* \), which measures the profitability of the capital stock. Note that the shock and the signal increase expected profits if and only
if they increase the profitability of the capital stock, \( z^* \). The component \( z^* \) is discussed below.

**The component** \( z^* \): Assume that \( \log z \) comprises a predictable component \( \mu \) and an unpredictable component \( u \):

\[
\log z = \mu + u
\]

where \( u \) is normally distributed with mean 0 and variance \( \sigma_u^2 \). It is assumed that the unpredictable component \( u \) summarizes an unexpected change in productivity and demand.

The firm cannot observe \( u \) before making production decisions, but it can observe the signal \( s \):

\[
s = u + \varepsilon
\]

where \( \varepsilon \) is normally distributed with mean 0 and variance \( \sigma_\varepsilon^2 \). This paper assumes that a firm’s adaptability is determined by its ability to recognize the unexpected change. Given this assumption, adaptability is captured by the accuracy of the signal, \( s \), which is used to predict the unexpected change \( u \).

Hence, we are applying the notion of prediction ability in Takii (2003) to measure a firm’s adaptability. Let \( \Phi(u|s) \) denote the conditional distribution of \( u \) given \( s \). Following Takii (2003), the accuracy of the signal \( s \) is measured in the following manner.

**Definition 1** The measure of the ability to recognize an unexpected change \( u \) (the measure of adaptability) is defined by:

\[
h = 1 - \frac{\text{Var}(u|s) dF_s(s)}{\sigma_u^2},
\]

where \( \text{Var}(u|s) = \int u^2 \phi(u|s) d\Phi(u|s) - (\int u \phi(u|s) d\Phi(u|s))^2 \).

This measure implies that the firm can accurately recognize \( u \) when it can reduce the average conditional variance having observed \( s \). To compare ability
in different environments, $\mathbb{R} Var(u|s) \, dF_s(s)$ is divided by $\sigma_u^2$, which is the unconditional variance of $u$. The measure $h$ spans 0 to 1. If the firm perfectly recognizes the changes, $h = 1$, and if the firm knows nothing about the change, $h = 0$. This is the measure of adaptability used in this paper.

The meaning of $h$ is more easily understood if we derive the posterior mean as a function of $h$:

$$E[u|s] = 0 \times (1 - h) + hs = hs.$$  

Since the noise term, $\varepsilon$, and the unexpected change, $u$, are normally distributed, the posterior mean is a weighted average of the prior mean and new information. As is apparent, $h$ measures weight attached to the signal, $s$. When the signal is informative, the firm will attach more weight to it, but when it is uninformative, the firm takes a more conservative view and keeps to its prior mean. Using the definition of $h$, the variance of the noise term is endogenously determined as follows:

$$\sigma_{\varepsilon}^2 = \frac{(1 - h) \sigma_u^2}{h}.$$  

This shows that when the firm more accurately recognizes an unexpected change, the variance of the noise term is smaller. When $h = 1$, the variance is 0 and when $h = 0$, the variance is infinite.

Using this measure, $z^*$ can be decomposed into productivity, risk and adaptability. The proof is given in the Appendix.

Lemma 2. The profitability of the capital stock, $z^*$, is an increasing function of productivity, $z^c$, risk, $\sigma_u^2$, and adaptability, $h$:

$$z^* = (z^c)^{\frac{1}{1+\alpha}} \exp \frac{\alpha \sigma_u^2 h}{2(1-\alpha)^2},$$

where $z^c = \exp \mu + \frac{\sigma_u^2 i}{2}$.
The impact of $z^e$ and $h$: Applying lemma 2 to the above problem produces the following theorem.

**Theorem 3** Expected profits are an increasing function of productivity, $z^e$, risk, $\sigma^2_u$, adaptability, $h$ and size $K$:

$$\Pi^e = (1 - \alpha) \alpha^{\frac{1}{2}} (z^e)^{1-\alpha} \exp \frac{\alpha \sigma^2_u h}{2(1 - \alpha)^2}.$$  

3 Empirical Strategy

The objective is to test theorem 3 and compare the relative importance of productivity, size, risk and adaptability. The following empirical equation can be derived from theorem 3.

$$\log \Pi^e = \phi_0 + \phi_z \log z^e + \phi_h \sigma^2_u h + \phi_k \log K + \varepsilon,$$  \hspace{1cm} (3)

where $\phi_0, \phi_z, \phi_h$ and $\phi_k$ are constant parameters, and $\varepsilon$ is an error term. Strictly speaking, the theory implies several restrictions on the parameters. However, the purpose of this empirical study is to investigate the impact of adaptability on the firm’s expected profits rather than to identify the model. Hence, we do not concern ourselves with these restrictions but focus instead on the economic importance of adaptability.

Given some econometric problems, some modification to equation (3) is required. Unobserved heterogeneous productivity may be correlated with the measure of productivity, $z^e$, and adaptability, $h$. This may cause the coefficients of these variables to be overestimated. Moreover, the capital stock may not be stationary. In this case, the regression might produce spurious correlations. To take account of these econometric issues, the first difference of equation (3) is estimated. Hence, the empirical equation is:

$$\Delta \log \Pi^e = \phi'_0 + \phi'_z \Delta \log z^e + \phi'_h \sigma^2_u \Delta h + \phi'_u \Delta \sigma^2_u h + \phi'_k \Delta \log K + \nu$$  \hspace{1cm} (4)
where $v$ is an error term. The estimation of this equation requires estimates of the variables, $z^e$, $\sigma_u^2$ and $h$, which are described below.

**Data Description:** The COMPUSTAT data set, 1980-1999, is used for the empirical analysis. This time period was chosen because many firms did not report quarterly data before 1980.

$Y_t$ and $X_t$ are measured as sales (DATA2 in the COMPUSTAT) and as the cost of goods sold (DATA30) in the Industrial Quarterly data set, respectively, and $K_{t-1}$ is the net value of property, plant and equipment (DATA8) in the previous year from the Industry Annual data set. Each variable is deflated by the implicit price deflator for GDP (for $Y$ and $X$) or the deflator for non-residential investment (for $K$), which is taken from the Bureau of Economic Analysis. COMPUSTAT was split into four periods: 1980-1984, 1985-1989, 1990-1995 and 1995-1999. For each period and each firm, we estimate $z^e$, $\sigma_u^2$ and $h$ by using $Y_t$, $X_t$ and $K_{t-1}$.

The cost of goods sold is all expenses directly allocated to production (e.g. material, labor and overheads). Hence, it is not a quantity measure. There are two justifications for using expenses to proxy inputs. First, many firms in the COMPUSTAT data set do not report quantity measures such as the number of workers. Therefore, the cost of goods sold is the best available measure for most firms. Second, there are many unobserved variable inputs. In particular, since it is difficult to fire workers, many firms might react to an unexpected change by varying working hours or workers’ effort levels. Expenses best capture these changes in unobserved inputs.

**Estimation of $z^e$, $\sigma_u^2$ and $h$:** If the parameters $\alpha$ and $\beta$ are known, the productivity of a firm is derived from the production function:

$$\log z_t = \log Y_t - \alpha \log X_t - \beta \log K_{t-1}.$$
Hence, we can obtain consistent estimators of \( z^e \) and \( \sigma_u^2 \) by using the sample means of \( z \) and the sample variance of \( \log z \) over time for each period and each firm provided \( \log z \) is stationary. Since there is only a mild trend for \( \log z \), we do not detrend \( \log z \). Some researchers find the absence of a strong trend unusual. Note, however, that \( X \) is expenditure. Since shifts in aggregate shocks change input prices in the same direction, \( \log z \) approximately excludes the effect of aggregate shocks.

The exclusion of the aggregate shocks provides another rationale for using expenditure to proxy inputs. This paper implicitly assumes that a major part of shocks is idiosyncratic. This assumption is innocuous. Davis and Haltiwanger (1999) review the literature and insist that idiosyncratic factors primarily explain the behavior of job flows. However, if aggregate shocks are important, the estimation of adaptability may be less accurate. For example, if every firm reacts to a boom and input prices rise, firms will not use as much input as theory predicts. Hence, the use of expenditure mitigates a potential mismeasurement of adaptability caused by aggregate shocks.

Using the first-order condition, it is shown that the parameter \( \alpha \) can be estimated from the factor share:

\[
\alpha = \frac{(X_t)^e}{(Y_t)^e} = \frac{(X_t/K_{t-1})^e}{(Y_t/K_{t-1})^e},
\]

where \((x)^e\) is the expected value of \( x \). Dividing \( X \) and \( Y \) by \( K \) is unusual. However, since the capital stock is fixed when the firm recognizes the unexpected shock, the second equality is also true in this model. When \( X \) and \( Y \) are divided by \( K \), the trends of \( X \) and \( Y \) are approximately eliminated. Hence we can estimate \((X/K)^e\) and \((Y/K)^e\) at the firm level by using the sample mean of \( \frac{X}{K} \) and \( \frac{Y}{K} \) over time for each firm and each period. For accurate estimation, observations that have less than 20 entries in a period are deleted; 20 is the maximum possible number in each period and for each firm.

To estimate the parameter \( \beta \), the following regression equation is derived
from the first-order condition (1):

\[
\log X_t = \psi_0 + \psi_1 \log K_{t-1} + \upsilon_t,
\]

where \( \psi_1 = \frac{\beta}{1 - \alpha} v_t \tilde{N}(0, \sigma^2) \).

Once \( \psi_1 \) has been estimated, \( \beta \) given \( \alpha \) can be recovered. Potentially, \( \psi_1 \) can be estimated at the level of the individual firm. However, the results are fairly unstable since there are only 20 entries for estimating \( \psi_1 \). Therefore, we assume that \( \psi_1 \) is the same at the one-digit industry level and estimate it for each one-digit industry and each period. Since unobserved firm-specific productivity is likely to be positively correlated with the level of the capital stock, a simple OLS will overestimate \( \psi_1 \). Hence, a fixed-effects regression is conducted for each period and each one-digit industry.

The other variable for which we require an estimate is \( h \). Intuitively, \( h \) can be estimated from the correlation between the unexpected shock and the reaction to the shock. If the firm’s response to the shock is appropriate, this correlation must be high. To verify this intuition, we define the reaction to the shock.

**Definition 4** The firm’s reaction to the shock \( R(X(s)) \), is defined by the logarithm of the deviation of the actual input, \( X(s) \), from the predicted input, \( X^* \):

\[
R(X(s)) = \log X(s) - \log X^*,
\]

where \( X^* \) can be estimated from the amount of input that would be used if there were no unexpected shock:

\[
X^* = (\alpha z^*) \frac{1}{1 - \alpha} K \frac{\beta}{1 - \alpha}.
\]  

Equation (5) is derived by replacing \( zdF(z|s) \) in the first-order condition (1) by \( z^* \). Since \( X^* \) can be estimated from data given the estimates of \( \alpha, \beta \) and \( z^* \), it is possible to estimate the firm’s reaction to the shock. Using this idea, we prove the following theorem. The proof is in the Appendix.
Theorem 5  The firm’s adaptability, $h$, can be estimated from the squared correlation between the unexpected shock, $u$, and the firm’s reaction to the shock, $R(X(s))$: 

$$h = \rho_{uR(x)}^2, \quad \rho_{uR(x)} \geq 0,$$

where $\rho_{uR(x)}$ is the correlation coefficient between $u$ and $R(X(s))$.

Since $u$ can be estimated from $\log z - \log zdF_z(z)$, we can estimate $h$ from data for each firm and each period. To save computing time and reduce estimation errors, the following corollary is useful.

Corollary 6  The correlation between the unexpected shock, $u$, and the firm’s reaction to the shock, $R(X(s))$ is equal to the correlation between $\log Y - \alpha \log X - \beta \log K$ and $\log X - \alpha \log X - \beta \log K$:

Proof.  Since the correlation coefficient is invariant to an affine transformation of a variable ( $\rho_{XY} = \rho_{X(\phi Y + \theta)}$, where $\phi$ and $\theta$ are constant ), the desired result is immediate.

There is one technical issue. Since quarterly data is used, the data exhibits seasonality. Hence, to the extent that seasonality is predicted, the simple correlation coefficient may not capture the reaction to an unexpected change. For this reason, we estimate the correlation for each firm, period and quarter, and calculate the average correlation for each firm and period.

Estimate of the dependent variable:  For the dependent variable, we use two different variables as proxies for expected profits. One is the average profit rate; the other is the market value of the firm.
The profit rates are defined by \( \frac{Y_t - X_t}{K_{t-1}} \) and the average profit rates are estimated by the sample mean profit rates over time for each firm and period. Since profits are divided by the capital stock, the empirical equation (3) must be changed to:

\[
\Delta \log \frac{\Pi_t^e}{K} = \phi_0 + \phi_\gamma \Delta \log z^e + \phi_\sigma \sigma_u^2 \Delta h + (\phi_k - 1) \Delta \log K + \nu. \quad (6)
\]

If the firm has the decreasing returns to scale, \( \phi_k - 1 \) is expected to be negative. The advantage of this measure is that it closely reflects the profits from the theory. The disadvantage is that the dependent variable is estimated from variables that are used to estimate independent variables. This may cause simultaneous equations bias.

To deal with possible simultaneous equations bias, we use the market value of a firm in the final quarter of each period as another proxy of expected profits. The market value of assets is defined as the book value of assets plus the market value of common equity less the book value of common equity \([\text{DATA44} + (\text{DATA61} \times \text{DATA14}) - \text{DATA59}]\). The market value of assets reflects the investors’ expectation of the present value of discounted income flows. We implicitly assume that investors base their predictions on the average accounting profits of the last five years. Given this assumption, the market value of assets in the final quarter is a function of average profits in the period. Since the future market value is less likely to affect decisions about \( Y_t \) and \( X_t \), we can mitigate the problem of simultaneous equations bias.

However, the use of market value causes another concern. Market value might be distorted by investor speculation. It is hoped that each of these two different dependent variables complements the weakness of the other variable.

**Instrumental variable for** \( K_{t-1} \): We estimate \( K_{t-1} \) by using the capital stock in the previous year of the initial year for each period and each firm. In the
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Table 1: summary statistics

The variable $K$ is the net capital stock. The variables $E$, $\frac{Y_i - X_i}{K_{t-1}}$, $MV$, $z^e$, $h$, $\rho_{uR(x)}$ and $\sigma_n^2$ are estimates of the average profit rate, the market value of a firm, productivity, adaptability, the simple correlation between the unexpected shock and the firm’s reaction, and risk. * indicates significant at 5%.
theory, we assumed that the capital stock was fixed. However, in practice it must be chosen optimally, in which case, future profitability can be expected to have a positive impact on $K_{t-1}$. To avoid simultaneous equations bias, we use $\Delta \log K_{t-2}$ as the instrumental variable for $\Delta \log K_{t-1}$. In theory, once $\Delta \log K_{t-1}$ is controlled for, future profits must be independent of $\Delta \log K_{t-2}$.

4 Empirical Results

This section reports the estimation results. First, summary statistics are reported. Contrary to the commonly-held expectation that many firms have improved their adaptability, these show that the measure of adaptability is not trended. Then the regression results are reported. They show that adaptability has a robust and significant impact on expected profits.

Summary Statistics: Table 1 shows the summary statistics for each variable. The first two rows report the summary statistics for the dependent variables, the growth rates of the average profit rate and the market value of a firm. On average, the growth rate of the average profit rate is negative throughout the period. On the other hand, the growth rate of the market value is positive. It is well known that a large firm is likely to have a small profit rate. Hence both pieces of evidence imply that on average firms in this sample are growing. This interpretation is confirmed by the positive growth rate of the capital stock in the final row.

The third row shows that the growth rate of productivity is positive throughout the period. However, the growth rate is much lower than the growth rate of the capital stock. This is expected because the impact of the aggregate shocks are excluded from this productivity measure.
The fourth row reports the measure of adaptability. It shows that the adaptability measure does not have any significant trend. The absence of a trend is confirmed by the simple correlation between the unexpected shock and the reaction to the shock [in the fifth row]. This evidence is contrary to the commonly-held view that many firms have recently improved their adaptability.

The evidence is not conclusive. In particular, the measure fails to capture firms’ reactions to plant-level shocks. However, the evidence indicates that organizational change may not have improved the ability of firms to react appropriately to firm-level shocks.

Note that the number of observations for $\Delta h$ is substantially lower than that for $\Delta \rho_{uR(x)}$. The estimation of adaptability requires that a firm should have a positive correlation coefficient. However, 22% of the observations in the sample do not satisfy this condition. Moreover, since first differences are used for the regression analysis, it is necessary to have a positive correlation in two consecutive periods. Unfortunately, 39% of observations do not satisfy this criterion.

This apparent irrationality among firms may indicate that some assumptions are unrealistic. In particular, we assume that all firms know the unconditional mean of the shock and define an unexpected change as the deviation from the unconditional mean. If a firm’s subjective belief about the unconditional mean of the shock differs from the objective one, a negative correlation might be possible. To check the robustness of the results, we used a simple correlation between an unexpected shock and the reaction to the shock as an alternative measure of adaptability. Using this simple correlation, it is possible to use all the observations and examine whether irrationality among firms affects the results.

The Impact on Average Profit Rates: Table 2 shows that a change in
\[ \Delta \log \frac{K_t}{K} = \phi_0 + \phi_z \Delta \log z_e + \phi_h \sigma_u^2 \Delta h + \phi_\sigma \sigma_u^2 h + (\phi_k - 1) \Delta \log K + v \]

<table>
<thead>
<tr>
<th>Regression #</th>
<th>(A)</th>
<th>(B)</th>
<th>(C)</th>
<th>(D)</th>
<th>(a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta \log K_{t-1})</td>
<td>-0.332***</td>
<td>-0.324***</td>
<td>-0.313***</td>
<td>-0.316***</td>
<td>-0.314***</td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
<td>(0.013)</td>
<td>(0.014)</td>
<td>(0.011)</td>
<td>(0.012)</td>
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<tr>
<td>(\Delta \log z_e)</td>
<td>0.853***</td>
<td>0.784***</td>
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<td>0.346***</td>
<td>0.998***</td>
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<td></td>
<td>(0.046)</td>
<td>(0.045)</td>
<td>(0.048)</td>
<td>(0.032)</td>
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</tr>
<tr>
<td>(\sigma_u^2 \Delta h)</td>
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<td>4.600***</td>
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<td>(\Delta \sigma_u^2 h)</td>
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<td>1.306***</td>
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<tr>
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<td>(0.220)</td>
<td>(0.170)</td>
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<tr>
<td>(\Delta h)</td>
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<td></td>
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<td>3.629***</td>
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<tr>
<td>(\Delta \sigma_u^2 \rho_{uR(x)})</td>
<td></td>
<td></td>
<td>0.713***</td>
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<td>(\Delta E[\Delta \log (X)])</td>
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</table>

Table 2: The Impact of Adaptability on the Average Profit Rate

The dependent variable is the growth rate of the average profit rate. Each regression includes a constant term and \(\Delta \log K_{t-1}\) is instrumented by \(\Delta \log K_{t-2}\). The variable \(z_e\), \(\sigma_u^2\), \(h\), \(\rho_{uR(x)}\), and \(E[\Delta \log (X)]\), are estimates of productivity, risk, adaptability, the simple correlation between the shock and the firm’s response, and the average growth rate of inputs, respectively. *, ** and *** indicate significance at the 5%, 1% and 0.5% levels, respectively.

Standard errors are reported in parentheses.
adaptability has a positive significant impact on the growth rate of the average profit rate.

Regression (A) in Table 2 reports that improvements in both productivity and adaptability raise the growth rate of the average profit rate, while an increase in the capital stock lowers the average profit rate and a rise in risk has no impact on the average profit rate. The negative impact of the capital stock on the average profit rate confirms existing results, which find that larger firms have lower profit rates. Note that since the coefficient of $\Delta \log K_{t-1}$ is larger than -1 (-0.33), an increase in the capital stock raises average profits. That is, size, productivity and adaptability are important contributory factors to increasing average profits.

Since the original regression cannot perfectly distinguish the impact of adaptability from that of risk, we also estimate the impact of a change in adaptability alone. Regression (B) reports the results. The positive coefficient of adaptability remains, which confirms that adaptability has a positive impact on the average profit rate.

Regression (C) examines whether irreversible inputs bias the coefficients of adaptability. If inputs are irreversible, firms that are contracting may have difficulty in adapting to unexpected negative shocks. This is possible since the major input for many firms is expected to be labor. If firms that are expanding have a higher growth rate of the average profit rate, the coefficients of adaptability may be overestimated. To examine this potential bias, we include the change in the average growth rate of inputs as an additional regressor. The coefficient of adaptability is almost the same, and remains significant. Hence, irreversible inputs cause little bias.

Regression (D) investigates whether including irrational firms changes the results. The results show that adaptability still has a significant impact on the average profit rate. The coefficient of adaptability is a little lower than when $h$ was used as the measure of adaptability: 4.8 when $h$ is used, and 3.6 when
\( \rho_{uR(x)} \) is used. This is expected because the variance of \( \rho_{uR(x)} \) is larger than the variance of \( h \) by construction.

Let us consider a different question: how important is adaptability? It depends on the level of risk that a firm faces. In this sample, average risk is 0.04, the top 10% level of risk is 0.07, and the top 1% level of risk is 0.58. Equation (A) implies that a firm that fully recognizes a previously unpredicted change in its environment (i.e., \( h \) rises from 0 to 1) increases its expected profit rate by 18% if the firm faces the average level of risk, by 33% if it faces the top 10% level of risk, and by 277% if it faces the top 1% level of risk.

The Impact on the Growth Rate of the Market Value: Table 3 reports the impact of adaptability on the growth rate of the market value of a firm. Equations (E), (F), (G) and (H) in Table 3 refer to the same regressions as equations (A), (B), (C) and (D) in Table 2.

The results are similar to the previous results: both productivity and adaptability have a positive, significant and robust impact on the growth rate of the market value. The coefficients of adaptability and productivity in equations (E), (F), (G) and (H) are all positive and significant. A comparison of equations (E) and (G) indicates that the irreversibility of inputs overstates the coefficient of adaptability in the original model. Since no bias due to irreversibility was found in the regressions for the average profit rate, this result implies that the growth rate of inputs may have more influence on the market value than on the expected profit rate. In fact, the simple correlation between a change in the average growth rate of inputs and the growth rate of the average profit rate is 0.21; the correlation between a change in the average growth rates of inputs and the growth rate of the market value is 0.39. It seems that although a higher growth rate of inputs does not increase current profits, it might be interpreted as a signal of higher future profits.
\[ \Delta \log \Pi^c = \phi_0 + \phi_x \Delta \log z^c + \phi_h \sigma_u^2 \Delta h + \phi_y \Delta \sigma_u^2 h + \phi_k \Delta \log K + v \]

<table>
<thead>
<tr>
<th>regression #</th>
<th>(E)</th>
<th>(F)</th>
<th>(G)</th>
<th>(H)</th>
<th>(b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta \log K )</td>
<td>-0.024</td>
<td>-0.013</td>
<td>0.164***</td>
<td>-0.015</td>
<td>-0.041*</td>
</tr>
<tr>
<td></td>
<td>(0.020)</td>
<td>(0.020)</td>
<td>(0.022)</td>
<td>(0.017)</td>
<td>(0.021)</td>
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<tr>
<td>( \Delta \log z^c )</td>
<td>1.863***</td>
<td>1.713***</td>
<td>1.361***</td>
<td>1.145***</td>
<td>2.041***</td>
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<td></td>
<td>(0.072)</td>
<td>(0.072)</td>
<td>(0.072)</td>
<td>(0.475)</td>
<td>(0.077)</td>
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<tr>
<td>( \sigma_u^2 \Delta h )</td>
<td>11.94***</td>
<td>6.903***</td>
<td>10.14***</td>
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<td></td>
<td>(1.059)</td>
<td>(1.019)</td>
<td>(0.868)</td>
<td></td>
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</tr>
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<td>( \Delta \sigma_u^2 h )</td>
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<td>2.507***</td>
<td>2.921***</td>
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<td></td>
</tr>
<tr>
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<td>(0.330)</td>
<td>(0.306)</td>
<td>(0.274)</td>
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<td>( \Delta h )</td>
<td>0.486***</td>
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<td></td>
<td>(0.050)</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>( \sigma_u^2 \Delta \rho_u R(x) )</td>
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<td>4.434***</td>
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<tr>
<td></td>
<td></td>
<td>(0.333)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Delta \sigma_u^2 \rho_u R(x) )</td>
<td></td>
<td>1.425***</td>
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<td></td>
<td></td>
<td>(0.174)</td>
<td></td>
<td></td>
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<tr>
<td>Adjusted-R²</td>
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<td>0.211</td>
<td>0.337</td>
<td>0.143</td>
<td>0.240</td>
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<td>extra-regressor</td>
<td>( \Delta E [\Delta \log X] )</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

Table 3: The Impact of Adaptability on the Market Value of a Firm

The dependent variable is the growth rate of the market value of a firm. Every regression includes a constant term and \( \Delta \log K_{t-1} \) is instrumented by \( \Delta \log K_{t-2} \). The variables \( z^c, \sigma_u^2, h, \rho_u R(x) \) and \( E [\Delta \log (X)] \), are estimates of productivity, risk, adaptability, the simple correlation between the shock and the firm’s response, and the average growth rate of inputs. *, ** and *** indicate significance at the 5 %, 1% and 0.5% levels, respectively. Standard errors are reported in parentheses.
Although most of the results are unchanged, there are two main differences. First, a rise in risk has a positive impact on the growth rate of the market value; the growth rate of the capital stock does not have a significant impact on market value. The results in Table 2 and Table 3 lead us to conclude that improvements in productivity and adaptability are the two robust factors that increase expected profits.

Second, the impact of both productivity and adaptability on the market value is larger than those on the average profit rate. Compare equations (C) and (G). The coefficients of productivity and adaptability increase substantially, from 0.8 to 1.4 for productivity and from 4.6 to 6.9 for adaptability. The coefficient of 6.9 implies that the firm that fully recognizes a previously unpredicted change in its environment increases its market value by 28% if the firm faces the average level of risk, by 48% if the firm faces the top 10% level of risk, and by 400% if the firm faces the top 1% level of risk.

**Further Robustness Check:** We conducted several other regressions to check the robustness of the results. First, the same regression was conducted using different sample periods and different samples of firms. For example, the same regressions using the full sample period, 1965-1999 were conducted. The same regressions were conducted on a complete panel, obtained by deleting observations with missing values during the 1980-1999 period. Most of the regression results produced significant and positive coefficients of productivity and adaptability.

As a final robustness check, we examined the partial correlation between the measure of adaptability and the growth rate of average advertising expenditure over sales. If advertising raises sales, greater adaptability may be the result of additional advertising. However, since no significant correlation between advertising and adaptability was found, this concern was dismissed.

In summary, the results in this paper pass several robustness checks.
**Discussion:** There are reasons to suspect that the impact of adaptability would be larger than these estimates. First, the COMPUSTAT data set does not allow for an examination of reactions within the firm. Hence, if a plant-specific shock is important, this paper underestimates the importance of adaptability.

Second, measurement errors may understate the impact of adaptability. Since independent variables are unobserved, they must be estimated for regression analysis. Estimation errors may bias the results. Unfortunately, the relation between error terms and the regressors in the model is highly non-linear, and to our knowledge, no econometric method deals with this type of a measurement error problem. To see how serious the measurement error is, we re-estimated the variables using only three years in each period: the initial year, the middle year and final year of each period. This gave 12 entries with which to estimate each variable and conduct the same regressions. Equation (a) in Table 2 and equation (b) in Table 3 report the results. Since 20 entries were used to estimate each variable for the original regressions, measurement errors in equations (a) and (b) must be larger than in the original regressions. In both cases, the coefficients of adaptability are smaller than the original ones [equations (A) and (E)], while those of productivity are larger. Moreover, the original coefficients are outside the 95% confidence intervals for productivity and adaptability under equations (a) and (b), but are close to the border for equation (b). This evidence suggests that measurement errors are likely to lead to underestimation of adaptability and overestimation of productivity, but the bias may not be too severe when the market value is the dependent variable\(^3\).

\(^3\)Since some additional firms exhibit a negative correlation between the unexpected shock and its reaction when adaptability is re-estimated, the number of observations for equations (a) and (b) is slightly less than for equations (A) and (E). However, the small difference in sample size is not the reason for the difference in the coefficients. Checking measurement errors using the same sample did not change the results.
Table 4: The partial correlation between the growth rates of risk, size and productivity, and a change in adaptability

<table>
<thead>
<tr>
<th></th>
<th>( \Delta \log z^e )</th>
<th>( \Delta h )</th>
<th>( \Delta \rho_{uR(x)} )</th>
<th>( \Delta \log \sigma_u^2 )</th>
<th>( \Delta \log K )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta \log z^e )</td>
<td>1 (5111)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Delta h )</td>
<td>0.009 (3117)</td>
<td>1 (3117)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Delta \rho_{uR(x)} )</td>
<td>-0.071*** (5111)</td>
<td>0.969*** (3117)</td>
<td>1 (5111)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Delta \log \sigma_u^2 )</td>
<td>0.029* (5111)</td>
<td>0.378*** (3117)</td>
<td>0.050*** (5111)</td>
<td>1 (5111)</td>
<td></td>
</tr>
<tr>
<td>( \Delta \log K )</td>
<td>0.123*** (5111)</td>
<td>-0.072*** (3117)</td>
<td>-0.083*** (5111)</td>
<td>-0.030* (5111)</td>
<td>1 (5111)</td>
</tr>
</tbody>
</table>

The variable \( K \) is the net capital stock. The variable \( z^e, \sigma_u^2, h, \) and \( \rho_{uR(x)} \) are estimates of productivity, risk, adaptability and the simple correlation between an unexpected shock and the firm’s response to it. *, ** and *** indicate significance at the 5%, 1% and 0.5% levels, respectively. The number of observations is reported in parentheses.

5 Risk, Size, Adaptability and Productivity

We have shown that adaptability is valuable, but why does adaptability differ between firms? In other words, what influences adaptability? We examine two hypotheses. The first is that larger firms are less adaptable. Top managers in large firms may be unable to observe signals that could be observed locally. Hence, an increase in size restricts adaptability. An alternative hypothesis is that risk demands adaptability. Takii (2003) argues that an increase in risk raises the importance of the ability to predict change. Hence, the larger is risk, the greater is adaptability.
The Growth Rate of Risk

A Change in Adaptability

-0.625508

6.62125

Figure 1: Adaptability vs. Risk

Table 4 reports the partial correlations between the growth rates of productivity, risk and the capital stock, and a change in adaptability. It shows that the growth rate of the capital stock is positively correlated with the growth rate of productivity, though the correlation is small (0.12). It weakly confirms the traditional view that large firms are more productive, as suggested by Lucas (1978).

The relationship between risk and adaptability is significant and positive: the partial correlation between $\Delta \log \sigma_u^2$ and $\Delta h$ is 0.38; that between $\Delta \log \sigma_u^2$ and $\Delta \rho_{uR(x)}$ is 0.04. Since the correlation between $\Delta h$ and $\Delta \rho_{uR(x)}$ is 0.97, the large difference in the correlation with a change in risk is peculiar and may indicate serious selection bias. This point is addressed later.

Table 4 also confirms the predicted relationship between size and adaptability, which is negative and significant. However, the correlation is relatively small: -0.07 when $h$ is adopted as the measure of adaptability; -0.08 when $\rho_{uR(x)}$ is used.

25
Note that the correlation between the growth rate of risk and the growth rate of size is negative and significant, at -0.03. Hence, a partial correlation may be misleading. That is, although there is no relationship between size and adaptability, since risk is positively correlated with adaptability, the partial correlation is negative, and vice versa. To identify the true relationship between size, risk and adaptability, further regressions are conducted.

\[ \Delta h = 0.002 + 0.067^{***} \Delta \log \sigma_u^2 - 0.005 \Delta \log K_{t-1} \]  
\[
(0.005) \quad (0.003) \quad (0.006) \]
\[
Adj - R^2 = 0.148 \quad \# \text{ of obs } = 2898
\]
\[ \Delta \rho_{uR(z)} = 0.004 + 0.016^{***} \Delta \log \sigma_u^2 - 0.040^{***} \Delta \log K_{t-1} \]  
\[
(0.008) \quad (0.005) \quad (0.011) \]
\[
Adj - R^2 = 0.008 \quad \# \text{ of obs } = 4781
\]

where $\Delta \log K_{t-1}$ is instrumented by $\Delta \log K_{t-2}$. The regression (7) shows that once the growth rate of risk is controlled for, the growth of the capital stock does not have any significant impact on a change in adaptability. That is, an increase in risk is the dominant factor in increasing adaptability. However, the regression (8) shows that both risk and size are significant. Note, however, that the adjusted $R^2$ is quite low in the regression (8). This indicates that the result might be quite sensitive to the choice of sample. The same regressions were run on a different sample. We found that the impact of size on the measure of adaptability is not robust once risk is used as a conditioning variable. However, the impact of risk is fairly robust.

Although $\Delta h$ and $\Delta \rho_{uR(z)}$ are highly correlated, the two measures have quite different relationships to risk. To understand why, we estimated the correlation between $\Delta \rho_{uR(z)}$ and $\Delta \log \sigma_u^2$ for separate samples. When $\rho_{uR(z)}$ is negative, the correlation is negative (-0.14) and significant. When it is positive, the correlation is positive (0.15) and significant. Hence, the hypothesis that
risk demands adaptability is true only for firms that react rationally to an unexpected change. When a firm reacts irrationally, an increase in risk simply worsens the situation.

6 Conclusion

This paper developed a model to empirically examine the impact of a firm’s adaptability on its expected profits. The regression analysis showed that adaptability has a robust, significant and positive impact on the average profit rate and the market value of a firm. There is also evidence that risk-taking firms are more adaptable when reacting rationally to changes in the environment.

Several extensions of this research are worthwhile. First, what improves adaptability? Obvious candidates are flexible organization, the use of information technology and the employment of skilled workers. This would require the measure of adaptability to be matched with labor-market data. Second, if firms can react to unpredicted shocks by allocating resources within the firm, this paper underestimates the impact of adaptability. To investigate resource allocation within the firm, more-disaggregated data is required.

7 Appendix

The proof of lemma 2: Define $\gamma_u = \frac{1}{\sigma_u^2}$ and $\gamma_\varepsilon = \frac{1}{\sigma_\varepsilon^2}$. Applying the standard Bayesian updating technique, it is easy to show that $Var(u|s) = \frac{1}{\gamma_u + \gamma_\varepsilon}$ and $R_{udF}(u|s) = \frac{\gamma_\varepsilon}{\gamma_u + \gamma_\varepsilon}$. Applying this result to the definition of $h$, we can show that:

$$h = \frac{\gamma_\varepsilon}{\gamma_u + \gamma_\varepsilon}.$$
With this result, we can rewrite $V(u|s)$ and $R \ud F(u|s)$ as:

$$Z \ud F(u|s) = hs,$$

$$Var(u|s) = \frac{1 - h}{\gamma_u}.$$ 

Using the above results, $R \zd F(z|s)$ can be expressed as follows:

$$Z \zd F(z|s) = z^e \exp \left[ hs - \frac{h}{2\gamma_u} \right],$$ (9)

where $z^e = \exp \left[ \mu + \frac{1}{2\gamma_u} \right]$. Since the variance of $s$ is $\frac{1}{\gamma_u} + \frac{1}{\gamma_e}$, this can be written as $\frac{1}{\gamma_u}$. Using this result, it is easy to show that:

$$Z \cdot Z \zd F(z|s) \cdot dF(s) = (z^e)^{\frac{1}{1-\alpha}} \exp \frac{\alpha h}{2(1-\alpha)^{2\gamma_u}}.$$

Q.E.D.

**The proof of theorem 5:** Define $\gamma_u = \frac{1}{\sigma_u}$ and $\gamma_e = \frac{1}{\sigma_e}$. Applying equation (9) to equation (1), it can be shown that:

$$X(s) = X^* \exp \left[ hs - \frac{h}{2\gamma_u} \right].$$

Hence the firm’s reaction to the shock $R(X(s))$ is $\frac{1}{(1-\alpha)} h s - \frac{h}{2\gamma_u} i$. Hence, it can be derived from the definition of a correlation coefficient that:

$$\rho_{uR(s)} = \sqrt{h}.$$

Q.E.D.

**References**


